Steady State and Transient Analysis of Grid Connected Doubly Fed Induction Generator Under Different Operating Conditions

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ABSTRACT

This paper presents steady state and dynamic (Transient) models of the doubly fed induction generator connected to grid. The steady state model of the DFIAG (Doubly Fed Asynchronous induction Generator) has been constructed by referring all the rotor quantities to stator side. With the help of MATLAB programming simulation results are obtained to depict the steady state response of electromechanical torque, rotor speed, stator and rotor currents, stator and rotor fluxes, active and reactive drawn and delivered by Doubly Fed Asynchronous Induction machine (DFAIM) as it is operating in two modes i.e. generator and motor. The mathematical steady state and transient model of the DFIAM is constructed for three basic reference frames such as rotor, stator and synchronously revolving reference frame using first order differential equations. The effect of unsaturated and saturated resultant flux on the mutual inductance is also taken into account to deeply understand the dynamic response of the machine. The steady state and dynamic response of the DFIAM are compared for different rotor voltage magnitudes. Also, the effect of variations in mechanical input torque, stator voltage variations are simulated to predict the stator and rotor currents, active and reactive power, electromagnetic torque and rotor speed variations.

1. INTRODUCTION

Doubly fed induction generators is also known as doubly fed asynchronous machine (DFIAM) with wound rotor construction. Nowadays DFIAMs are most popular machines for wind turbines as generator due to the applicability in the wide range of speed variations. The fixed speed i.e. synchronous generator the power control (active and reactive) electronic equipment required should be of 100% rating of synchronous generator. The power control equipment is generally installed to the stator side to control the voltage profile pertaining to the contingencies due to on load and off load variations. The voltage stability control is mainly linked with the reactive power control. It may be provided either by the control of the reactive power given by capacitor banks on the basis of error signal generated through comparator between a specified voltage reference and the measured voltage of the grid side phase to phase or phase to ground per unit voltage. So, the power control equipment employed for synchronous fixed speed generator is still very costlier which would take major part of the total cost of the synchronous generator based wind turbine installation. On the other hand in the case of doubly fed asynchronous induction generator has the wider control options i.e. from stator and rotor side control. It also has pitch angle control of the wind turbine which controls the angular blade movement depends on the error signal generated due the difference between the preset reference speed
and generator’s shaft speed when wind turbine undergo different wind patterns with respect to variable time durations. The main advantage of the DFIAM is that near about 70% of power control is directly through stator side with FACT (Flexible AC Transmission) devices such as Static Synchronous Compensator (STATCOM)—Controls voltage, Static VAR Compensator (SVC) —Controls voltage. The operational difference between a STATCOM and an SVC is that a STATCOM works as a convenient under control voltage source on the other hand a SVC works on the principle of dynamically regulated the magnitude of reactance connected in parallel with DIAM’s Stator. STATCOM has the provision of maximum reactive power feed to grid at under voltage conditions [1, 2]. The rest of the 30% power is fed to the grid with the help of power electronic converter connected to rotor side so; its rating is only 25%-30% of DFAIG’s 100% power rating and as compared to fixed speed synchronous generator [2-5].

As the DFIG or DFIAM is an important component of wind turbine based power system so it require very steak and an accurate dynamic mathematical model to study and predict the response of the DFIAM under abnormal loading conditions and at switching instants. Dynamic model to global rotating reference frame of the DFIG system by a d (direct) and q (quadrature)-axis rotating at the angular frequency of ωs [6]. The classical d–q magnetization of the squirrel cage induction generator was modeled with non zero rotor voltage in the Park reference frame [7].Presented steady state and dynamic models and control strategies of wind turbine generators, [8][9].Dynamic time domain DFIG model with synchronously rotating reference frame was made [10][11].A brushless model of DFIG was used [12]. However there are many techniques to develop dynamic models of DFIAM but still there is need to study and explore the dynamic models with time domain and z-domain by consideration of effect of magnetic saturation. Implementation of different discrete models to show the simulation work much near to the practical condition. In this paper steady state and dynamic models are implemented. Discrete dynamic models are implemented based on various z-domain methods. The mathematical steady state and transient model of the DFIAM is constructed for three basic reference frames such as rotor, stator and synchronously revolving reference frame using first order deferential equations. The steady state and dynamic response of the DFAIG are compared for different rotor voltage magnitudes. Also, the effect of variations in mechanical input torque, stator voltage variations are simulated in MATLAB simulator to predict the stator and rotor currents, active and reactive power, electromagnetic torque and rotor speed variations.

