Optimal Placement of Phasor Measurement Unit for Better Power System Observability

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Abstract

Secure and reliable operation of the grid requires real time monitoring and control of entire power system. Phasor measurement units (PMU) are being deployed for real time monitoring and analysis of power system. The most important factor to be considered in placement problem is observability of power system with minimum number of PMUs. This paper proposed an Integer Linear Programming (ILP) based optimization approach to minimize the number of PMUs in the network. The proposed work determines the number of PMUs and its placement, while maximizing the system observability in normal operating condition. In this paper modeling of zero-injection bus has been formulated to reduce the number of PMUs further. Simulations are carried out in Standard IEEE 14 and IEEE 30 bus system, the results indicate the ILP approach determines the minimum number of PMUs and improves Observability of the Power System.

Keywords: phasor measurement units (PMU), zero injection constraints, optimal placement, integer linear programming (ILP)

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1. Introduction

Phasor measurement unit is monitoring device and being used effectively in real-time monitoring system to assure reliable and secure supply to end users. In Phasor measurement unit (PMU) all the electrical parameters are measured in frequency domain with both magnitude and phase angle of voltage and current [1]. Through Global Positioning System (GPS) all the measurements of PMUs are time stamped with common time reference signal. Synchronization of power system measurements is achieved by Global positioning system with time of less than 1μs. The PMU has roles for specific applications such as monitoring, protection, state estimation [2] and control in power systems [3]. A rapid development of processor and information technology, computer aided tools and data collection techniques are being used widely for power plant monitoring and control [4].

The use of PMU has been increased worldwide in electrical utilities. The major issues of PMUs are site location and its placement. Due to the association of huge costs involved in PMUs and its communication infrastructure, It is not necessary and also it will not be economically to place PMU in all buses of the connected network. PMUs installed on one bus can able to measure nearby buses. As result, problem has been raised for number of PMUs to be installed in power system. Optimization of PMU placement with complete observability of system will help the utility to operate the network with more reliability. Many investigation has been carried out by using different methods for placement problem using both evolutionary algorithms and mathematical programming approaches [5], such as canonical genetic algorithm [6], non-dominated sorting genetic algorithm [7], simulated annealing [8], exhaustive search [9]. Tabu search [10], optimal placement of PMU (OPP) is verified with topological observability of the network by an recursive Tabu search (RTS) [11] which is faster than traditional Tabu search (TS), particle swarm optimization [12]. Paper investigated full observability of PMU placement with Iterated Local Search (ILS) [13] and truncated the number of PMUs in configuration of the network. Paper outlined GA and immune algorithm (IA) [14], opted IGA for optimal placement of PMUs. IGA takes longer time for its execution than GA and binary search algorithm [15], OPP solved with normal operating conditions and single branch outages for complete observability of the power system.
The present paper recommends mathematical optimization technique as integer linear programming to deduce the minimum number of PMUs to be installed in the network for complete observability. The proposed method incorporates modeling of zero-injection constraints with optimal placement problem to ensure system observability with minimum number of pmu. This method brings out optimal solution compared with other evolutionary algorithms and other optimization techniques. Integer linear programming algorithm technique is implemented in this paper, to determine the total number of PMUs required making the system observable. Simulation results are analyzed for the standard IEEE 14-bus and 30-bus systems are presented.

The rest of the paper is organized as follows Section II will give detailed explanation of optimal placement of PMU and formulation of Zero-Injection Constraints. Simulation result of PMU placement in Section III, and the paper concludes in Section IV.

2. Optimal Placement of PMU

Formulation of Optimal placement of PMU enhances observability of power system. PMU Observability may be classified as Numerical Observability and Topological Observability [16]. In this paper topological Observability is evaluated through following rules.

Rule 1:
When a PMU is placed at a bus, the voltage and current phasor is known for that particular branch. If Voltage and Current phasor of one end of the branch is known, then the other side may be computed easily by using Ohm's Law [17]. This shows if a PMU placed at one bus, then the buses incident to PMU installed bus also become observable.

Rule 2:
When there is no current injection at a bus, the power flow in any one of the incident lines can theoretically be calculated by using Kirchhoff's current law (KCL), when the power flow in the remaining of the connected lines are known.

Thus zero-injection modeling approach is used for minimizing optimal PMU locations. Optimal PMU placement problem i.e., minimum PMU placement for system observability, can be formulated as a combinatorial optimization problem using an Integer Linear Programming (ILP) method.

The PMU placement method has been presented in this paper serves the following objective: It minimizes the total number of PMUs required to be installed in the system considered to make it completely observable. Design of zero-injection constraint reduces the total number of PMUs. The integer linear programming method is used to achieve these main objectives.

