Extended Kalman observer based sensor fault detection

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ABSTRACT

This article discusses the Kalman observer based fault detection approach. The calculation of the residues can detect faults, but if there are noises, uncertainties become very important. To reduce the influence of these noises, a calculation of the instantaneous energy of the residues gave a better precision. The Kalman observer was used to estimate system performance and eliminate unknown noise and external disturbances. Instantaneous Power Calculation (IPCFD) based fault detection can detect potential sensor faults in hybrid systems. The effectiveness of the proposed approach is illustrated by the main application.

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1. INTRODUCTION

In recent years, fault detection received more attention than ever before due to the increasing demand for more reliable dynamic systems because the failure of actuators, sensors and other components can result in performance degradation, severe damage of systems with the possibility of the loss of human lives. Fault detection and isolation (FDI) techniques for discrete systems and switching systems have gained increasing consideration world-wide. Fault detection concerned with extraction of relevant features that indicate the existence of a fault, whereas, fault identification refers location and categorization of a fault or a set of faults. To avoid these consequences in the control systems, it is critical to detect and identify any possible faults at the earliest stage [1]. Among all the methods for fault diagnosis, one of the particular interesting techniques is the model-based fault detection observer approach [2–7]. When addressing the problem of fault detection, two strategies can be found in the literature: hardware redundancy and software (or analytical) redundancy [8], [9]. However, the use of hardware redundancy is very expensive. FDI techniques based on fault detection observer were proposed [10–12] as the fault detection observer design based on the $H_\infty/H_2$ index criterion [13], [14]. Furthermore, Kalman filters were used in model-based fault diagnosis [15], [16]. However, several deficiencies were detected in either the fault detection or isolation stages when the standard FDI theory was applied. In this optic, a robust fault diagnosis method based on instantaneous power calculation (IPCFD) combined with Kalman observer is presented.

2. OVERVIEW ON EXISTING FDI METHODS

Model-based FDI [17], [6], [7] is based on a certain set of numerical fault indicators, known as residues $r(k)$, which are computed using the measured inputs $u(k)$ and outputs $y(k)$ of the monitored system.

$$r(k) = \Omega(y(k), u(k))$$

where $\Omega$ is the residue generator function. This function allows computing the residue set at every time instant using the measurements of the system inputs and outputs. Ideally, residues should be zero or less than a threshold.
that takes into account noise and model uncertainty when no fault is affecting the system.

The fault detection task consists of deciding if there is a fault affecting the monitored system by checking each residue $r_i(k)$ of the residue set against a threshold that takes into account model uncertainty, noise, and the unknown disturbances.

The result of this test applied to every residue $r_i(k)$ produces an observed fault signature

$$
\psi(k) : \psi(k) = [\psi_1(k), \psi_2(k), ..., \psi_n(k)].
$$

Basic way of obtaining these observed fault signals could be through a binary evaluation of every residue $r_i(k)$ against a threshold $\delta_i$.

$$
\psi_i(k) = \begin{cases} 
0 & |r_i| \leq \delta_i \\
1 & |r_i| > \delta_i 
\end{cases}
$$

The observed fault signature is then supplied to the fault isolation module that will try to isolate the fault so that a fault diagnosis can be given. This module is able to produce such a fault diagnosis since it has the knowledge about the binary relation between the considered fault hypothesis set $f(k) = f_1(k), f_2(k), ..., f_n(k)$ and the fault signal set $\psi(k)$.

This relation is stored in the so-called theoretical binary fault signature matrix ($FSM$). Thereby, an element $FSM_{ij}$ of this matrix is equal to 1 if the fault hypothesis $f_j(k)$ is expected to affect the residue $r_i(k)$ such that the related fault signal $\psi_i(k)$ is equal to 1 when this fault is affecting the monitored system.

Otherwise, the element $FSM_{ij}$ is zero valued. However, this basic scheme, has the following drawbacks:

1. The threshold $\delta_i$ should be determined and adapted online.
2. The presence of the noise produces chattering if a binary evaluation of the residue is used.
3. Restricting the relation between faults and fault signals to a binary one causes loss of useful information that can add fault distinguish ability and accurateness to the fault isolation algorithm, preventing possible wrong fault diagnosis results.

The occurrence of a fault causes the appearance of a certain subset of fault signals such as each of them has dynamic properties characteristic of this defect which can improve the performance of the fault isolation algorithm if they are taken into account.