2. STEADY STATE MODEL OF DFIAM

The steady state model of the three phase DFIAM machine is obtained from an equivalent circuit diagram as shown in the figure 1, 2 and figure 3. The mutual reactance Xls is moved to stator voltage supply source Vs and the simplified model of the induction machine is shown by the figure 3. To obtain the torque equation from the equivalent circuit, the rotor current Ir is expressed as the following set of equations as [8].

\[ I_{sd}^r = \frac{V_{sd}^s - V_{sd}^r / S_{slip}}{(R_s + \frac{R_p^r}{S_{slip}}) + j(X_{ls} + X_{lr})} \]  

(1)

The electromagnetic torque in an induction machine is the sum of air gap power and rotor fed power.

\[ V_{sd}^s = \text{Stator steady state voltage (p.u)}; \quad X_{ls} = \text{Stator reactance (p.u)}; \quad X_{lr} = \text{Rotor reactance (p.u)}; \]

\[ R_s = \text{Stator resistance (p.u)}; \quad R_p^r = \text{Rotor resistance (p.u)}; \]

\[ S_{slip} = \text{Slip (Stator’s flux speed-Rotor’s speed)/ Stator’s flux speed} \]

\[ T_{e, stator} = \text{Electromechanical torque from stator side (p.u)}; \]

\[ \Psi_s^s = \text{Stator side flux (p.u)}; \quad \Psi_r^s = \text{Rotor side flux (p.u)} \]

\[ P_R = \text{Rotor fed active power} \]

\[ T_{e, std} = \left( I_{sd}^r \right)^2 * \left( \frac{R_p^r}{S_{slip} + R_s} \right) \]

(2)

Rotor fed active power.

\[ P_R = \frac{V_{sd}^s}{S_{sd}^s} * I_{sd}^r * \cos \phi_p \]

(3)
Figure 1. Equivalent Circuit of DFIAM with Stator and Rotor side.

Figure 2. Equivalent Circuit of DFIAM as rotor side is referred to stator.

Figure 3. Steady state equivalent Circuit of DFIAM as rotor is referred to stator.

\[ T_{e, sd} = \left( s^2(V_{sd}^* - V_{sd}) \right) \frac{R_s + R_s}{s(R_s + R_s)} \]

(4)

\[ I_m^s = \frac{V_{sd}}{X_m} \]

(5)

\[ I_{sd}^s = I_m^s + I_r \]

(6)

\[ \Psi_m^s = L_{ls} * I_{sd}^s + L_m * (I_{sd}^s - I_{sd}^s) \]

(7)

Stator’s active power

\[ P_{sd}^s = V_{sd}^s * I_{sd}^s \cos \Theta_s \]

(8)

Where

\[ \Theta_s = \cos^{-1} \left( \frac{sV_{sd}^s + \left( s^2(R_s + R_s) + X_s^s + X_r^s \right) \left( X_s^s + X_r^s + 1 \right)}{s^2(R_s + R_s)} \right) \]

(9)

Alternatively electromagnetic torque is given by [8] the following equation:
Steady state and transient analysis of grid connected doubly fed induction machine (Sukhwinder Singh Dhillon)

\[ T_{em} = \frac{3 p_{par} R_s T_r^2}{S_{imp} \Omega_{syn}} \]  

(10)

\( L_s \) = Stator leakage self inductance (p.u);
\( L_m \) = Mutual inductance from Stator side (p.u);
\( \Omega_{syn} \) = Synchronous or angular speed at supply frequency (p.u);
\( S_{slip} \) = Slip (Stator’s flux speed - Rotor’s speed) / Stator’s flux speed
\( T_{e} \) = Electromechanical torque from stator side (p.u).
\( \Psi_s \) = Stator side flux (p.u.)
\( \Psi_m \) = Rotor side flux (p.u.)
\( \psi_{m flux} \) = Equivalent magnetic flux from stator.
\( \phi \) = power factor of the equivalent ckt. As shown in figure 3.

Steady state simulation analysis pertaining to the above given mathematical equations of a 1.5 MW DFIAM whose circuit constant values are tabulated in Appendix-A. The following figures show the steady state response of the induction machine for the equations (1)-(10).

Figure 4. Steady state electromagnetic torque versus slip curves for different rotor voltages.

Figure 3 shows the steady state variations of electro- magnetic torque corresponding to different rotor fed voltages and slip factors. Operating region of the doubly fed Asynchronous machine lies between the slip factors 0.2 and -0.2.

Figure 5. Rotor current versus slip to different rotor voltages (p.u.).

Above figure shows the rotor current variations for different voltages corresponds to the different slip factors. The rotor current follows the Equation (1).
The variations of rotor active and reactive power as per rotor voltage magnitudes for slip factors are shown in the Figure 6 and Figure 7.

Figure 6. Rotor active power ($I^2R_r$) versus slip to different rotor voltages.

Figure 7. Rotor reactive power ($I^2X_{lr}$) versus slip to different rotor voltages.

Figure 8. Magnetizing current versus slip for different voltages and.
Above figure shows that magnetizing current remains constant over slip and rotor voltage variations. The magnetizing reactance considered to be constant i.e. unsaturated 2.9(p.u.). The angular speed is considered to be 1(p.u).

![Figure 9. Stator's flux versus slip for different rotor voltages](image)

The stator’s flux variations correspond to different rotor fed voltages and slip values are shown in the above Figure.

3. TRANSIENT MODEL OF DFIAM WITH VARIOUS REFERENCE FRAMES.

The per unit dynamic model of the three phase doubly fed asynchronous machine is derived by the transformation of three variable phase quantities to a set of two stationary vectors known as α-axis and β-axis (clark’s transformation). Then these stationary vectors are transformed to rotating frame with d-axis and q-axis coordinates. The three phase supply voltages are alternating quantities which are transformed to α and β-axis with the help of Clark’s transformation (stationary reference frame) is as follows.