Objective function for the minimum PMU placement problem can be defined as follows:

$$ F = \min \sum_{i=0}^{n} c_i b_i $$

If the PMU placement vector having elements defines chance of PMUs at a bus, i.e:

$$ b_i = \begin{cases} 1, & \text{if a PMU is installed at bus } i \\ 0, & \text{otherwise} \end{cases} $$

And if $c_i$ is cost related to placement of PMU at bus, then subject to the following constraints:

$$ T b \geq e $$

Where $e$ is a unit vector of length, i.e:

$$ e = [1 \ 1 \ 1 \ ... \ 1]^T $$

$$ b = [b_1 \ b_2 \ ... \ b_n] $$

And $A$ is the Network Connectivity matrix of the system, i.e:
This objective can be formulated as for an $n$ bus system. Design step for an optimal placement of PMUs.

**Step 1:**
By using an line data of an bus system form an network observability matrix. Assign value 1 to variable when two adjacent node are connected. Otherwise assign as zero.

$$t_{ij} = \begin{cases} 1, & \text{if either } i = j \text{ are adjacent node} \\ 0, & \text{otherwise} \end{cases}$$

**Step 2:**
Create an observability constraint matrix and objective Function.

$$T_b \geq e$$

**Step 3:**
Incorporate Zero-injection constraints along with observability constraints.

**Step 4:**
Formulated problem can be resolved through optimization technique by Integer linear programming.

**Step 5:**
The variable $x_i$ is assign to one if PMU is installed in at bus $i$ otherwise assign zero.

### 3. Simulation
Consider standard IEEE 14 bus system the objective function of optimal placement can be formulated as Equation (1). Observability constraints can be formulated using line data of bus system equations are (2)-(15).

Optimal PMU placements problem are being carried out for the IEEE 14 bus system and IEEE 30 bus system using MATLAB ILP solver. In order to reduce usage of PMUs in site location includes modelling of Zero injection buses considered fo various systems are: $Z_{14BUS} = \{7\}$, $Z_{30BUS} = \{6,9,22,25,27,28\}$.
Table 1. Line Data for IEEE-14 bus system

<table>
<thead>
<tr>
<th>Sending end bus</th>
<th>Receiving End Bus</th>
<th>Receiving End Bus</th>
<th>Reactance p.u</th>
<th>Half susceptance p.u</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
<td>0.05917</td>
<td>0.0264</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>4</td>
<td>0.17632</td>
<td>0.0187</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>5</td>
<td>0.22304</td>
<td>0.0246</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>4</td>
<td>0.17388</td>
<td>0.017</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>5</td>
<td>0.17103</td>
<td>0.0173</td>
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<td>4</td>
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<td>6</td>
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<td>5</td>
<td>6</td>
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<td>0.25202</td>
<td>0</td>
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<tr>
<td>6</td>
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</tr>
<tr>
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<td>9</td>
<td>9</td>
<td>0.55818</td>
<td>0</td>
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<tr>
<td>6</td>
<td>12</td>
<td>12</td>
<td>0.25581</td>
<td>0</td>
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<td>9</td>
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<tr>
<td>10</td>
<td>11</td>
<td>11</td>
<td>0.19207</td>
<td>0</td>
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<td>12</td>
<td>13</td>
<td>13</td>
<td>0.19988</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>14</td>
<td>14</td>
<td>0.34802</td>
<td>0</td>
</tr>
</tbody>
</table>

The objective function for an IEEE-14 bus system:

\[ \text{OPP: Min } b_1 + b_2 + b_3 + \ldots + b_{14} \]  \hspace{1cm} (1)

Subject to bus observability constraints defined as follows:

Bus 1: \[ b_1 + b_2 + b_5 \geq 1 \]  \hspace{1cm} (2)

Bus 2: \[ b_1 + b_2 + b_3 + b_4 + b_5 \geq 1 \]  \hspace{1cm} (3)

Bus 3: \[ b_2 + b_3 + b_4 \geq 1 \]  \hspace{1cm} (4)

Bus 4: \[ b_2 + b_3 + b_4 + b_5 + b_7 + b_9 \geq 1 \]  \hspace{1cm} (5)

Bus 5: \[ b_1 + b_2 + b_4 + b_5 \geq 1 \]  \hspace{1cm} (6)

Bus 6: \[ b_6 + b_{11} + b_{12} + b_{13} \geq 1 \]  \hspace{1cm} (7)

Bus 7: \[ b_4 + b_7 + b_8 + b_9 \geq 1 \]  \hspace{1cm} (8)

Bus 8: \[ b_7 + b_8 \geq 1 \]  \hspace{1cm} (9)