### 3. EXTENDED KALMAN OBSERVER

Consider the discrete time system described by the state space representation as follows:

$$
\begin{align*}
x[k+1] &= Ax[k] + Bu[k] + w[k] \\
y[k] &= Cx[k] + v[k]
\end{align*}
$$

Where matrices $A, B, C$ are the parameters of the system, $w \in \mathbb{R}^{n_w}$ is the input variable, $x \in \mathbb{R}^{n_x}$ and $y \in \mathbb{R}^{n_y}$ represent the state and the output of the system respectively.

The process noise $w$ and the measurement noise $v$ are assumed to be white and uncorrelated with the input $u$.

When sensor and actuator faults are considered, the state space representation (3) becomes

$$
\begin{align*}
x[k+1] &= Ax[k] + Bu[k] + f_u(k) + w[k] \\
y_u[k] &= Cx[k] + f_y(k) + v[k]
\end{align*}
$$

Where $f_u$ and $f_y$ are possible actuator and sensor faults and are assumed to be unknown constant additive numbers. The equations of the steady-state discrete Kalman filter to estimate the state are given as follows.

$$
\hat{x}[k | k] = \hat{x}[k | k-1] + M(y_u[k] - C\hat{x}[k | k-1])
$$

Where $\hat{x}[k | k]$ is the estimate of $x[k]$ given past measurement up to $y_u[k-1]$ and $\hat{x}[k | k-1]$ is the updated estimate based on the last measurement $y_u[k]$.

Given the current estimate $\hat{x}[k | k]$, the time update predicts the state value at the next sample $k+1$ (one-step-ahead predictor). The measurement update then adjusts this prediction based on the new measurement $y_u[k+1]$. The correction term is a function of innovation, that is, the discrepancy between the measurement and predicted
values of \( y_e[k + 1] \) is given as

\[
y_e[k + 1] = c\hat{x}[k + 1 \mid k]
\]

The innovation gain \( M \) is chosen to minimize the steady-state covariance of the estimation error given the noise covariances given as \( E(w[k]w[k]^T) = Q \); \( E(v[k]v[k]^T) = R \) and \( E(u[k]v[k]^T) = O \). We combine the time and measurement update equations into one state-space model as:

\[
\hat{x}[k + 1 \mid k] = A(I - MC)\hat{x}[k \mid k - 1] + [B \ AM]\begin{bmatrix} u[k] \\ y_n[k] \end{bmatrix} \quad \hat{y}[k \mid k] = C(I - MC)\hat{x}[k \mid k - 1] + CMv_n[k]
\]

This observer generates an optimal estimate \( \hat{y}[k \mid k] \) of \( y[k] \). The robustness of a fault detection algorithm is given by the degree of sensitivity to faults compared with the degree of sensitivity to uncertainty.

The algorithm of the IPCFD module is given as follows, \( p_{\text{max}} \) represents the instantaneous power in faultless operating system and \( \text{Ind} \) is the indicator signature generated by the IPCFD module.

\[
\text{residual} = r(k) = y_n(k) - y_e(k)
\]

For \( n \),

\[
pn = \frac{1}{N} \sum_{k=n-N+1}^{n} r^2(k)
\]

\[
pn_s = pn(r_s(k))
\]

\[
p_{\text{max}} = \max(pn_s)
\]

If

\[
(pn - pn) > p_{\text{max}}
\]

Then \( \text{Ind} = pn - pn \)

Else \( \text{Ind} = 0 \)

So, the residue allows to compare the measurements of real variables \( y_n(k) \) of the process with their estimation \( \hat{y}(k) \) provided by the associated system model is generated:

\[
r(k) = y_n(k) - y_e(k)
\]

Where \( y_n \) is the true output and \( y_e \) is the estimated output. \( r \in \mathbb{R}^{nv} \) is the residue set then, in a non faulty scenario, \( r(k) \) should be zero valued at every time instant \( k \) considering an ideal situation. Nevertheless, it will never be satisfied since the system can be affected by unknown inputs (i.e., noise, nuisance disturbances, etc.) and the model might be affected by some error assumptions (model errors) apart from its considered parameter uncertainty. Thus, the residue generated cannot be expected to be zero valued in a non faulty scenario. A generic form of a residue generator can be obtained using (4) et (7) and the instantaneous power of the residue \( r(k) \) is estimated.

The expression of instantaneous power \( pn \) is given as follows.

\[
pn = \frac{1}{N} \sum_{n=k-N+1}^{k} r^2(k)
\]

The instantaneous power of a physical system is defined, monotone and bounded variations for each interval of the system \([a, b]\), i.e. it must verify the proposition 1 [15]. Let \( u \in L^1(\Omega) \), then \( u \in BV(\Omega) \) if and only if its derivative in the sense of distributions \( Du \) is a Radon measure over. In addition, the total variation of \( Du \) is then equal to \( \int_{\Omega} | Du | \) (recorded | du | \( \Omega = \int_{\Omega} | Du | \)). We denote by \( \Omega \), an open set of \( \mathbb{R}^m \).