A) abc to αβo (stationary):

\[
\begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix}
= \begin{bmatrix}
2/3 & -1/3 & -1/3 \\
0 & 1/\sqrt{3} & -1/\sqrt{3} \\
1/3 & 1/3 & 1/3
\end{bmatrix}
\begin{bmatrix}
V_d \\
V_q \\
V_o
\end{bmatrix}
\]

(11)

![Figure 10. Stator's phase voltages along dq and αβ axis.](image)
B) $\alpha \beta \theta$ (stationary) to $dq\theta$ reference:

$$
\begin{bmatrix}
V_d \\
V_r \\
V_s
\end{bmatrix}
=
\begin{bmatrix}
\sin \omega t & -\cos \omega t & 0 \\
\cos \omega t & \sin \omega t & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
V_\alpha \\
V_\beta \\
V_\theta
\end{bmatrix}
$$

(12)

C) $\alpha \beta \theta$ (stationary) to $dq\theta$ reference:

$$
\begin{bmatrix}
V_d \\
V_r \\
V_s
\end{bmatrix}
=
\begin{bmatrix}
\sin (\omega t - 2\pi/3) & \cos (\omega t - 2\pi/3) & 1 \\
\sin (\omega t - 4\pi/3) & \cos (\omega t - 4\pi/3) & 1 \\
\sin (\omega t - 2\pi) & \cos (\omega t - 2\pi) & 1
\end{bmatrix}
\begin{bmatrix}
V_d \\
V_r \\
V_s
\end{bmatrix}
$$

(13)

Assumptions for the Dynamic Model:

a. No magnetic flux saturation so the mutual inductance is constant i.e unsaturated.

b. Machine windings are connected in star configuration on stator and rotor hence no $(0)$ component.

c. $V_a = V_{ph} \sin(\omega t)$ (i), $V_b = V_{ph} \sin(\omega t - 120)$ (ii), $V_c = V_{ph} \sin(\omega t + 120)$ balanced three phase voltages.

As per Park’s transformation the three phase stator and rotor voltages transformed to a $dq\theta$ rotating frame is given the following equations:

$$
\begin{bmatrix}
V_d \\
V_q \\
V_\theta
\end{bmatrix}
=
\begin{bmatrix}
\sin \theta & \cos \theta & 1 \\
\sin(\theta - 2\pi/3) & \cos(\theta - 2\pi/3) & 1 \\
\sin(\theta - 4\pi/3) & \cos(\theta - 4\pi/3) & 1
\end{bmatrix}
\begin{bmatrix}
V_{\alpha} \\
V_{\beta} \\
V_{\theta}
\end{bmatrix}
$$

(14)

In terms of line voltages

$$
\begin{bmatrix}
V_d \\
V_q \\
V_\theta
\end{bmatrix}
=
\begin{bmatrix}
(\sqrt{3} \sin \theta + \cos \theta) V_{b_1} + 2 \cos \theta V_{c_1} \\
(-\sqrt{3} \cos \theta + \sin \theta) V_{b_1} + 2 \sin \theta V_{c_1} \\
0 -1/3
\end{bmatrix}
$$

(15)

$$
\begin{bmatrix}
V_d \\
V_q \\
V_\theta
\end{bmatrix}
=
\begin{bmatrix}
(\sqrt{3} \sin \Gamma + \cos \Gamma) V_{b_1} + 2 \cos \Gamma V_{c_1} \\
(-\sqrt{3} \cos \Gamma + \sin \Gamma) V_{b_1} + 2 \sin \Gamma V_{c_1} \\
0 -1/3
\end{bmatrix}
$$

(16)

$$
\Gamma = \theta_s - \theta_r
$$

(17)

$\theta_s$ angle of reference frame and $\Gamma$ is the angle between reference frame and position of rotor. As $w_s$ is the speed of stator flux and $w_c-w_r$ is the relative speed between stator’s flux and rotor’s actual angular speed. So the $W$ speed (p.u.) matrix for stator and rotor ckt is given by

$$
W =
\begin{bmatrix}
0 & -w_s & 0 & 0 \\
w_s & 0 & 0 & 0 \\
0 & 0 & 0 & -(w_c - w_r) \\
0 & 0 & (w_c - w_r) & 0
\end{bmatrix}
$$

(18)
4. ROTOR REFERENCE FRAME

In this reference frame $\Theta_s = \Theta_r$, $\Gamma = \Theta_r - \Theta_i = 0$ stator and rotor dq component corresponds to line voltage given by equation (19) and (20). $\Theta_r$ is the position of phase ‘a’ of rotor abc frame in electrical degree w.r.t phase ‘a’ of stator abc reference frame.