Bus 9: \[ b_4 + b_7 + b_9 + b_{10} + b_{14} \geq 1 \]  \hspace{1cm} (10)

Bus 10: \[ b_9 + b_{10} + b_{11} \geq 1 \]  \hspace{1cm} (11)

Bus 11: \[ b_6 + b_{10} + b_{11} \geq 1 \]  \hspace{1cm} (12)

Bus 12: \[ b_6 + b_{12} + b_{13} \geq 1 \]  \hspace{1cm} (13)

Bus 13: \[ b_9 + b_{12} + b_{13} \geq 1 \]  \hspace{1cm} (14)

Bus 14: \[ b_9 + b_{14} + b_{13} \geq 1 \]  \hspace{1cm} (15)

Zero-Injection Constraints:

Bus 7: \[ b_4 + b_7 + b_8 + b_9 \geq 3 \]  \hspace{1cm} (16)
Revised observability constraint:

Bus 1: \( b_1 + b_2 + b_5 \geq 1 \) \hspace{1cm} (17)

Bus 2: \( b_1 + b_2 + b_3 + b_4 + b_5 \geq 1 \) \hspace{1cm} (18)

Bus 3: \( b_2 + b_3 + b_4 \geq 1 \) \hspace{1cm} (19)

Bus 4: \( b_2 + b_3 + b_4 + b_5 + b_7 + b_9 \geq 1 \) \hspace{1cm} (20)

Bus 5: \( b_1 + b_2 + b_4 + b_5 \geq 1 \) \hspace{1cm} (21)

Bus 6: \( b_6 + b_{11} + b_{12} + b_{13} \geq 1 \) \hspace{1cm} (22)

Bus 7: \( b_4 + b_7 + b_8 + b_9 \geq u_1 \) \hspace{1cm} (23)

Bus 8: \( b_7 + b_8 \geq 1 \) \hspace{1cm} (24)

Bus 9: \( b_4 + b_7 + b_9 + b_{10} + b_{14} \geq 1 \) \hspace{1cm} (25)

Bus 10: \( b_9 + b_{10} + b_{11} \geq 1 \) \hspace{1cm} (26)

Bus 11: \( b_6 + b_{10} + b_{11} \geq 1 \) \hspace{1cm} (27)

Bus 12: \( b_6 + b_{12} + b_{13} \geq 1 \) \hspace{1cm} (28)

Bus 13: \( b_9 + b_{12} + b_{13} \geq 1 \) \hspace{1cm} (29)

Bus 14: \( b_9 + b_{14} + b_{13} \geq 1 \) \hspace{1cm} (30)

The objective function in (1) represents the minimum number of PMUs required for optimal system observability of buses. General observability constraints (2)-(15) provides solution to the problem for optimal system observability.

Solving the ILP problem (1), (2)-(14) we required four PMU to place in buses. Identified buses are 3,10,12 in an IEEE 14 bus. In order to reduce the total number of PMUs, solving ILP problem (1) with modeling of zero-injection constraints (16)-(30). Therefore, thus the number of PMUs to be installed is reduced by one. The optimal system observability is attained with three PMUs. Identified buses are 3, 10, 12 in an IEEE 14 bus.

Table 2 brings out the simulation result of IEEE-14 and 30 bus system for optimal placement of PMU including zero injections using various mathematical optimization algorithms. System observability is maintained with minimum number of PMUs by considering modeling of zero-injection constraints. Optimal placement of PMU has been attained with minimum number of PMUs and nearest observability of buses is shown in Figure 2 for IEEE-14 bus system. Three PMUs are placed at respective bus to achieve optimal observability.
Table 2. Study on standard IEEE bus system

<table>
<thead>
<tr>
<th>IEEE bus system</th>
<th>No. of PMUs with Zero-Injection bus</th>
<th>No. of PMUs without Zero-Injection bus</th>
<th>Observability %</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE 14 bus</td>
<td>3</td>
<td>4</td>
<td>85</td>
</tr>
<tr>
<td>IEEE 30 bus</td>
<td>8</td>
<td>9</td>
<td>93</td>
</tr>
</tbody>
</table>

4. Conclusion

The proposed technique has been implemented by using MATLAB as a programming tool. Integer linear programming provides feasible solution and minimizes computing efforts. Simulation result shows that there is a reduction in the number of PMUs to be placed in the network and the number is further reduced by the inclusion of ZIC in the problem formulation. Optimization algorithm shows the effectiveness in the proposed Standards IEEE 14 as shown in Figure 2. The future work can be done with hybrid techniques such as hybridizing optimization technique with evolutionary algorithms. Some additional constraints can be combined account for realistic implementation of optimization techniques.

References