Otherwise, let \( f \) is a function defined on a compact \([a, b]\) and real valued function.

A function \( f \) is said to be bounded variations if \( M \) is a constant such that for each subdivision \( \sigma = (a = x_0, \quad x_1, \quad x_2, \ldots, \quad x_n = b) \in S([a, b]) \), we define \( V(f, \sigma) \) by

\[
V(f, \sigma) = \sum_{i=1}^{n} | f(x_i) - f(x_{i-1}) |
\]
We call total variation of \( f \) defined by the value \( V_b^a \) given as follows.

\[
V_b^a(f) = \sup_{\sigma \in \mathcal{S}(a,b)} V(f, \sigma)
\]  

(11)

It says that \( f \) is of bounded variation if \( V_b^a(f) \) is finite. Checking the condition cited in the definition, the instantaneous power \( p_m \) verifies this regularity condition. \( V_b^a(f) \) represents the maximum power of the instantaneous power of faultless operating system and represents the threshold of our detection algorithm.

In a real operating system that will be affected by disturbances and faults, instant verification of the difference provides information on changes in the status of the system and allow us to detect faults.

4. RESULT AND ANALYSIS

The proposed IPCFD algorithm has been applied to the discrete time system with a state space model

\[
\begin{align*}
x[k+1] &= Ax[k] + Bu[k] + f_u(k) + w[k] \\
y[k] &= Cx[k] + f_y(k) + v[k]
\end{align*}
\]

Where

\[
A = \begin{bmatrix}
0.156 & -0.310 & 0.216 \\
1.000 & 0 & 0 \\
0 & 1.000 & 0
\end{bmatrix},
B = \begin{bmatrix}
-0.725 \\
0.140 \\
0.681
\end{bmatrix}
\]

and

\[
C = \begin{bmatrix}
1 & 0 & 1
\end{bmatrix}
\]

\( v[k] \) and \( w[k] \) are Gaussian white noises.

In the case study, first the output is estimated assuming no sensor fault is affecting the system. Figure 1 shows the true and estimated outputs and Figure 2 gives the residue when no fault is affecting the system. We note that the residue is not null, due to the presence of the process noise \( w \) and the measurement noise \( v \) which are assumed to be white.

The innovation gain \( M = \begin{bmatrix}
0.3675 \\
0.0686 \\
-0.3957
\end{bmatrix} \)

Figure 1. The true and estimated output when no fault is affecting the system

Figure 2. Residue when no fault is affecting the system
The instantaneous power $\pi_n$ represented in Figure 3 is monotone and regular. In case of occurrence of a fault ($k = 30$), the true and estimated outputs and the residue are shown in Figure 4 and Figure 5. We note a presence of a peak at the time of the occurrence of fault. The instantaneous power $\pi_n$ is represented in Figure 6. This figure shows a presence of a jump. The IPCFD indicator $Ind$ detects this jump (see Figure 7).

![Figure 3. Instantaneous power when no fault is affecting the system](image1)

![Figure 4. The true and estimated output when fault is affecting the system](image2)

![Figure 5. Residue when fault is affecting the system](image3)

![Figure 6. Instantaneous power when fault is affecting the system](image4)
5. CONCLUSION

In this article, a new approach to detecting sensor defects based on calculating residues using the Kalman filter and the instantaneous power of these residues is presented. It reduces the influence of noise on defects. The results show the robustness of this approach. The analytical structure of the fault signature generator is designed around a hybrid observer. It is configured to generate structured residuals sensitive only to defects. These residuals will be evaluated later for determine the signatures of the faults. This approach makes it possible to give decision-making power, concerning the good system operation, to the diagnostic module whose inputs are the actual and estimated outputs and the output being the system status indicator. It is also designed so that it is sensitive only to sensor and actuator faults. The main idea of this contribution is to take into account, from the beginning of the design of the hybrid observer, the robustness against unknown inputs and sensitivity to defects. The proposed synthesis approach takes place in two phases. First phase, we are dealing with the synthesis of the hybrid observer without taking into account unknown inputs or faults. The convergence conditions ensuring the limits of the estimation error have been taken into account. The second phase proposes an iterative procedure to find a compromise between robustness against unknown inputs and fault sensitivity of the hybrid observer. Finally, through the results obtained, the relevance of the proposed diagnostic synthesis approach is proven.

REFERENCES


BIOGRAPHY OF AUTHORS

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