Stator voltage equations:

$$\begin{bmatrix} V_{\alpha} \\ V_{\beta} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} (\sqrt{2} \sin \Theta_r + \cos \Theta_r) V_{\alpha s} + 2 \cos \Theta_r V_{\beta s} \\ (\sqrt{2} \cos \Theta_r - \sin \Theta_r) V_{\alpha s} + 2 \sin \Theta_r V_{\beta s} \end{bmatrix}$$

(19)

Rotor voltage equations:

$$\begin{bmatrix} V_{\alpha} \\ V_{\beta} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} V_{\alpha s} + 2V_{\beta s} \\ (\sqrt{3}) V_{\alpha s} \end{bmatrix}$$

(20)

For rotor ref. frame ($w_c = w_r$) for pu system $w_c = 1$ p.u.

$$\Psi = \begin{bmatrix} 0 & -w_c & 0 & 0 \\ w_c & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(21)

In general stator’s dqo components in terms of voltage drops are as follows:

$$\begin{aligned}
V_{\alpha s} &= \frac{d}{dt} \phi_{\alpha s} - \omega \phi_{\beta s} + R_s i_{\alpha s} \\
V_{\beta s} &= \frac{d}{dt} \phi_{\beta s} + \omega \phi_{\alpha s} + R_s i_{\beta s} \\
V_{\omega s} &= \frac{d}{dt} \omega_{\omega s} + R_s i_{\omega s}
\end{aligned}$$

(22)

Also, rotor dqo components in terms of voltage drops are as follows:
From Equation (21), (22) and (23) stator and rotor voltages for Rotor reference frame are as follows:

Stator and rotor dqo voltages are:

\[
\begin{align*}
V_{st} &= \frac{d\phi_{st}}{dt} - \omega_s \phi_{qst} + R_s i_{st} \\
V_{qs} &= \frac{d\phi_{qs}}{dt} + \omega_s \phi_{qst} + R_s i_{qs} \\
V_{op} &= \frac{d\phi_{op}}{dt} + R_p i_{op}
\end{align*}
\]

(24)

\[
\begin{align*}
V_{st} &= \frac{d\phi_{st}}{dt} - R_s i_{st} \\
V_{qs} &= \frac{d\phi_{qs}}{dt} + R_s i_{qs} \\
V_{op} &= \frac{d\phi_{op}}{dt} + R_p i_{op}
\end{align*}
\]

(25)

\[
\begin{align*}
\phi_{qs} (t) &= \begin{bmatrix} V_{st} \\ V_{qs} \end{bmatrix} - I_q(t) \\
\phi_{qst} (t) &= \begin{bmatrix} V_{st} \\ V_{qs} \end{bmatrix} \\
\phi_{qst} (t) &= \begin{bmatrix} V_{st} \\ V_{qs} \end{bmatrix}
\end{align*}
\]

(26)

Where the relation between flux and current is given by:

\[
\begin{bmatrix}
i_{qst}(t)
i_{qst}(t)
i_{qst}(t)
i_{qst}(t)
\end{bmatrix} = \begin{bmatrix}
L_{bs} + L_M & 0 & L_M & 0 \\
0 & L_{bs} + L_M & 0 & L_M \\
L_M & 0 & L_{bs} + L_M & 0 \\
0 & L_M & 0 & L_{bs} + L_M
\end{bmatrix}^{-1} \begin{bmatrix}
\phi_{qst}(t) \\
\phi_{qst}(t) \\
\phi_{qst}(t)
\end{bmatrix}
\]

(27)

By putting the equation (27) into equation (26) gives a relation of flux that

Fluxes:

\[
\phi_{ds} (t) = [V_d + s[L^{-1}R]] \phi_{ds} (t)
\]

\[
\theta_q = \int \left[ [V_d + s[L^{-1}R]] \phi_{ds} (t) \right] dt
\]

(28)

Flux is derived from the equation (28), Electromagnetic torque and mechanical model are given by the same set of equations (43)-(45)
5. **STATOR OR STATIONARY REFERENCE FRAME**

For stationary frame phase A of the stator voltage is in phase with the d-axis so, $\theta_s = 0$, $\Gamma = -\theta_r$ and stator and rotor voltages to dq component are given by equation (29) and equation (30). $\bar{\theta}_r$ = the position of phase ‘a’ of rotor abc frame in electrical degree w.r.t phase ‘a’ of stator abc reference frame. Stator voltage eqns:

$$
\begin{bmatrix}
V_{d_s} \\
V_{q_s}
\end{bmatrix} = 
\frac{1}{3}
\begin{bmatrix}
v_{d_{sa}} + 2v_{q_{sa}} \\
(-\sqrt{3})v_{d_{sb}}
\end{bmatrix}
$$

(29)

Rotor voltage are given by equation (30) as:

$$
\begin{bmatrix}
V_{d_r} \\
V_{q_r}
\end{bmatrix} = 
\frac{1}{3}
\begin{bmatrix}
(-\sqrt{3}\sin\bar{\theta}_r + \cos\bar{\theta}_r)v_{d_{ra}} + 2\cos\bar{\theta}_r v_{q_{ra}} \\
(-\sqrt{3}\cos\bar{\theta}_r - \sin\bar{\theta}_r)v_{d_{rb}} - 2\sin\bar{\theta}_r v_{q_{rb}}
\end{bmatrix}
$$

(30)

As $w_s$ is the speed of stator flux and $w_{s-w_r}$ the relative speed between stator’s flux and rotor’s actual speed. So the W speed matrix for stator and rotor ckt is given by equation (31). For stationary ref. frame ($w_s$=0) for pu system

$$
W =
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & (w_s) \\
0 & 0 & (w_r) & 0
\end{bmatrix}
$$

(31)

The stator dqo voltages:

$$
\begin{align*}
V_{d_a} &= \frac{d\theta_s}{dt} + R_{s*}i_{a} \\
V_{d_\alpha} &= \frac{d\theta_s}{dt} + R_{s*}i_{\alpha} \\
V_{d_\beta} &= \frac{d\theta_s}{dt} + R_{s*}i_{\beta}
\end{align*}
$$

(32)

Stator and rotor fluxes are given by:
Rotor dqo voltages:

\[
\begin{align*}
V_{e_r} &= \frac{d\theta_e}{dt} + o_e \phi_{e_r} + R_e i_{e_r} \\
V_{e_q} &= \frac{d\theta_q}{dt} + (-o_e)\phi_{e_q} + R_q i_{e_q} \\
V_{0_r} &= \frac{d\theta_0}{dt} + R_e i_{0_r}
\end{align*}
\]  

(33)

\[
\begin{bmatrix}
\phi_{d_1}(t) \\
\phi_{q_1}(t) \\
\phi_{d_2}(t) \\
\phi_{q_2}(t) \\
\phi_{d_3}(t) \\
\phi_{q_3}(t)
\end{bmatrix} =
\begin{bmatrix}
V_{e_1} \\
V_{e_2} \\
V_{e_3}
\end{bmatrix} 
- \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} 
\begin{bmatrix}
\phi_{d_1}(t) \\
\phi_{q_1}(t) \\
\phi_{d_2}(t) \\
\phi_{q_2}(t) \\
\phi_{d_3}(t) \\
\phi_{q_3}(t)
\end{bmatrix} 
- \begin{bmatrix}
R_e & 0 & 0 & 0 & 0 & 0 \\
0 & R_e & 0 & 0 & 0 & 0 \\
0 & 0 & R_e & 0 & 0 & 0
\end{bmatrix} 
\begin{bmatrix}
i_{d_1}(t) \\
i_{q_1}(t) \\
i_{d_2}(t) \\
i_{q_2}(t) \\
i_{d_3}(t) \\
i_{q_3}(t)
\end{bmatrix}
\]  

(34)

From Equation (27)

\[
\begin{bmatrix}
\phi_{d_1}(t) \\
\phi_{q_1}(t) \\
\phi_{d_2}(t) \\
\phi_{q_2}(t) \\
\phi_{d_3}(t) \\
\phi_{q_3}(t)
\end{bmatrix} =
\begin{bmatrix}
V_{e_1} \\
V_{e_2} \\
V_{e_3}
\end{bmatrix} 
- \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} 
\begin{bmatrix}
\phi_{d_1}(t) \\
\phi_{q_1}(t) \\
\phi_{d_2}(t) \\
\phi_{q_2}(t) \\
\phi_{d_3}(t) \\
\phi_{q_3}(t)
\end{bmatrix} 
- \begin{bmatrix}
L_{d_1} & L_{d_2} & L_{d_3} & 0 & 0 & 0 \\
L_{q_1} & L_{q_2} & L_{q_3} & 0 & 0 & 0 \\
L_{d_1} & L_{d_2} & L_{d_3} & 0 & 0 & 0
\end{bmatrix} 
\begin{bmatrix}
i_{d_1}(t) \\
i_{q_1}(t) \\
i_{d_2}(t) \\
i_{q_2}(t) \\
i_{d_3}(t) \\
i_{q_3}(t)
\end{bmatrix}
\]  

(35)

Flux is derived from the equation (28). Electromagnetic torque model are given by the same set of equations (43)-(45).

6. SYNCHRONOUSLY ROTATING FRAME

In this frame \( \theta_s = \theta_a \), \( \Gamma = \theta_a - \theta_s \) stator and rotor dq component corresponds to line voltage given by the set equation (37) and (39). \( \theta_a \) is the position of phase ‘a’ of rotor abc frame in electrical degree w.r.t phase ‘a’ of stator abc reference frame.

Stator voltage equations:

\[
\begin{bmatrix}
V_{s_1} \\
V_{s_2} \\
V_{s_3}
\end{bmatrix} = -1/3 \begin{bmatrix}
(\sqrt{3}\sin\theta_a + \cos\theta_a) v_{s_1} + 2\cos\theta_a v_{s_2} + 2\cos\theta_a v_{s_3} \\
(-\sqrt{3}\cos\theta_a + \sin\theta_a) v_{s_1} + 2\sin\theta_a v_{s_2} + 2\sin\theta_a v_{s_3}
\end{bmatrix}
\]  

(36)

Rotor voltage equations:

\[
\begin{bmatrix}
V_{e_1} \\
V_{e_2} \\
V_{e_3}
\end{bmatrix} = -1/3 \begin{bmatrix}
(\sqrt{3}\sin\theta_e - \theta_e) v_{e_1} + 2\cos\theta_e v_{e_2} + 2\cos\theta_e v_{e_3} \\
(-\sqrt{3}\cos\theta_e + \sin\theta_e) v_{e_1} + 2\sin\theta_e v_{e_2} + 2\sin\theta_e v_{e_3}
\end{bmatrix}
\]  

(37)

As \( w_s \) is the speed of stator flux and \( w_e - w_s \) is the relative speed between stator’s flux and rotor’s actual speed. For synchronously rotating ref. frame (\( w_s = 1 \)) for pu system \( w_e = 1 \) p.u. Matrix \( w \) is given by:
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\[
W = \begin{bmatrix}
0 & -1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & -(1 - w_r) \\
0 & 0 & (1 - w_r) & 0
\end{bmatrix}
\]

(38)

Stator dqo voltages:

\[
\begin{align*}
V_{ds} &= \frac{d\psi_d}{dt} + \frac{1}{w_r} \psi_q + R_s i_{ds} \\
V_{d} &= \frac{d\psi_d}{dt} + \frac{1}{w_r} \psi_q + R_s i_{ds} \\
V_{iq} &= -\frac{d\psi_q}{dt} + R_\gamma i_{iq}
\end{align*}
\]

(39)

Figure 13. Stator and rotor phase voltages along dq axis for a stationary reference frame

Te= Electromagnetic torque (p.u)
H= Damping constant (p.u)
F= Friction constant (p.u)
p= Pair of poles.
w_m= Generator shaft speed (p.u)
T_m= Mechanical torque (p.u)
\( \theta_i \)= Rotor angle electrical radian

Rotor dqo voltages:

\[
\begin{align*}
V_{ds'} &= \frac{d\psi_{ds'}}{dt} - (1 - \alpha) \frac{d\psi_{qs'}}{dt} + R_s i_{ds'} \\
V_{d'} &= \frac{d\psi_{ds'}}{dt} - (1 - \alpha) \frac{d\psi_{qs'}}{dt} + R_s i_{ds'} \\
V_{qs'} &= \frac{d\psi_{qs'}}{dt} - (1 - \alpha) \frac{d\psi_{qs'}}{dt} + R_s i_{qs'} \\
V_{q'} &= \frac{d\psi_{qs'}}{dt} + R_\gamma i_{qs'}
\end{align*}
\]

(40)

Stator and Rotor fluxes:
From Equation (27)

\[
\begin{bmatrix}
\phi_{d}(t) \\
\phi_{q}(t) \\
\phi_{d*}(t) \\
\phi_{q*}(t)
\end{bmatrix}
= 
\begin{bmatrix}
V_{d} \\
V_{q} \\
V_{d*} \\
V_{q*}
\end{bmatrix}
- 
\begin{bmatrix}
0 & 1 & 0 & 0 \\
-1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1-w_{c} \\
0 & 0 & -(1-w_{c}) & 0
\end{bmatrix}
\begin{bmatrix}
\phi_{d}(t) \\
\phi_{q}(t) \\
\phi_{d*}(t) \\
\phi_{q*}(t)
\end{bmatrix}
- 
\begin{bmatrix}
R_{s} & 0 & 0 & 0 \\
R_{s} & 0 & 0 & 0 \\
0 & R_{s} & 0 & 0 \\
0 & 0 & R_{s} & 0
\end{bmatrix}
\begin{bmatrix}
i_{d}(t) \\
i_{q}(t) \\
i_{d*}(t) \\
i_{q*}(t)
\end{bmatrix}
\]

(41)

Flux is derived from the equation (28), Electromagnetic torque and mechanical model are given by the same set of equations (43)-(45).

**Electromagnetic torque:**

\[
T_{e} = (\Phi_{d}i_{d} - \Phi_{q}i_{q})
\]

(43)

**Mechanical model:**

\[
\frac{d}{dt} \omega_{m} = \frac{1}{2H} (T_{e} - F_{e} - T_{n})
\]

(43)

\[
w_{r}(p.u) = \omega_{m} = \int \frac{1}{2H} (T_{e} - F_{e} - T_{n}) dt
\]

(44)

\[
\theta_{e} = \int w_{r} * \omega_{m} dt \quad \text{so} \quad \theta_{e} = \theta_{m} / p
\]

7. **SIMULINK MODEL OF GRID CONNECTED 1.5 MW DFIAM MACHINE**

To analyze the transient behavior of the doubly fed asynchronous machine a simulink test model is constructed in the matlab environment. A 1.5MW doubly fed asynchronous induction generator is connected to nominal line to line 575 V grids. A 1.5 MW wind turbine is coupled to the machine through drive train model. The single line diagram of the grid is shown in the figure 14.

Figure 14. Test model for grid connected standalone DFIAM
The test model shown above is used to test DFIAM performance based on electrical torque, generator’s shaft speed, and stator and rotor currents for various operating conditions such as:

Case 1. A 0 p.u. Mechanical torque input is applied to the DIAM without the use of Wind turbine to test the output of DIAM. Also, set initial slip $s=1$ and rotor side line-line rms voltage to zero.

Case 2. A 0.1 p.u. Mechanical torque input is applied to the DIAM without the use of Wind turbine to test the output of DIAM. Also, set initial slip $s=1$ and rotor side line-line rms voltage to zero.

Case 3. A -0.1 p.u. Mechanical torque input is applied to the DIAM without the use of Wind turbine to test the output of DIAM. Also, set initial slip $s=0$ and rotor side line-line rms voltage to zero.

Case 4. A -0.1 p.u. Mechanical torque input is applied to the DIAM without the use of Wind turbine to test the output of DIAM. Also, set the initial slip $s=-0.2$ and rotor side line-line rms voltage to zero.

8. WIND TURBINE MODEL

The wind turbine model is described by the following set of equations:

8.1. Aerodynamic model

The mechanical power produced by the turbine on interaction with wind depends upon swept area $S$ of the air disk formed by the rotation of the turbine’s blades $S$, power coefficient of performance $C_p$ (depends upon collective blade pitch angle $\beta$ and tip to speed ratio $\lambda$), air density and wind speed $V$. The expression for power extracted from wind is given by [13]-[15] following equation.

$$P_r = \frac{1}{2} \rho C_p(\beta, \lambda)SV^3$$  \hspace{1cm} (46)

Similarly torque produced is given by the equation as follows.

$$T_r = \frac{1}{2} \rho C_q R(\beta, \lambda)SV^2$$  \hspace{1cm} (47)

Hence the equation for thrust exerted on the tower $F_t$ is given as follows.

$$F_T = \frac{1}{2} \rho C_t (\beta, \lambda)SV^2$$  \hspace{1cm} (48)

Where $C_t$ thrust coefficient, $C_q$ is the torque coefficient, $R$ is the radius of the rotor.

![Figure 15](image)

Figure 15. (a), (b) Variations of $C_p$ and $C_t$

$C_q = C_p / \lambda$ and $\lambda = \text{Wrt}/V$. Where Wrt is the rotor speed of turbine. Figure 1 illustrate the variation of performance coefficients of $C_p$ and $C_t$. Now, The Wind turbine is coupled to the shaft of DIAM and the
performance of the DFIAM is tested with Slip $s=0$ and $s=-0.2$. Note that again there is no rotor voltage is applied to the rotor. To control the electromagnetic torque and speed a direct discrete PID controller is implemented to control the pith of the turbine blade. The results show the performance of wind turbine based DFIAM connected to grid.

8.2. Proposed PID based pitch controller:

A simple proportional gain of discrete PID controller is used which actuates for an error signal generated by reference speed (p.u) and generator’s speed (p.u). Then the output of the controller is optimized through limiter having maximum value of the pitch angle is 27 deg.

9. RESULTS AND DISCUSSIONS

In this section results of different and case studies are shown and discussed as follows: CASE 1 When slip $s=1$, $T_m=0$, $V_r=0$.

In this case DFIAM is set to work as squirrel cage induction machine as zero input voltage to the rotor. The mechanical input and load on the shaft is also set to be zero. The machine is working as a motor with slip...
s=1. Figure 16(a) shows the dynamic response of stator and rotor currents with magnitude of 5p.u at t=0, at t=4s rotor current attain minimum value 0.01p.u and stator current attains steady state value of 0.4p.u. Figure 16(b) shows the transient response of electromagnetic torque (Te) and generator shaft speed (wr) in p.u.CASE 2. With slip s=0, Tm=0.1, Vr=0.

Due to the mechanical shaft input is 0.1p.u and the shaft is running at synchronous speed (slip=0) DFIAM is still working as squirrel cage induction motor. The mechanical input helps to reduce the transients in Te and wr at t=0. Figure 17(a) shows the small time period dynamic deviations in the rotor and stator currents before reached to steady state. From Figure.17 (a) and 17(b) it is observed that the machine achieves the steady state response at t=0.2s.
CASE 3. When \( s=0, \ Tm=-0.1 \) and \( Vr=0 \).

![Graph showing stator currents, rotor currents, and stator voltages](image1)

**Figure 18 (a).** Stator currents, rotor currents and stator voltages (p.u)

![Graph showing transient response of \( Te(p.u) \), \( \theta(rad) \), and generator’s speed (p.u).](image2)

**Figure 18 (b).** Transient response of \( Te(p.u) \), \( \theta(rad) \) and generator’s speed (p.u).

In this mode DFIAM is working as squirrel cage induction generator as shaft of the machine is running above the synchronous speed. Again the time taken by the machine to attain steady state is 0.3sec. Figure 18(a) shows that stator and rotor currents has some transients during \( t=0 \) to \( t=0.3 \)sec also the response of \( Te \) and \( wr \) are shown in the Figure 18(b).

CASE 4. When \( s=-0.2, \ Tm=-0.1 \) and \( Vr=0 \).
In this mode DFAIM is working as squirrel cage induction generator. The initial speed of the generator shaft is 1.2p.u. When supply voltage is applied to the stator’s terminals the stator and rotor current transients are shown in the figure 19(a) also, the dynamic response of the (Te) and (wr) is shown in the Figure 19(b). At t=0.6 steady state is reached and all the above variables are settle down to steady state.

CASE 5. When s=-0.2, Tm=-0.1 and Vr=0.173V arbitrary taken the values of Vr to show the working of a DFAIM (without any control mechanism)
In this mode DFIAM is working as wound rotor induction generator. The initial speed of the generator shaft is 1.2p.u. When supply voltage is applied to the stator’s terminals at t=0s, the stator and rotor current transients are shown in the Figure 20(a) also, the dynamic response of the (Te) and (wr) is shown in the Figure 20(b). At t=0.6 steady state is reached and all the above variables are settle down to steady state. Corresponding to negative mechanical torque the electromagnetic torque is also becomes negative to show the working of DFIAM as generator. As from the Figure 20(b) the Te does not settle down to steady state value which is the major concern for control need to apply to the rotor side.

CASE 6. When s=1, Tm=-0.1 and Vr=0 p.u. to show the transient case as compared to steady state:
In this mode DFIAM is working as squirrel rotor induction motor and generator both. The initial speed of the generator shaft is 0. Slip is 1. When supply voltage is applied to the stator’s terminals at t=0s, the stator and rotor current transients are shown in the Figure 21(a) also, the dynamic response of the (Te) and (wr) is shown in the Figure 21(b). At t=1.2 motoring mode is shifted to generator mode as the mechanical torque is -1 p.u. and this mode is used to show the comparison of steady state(Figure 3) and transient (Figure 15b).

Case 7. When 1.5MW wind turbine is coupled with DFIAM and set s=-0.2, Vr=0. A direct PID pitch controller is implemented.
In this mode initial slip $s=0.2$ which shows the initial speed DFIAM is 1.2 p.u. The machine working as squirrel rotor induction motor. When supply voltage is applied to the stator’s terminals at $t=0$, the stator and rotor current transients are shown in the Figure 22(b) also, the dynamic response of the $(Te)$ and $(wr)$ is shown in the Figure 22(a). At $t=0.8$ motoring mode reaches to steady state condition.

Case 8. When 1.5MW wind turbine is coupled with DFIAM and set $s=-0.2$, $V_r=0.173V$. A direct PID pitch controller is implemented.
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(Sukhwinder Singh Dhillon)

In this mode rotor voltage of 0.173 p.u. is applied and also the shaft of the generator is coupled with the 1.5MW wind turbine having a simple discrete PID controller for pitch control based on the error signal of generator speed and reference speed. Initial slip $s=-0.2$ which shows the initial speed DFIAM is 1.2 p.u. The machine working as DFIG rotor induction generator. When supply voltage is applied to the stator’s terminals at $t=0s$, the stator and rotor current transients are shown in the Figure 23(b) also, the dynamic response of the $(Te)$ and $(wr)$ is shown in the Figure 23(a). At $t=0.8$ motoring mode reaches to steady state condition.

CASE 9. When $s=-0.2$, wind speed step change from 12m/s to 15 m/s and $V_r=0$. 

Figure 23(a). Stator currents, rotor currents and stator voltages (p.u)

Figure 23(b). Transient response of $Te$ (p.u), theta (rad) and generator’s speed (p.u).
In this mode rotor voltage is set to 0 and also the shaft of the generator is coupled with the 1.5MW wind turbine having a simple discrete PID controller for pitch control based on the error signal of generator speed and reference speed. Initial slip \( s = -0.2 \) which shows the initial speed DFIAM is 1.2 p.u. The step change in wind speed is applied which drift from 12m/s to 15m/s. The machine is working as squirrel cage rotor induction generator. When supply voltage is applied to the stator’s terminals at \( t=0 \)s, the stator and rotor current transients are shown in the Figure 24(b) also, the dynamic response of the \( (Te) \) and \( (w_r) \) is shown in the Figure 24(a). At \( t=1.8 \) generating mode reaches to steady state condition.

CASE 9. When \( s=-0.2 \), grid voltage changes to 0.3pu, wind speed =12m/s and \( V_r=0 \)
In this mode rotor voltage is set to 0 and the stator voltage dip of 0.3p.u is applied at t=0.3s to t=0.6s. The shaft of the generator is coupled with the 1.5MW wind turbine having a simple discrete PID controller for pitch control based on the error signal of generator speed and reference speed. Initial slip $s=-0.2$ which shows the initial speed DFIAM is 1.2 p.u. The machine is working as squirrel cage rotor induction generator. When supply voltage is applied to the stator’s terminals at $t=0s$, the stator and rotor current transients are shown in the Figure 24(a) also, the dynamic response of the $(T_e)$ and $(\omega_r)$ is shown in the Figure 24(b). At $t=1.4$ generating mode reaches to steady state condition.

10. CONCLUSIONS AND FUTURE SCOPE

In this work a fourth ordered continuous dynamic model of DFIAM is constructed using first order differential conditions. The steady state and dynamic performance of the machine is tested and compared. The various state space models of (DFIAM continuous type) based on rotor reference frame, stationary reference and synchronously rotation frames has been made. The simulation results shows that when DFIG is connected to grid with rotor voltage input at $t=0$, under various conditions when DFIAM is acting as squirrel cage rotor induction generator...
cage motor and generator also, conditions are applied to test the model for wound rotor generators. From the results obtained for wound rotor generator when rotor input is given at slip $s=0.2$ there is stability problems for rotor currents and electromagnetic torque $T_e$ as the transients are present in the curves. These variations in the torque at steady state need to be controlled. In this paper only the steady state and dynamic response is represented. The control aspects and various uncertainties are to be discussed in the coming research papers. Results show the performance of a standalone DFIAM connected to grid for the various cases of section VIII. Also the analysis of wind turbine based DFIAM for step change in wind speed and voltage dip is covered.

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