Sparsity-based Angle of Arrival Estimation for Emitter Localization

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Abstract
Angle of arrival (AOA) is able to achieve high accuracy when the antenna arrays are deployed much closer to the emitter. However, spatial resolution problem still exists. This paper presents a novel AOA estimation method called sparsity angle sensing (SAS) to improve the resolution. It integrates compressive sensing theorem into the parameter estimation formula. Traditional approaches for AOA estimation such as beamforming (BF), minimum variance distortionless response (MVDR), and multiple signal classification (MUSIC) are compared with SAS, and simulation results are discussed. It is shown that SAS method outperforms the other three methods in spatial resolution and robustness.

Keywords: antenna arrays, AOA estimation, localization, sparsity angle sensing

1. Introduction
Wireless localization has emerged as a popular research area. As one of its branches, wireless sensor network (WSN) is a promising issue [1], [2]. Advances in radio frequency (RF) and Micro-Electro-Mechanical Systems (MEMS) integrated circuit (IC) design have made it possible to deploy a large number of wireless sensors much closer to the emitters, so that AOA estimation is able to achieve higher accuracy [3], [4].

In the localization via the WSN, each sensor is equipped with antenna arrays. Array signal processing intelligently combines the signals received at each antenna element to perform the estimation of AOA. The AOA estimation is extensively studied in array signal processing [5], [6]. Many well-know existing methods include beamforming (BF) [7], minimum variance distortionless response (MVDR) [8], [9], subspace-based methods such as multiple signal classification (MUSIC) [10], [11]. The BF is a very robust and simple source localization technique, but it suffers from the Rayleigh resolution limit, which is independent of signal-to-noise ratio (SNR). The MVDR improves the performance of BF by minimizing the variance due to noise while keeping the gain in the direction of steering equal to unity. The MUSIC is the most prominent method. The underlying idea is to separate the eigen space of the covariance matrix of sensor outputs into the signal and noise components using the knowledge about the covariance of the noise. The MVDR and MUSIC [12] methods go a long way to improve the resolution capabilities of the BF. However, when the sources are close and the SNR is low, they also lose solution and eventually are unable to separate the sources. Our approach tries to transforms a parameter estimation problem into sparse representation problem.
Similar to our paper, other sparse approximation approaches to localization have been proposed before. In [13], spatial sparsity framework is used to improve the AOA estimation, however, the signal reconstruction algorithm has high computational complexity. In [14], [15], [16], the problem of finding the AOA is equivalent to finding the relative time delay which is not a universal method and the performance comparison with conventional methods are not given and discussed. And in all the above methods, the compressive sensing (CS) principle is not presented.

The remainder of the paper is organized as follows. In section II, the SAS method is explicitly described. Firstly, the sparsity representation framework for the SAS is formulated. In section III, the compressive sensing (CS) theorem is employed to solve the problem. In section IV, the performance of the SAS and other three conventional approaches are compared. And the high spatial resolution and robustness of the SAS is demonstrated. The conclusion is given in section V.

2. Signal model and problem formulation

2.1. Notations and definitions

Matrices and vectors are denoted by bold font, with lower-case letters corresponding to vector and uppercase letters to matrices. The nth element of a vector a is written as an, and the ijth element of a matrix A is denoted by Aij. Superscripts (.)*, (.)T, and (.)H represent complex conjugation, transposition and conjugate transposition, respectively.

2.2. Signal model

Consider the basic case, the narrowband sources in the farfield of a uniform linear arrays (ULA). ULA consists of M omni-directional sensors with equal spacing d, residing on the x-coordinate axis. Taking the phase center of the array at the origin, the position of the m-th sensor is pm=(m-(M+1)/2)d, m ∈ {1,...M}. The modulated signal in narrowband case can be expressed as s(t)exp(jω0t), where s(t) is the baseband signal, ω0 is the frequency of the carrier. The output of sensor m is

yt(t) = s(t−τcenter)exp(jωk(t−τcenter)−k²p),

where τcenter is the delay from the source to the phase-center. k²p_m = −(ω_k t c)(m−(M+1)/2)d cos(θ_m), where c is the speed of light, and vector θ contains arrival angels from all the K source locations: θ = [θ_1,...,θ_K].

By measuring the time relative to the phase center, τcenter can be dropped. Thus, for a single source the complex envelope of the sensor outputs has the following form:

y(t) = a(θ)s(t), where a(θ) = exp(−jk²p).The model for K narrowband sources can be written as

y(t) = A(θ)s(t) (1)

The matrix A(θ) = [a(θ_1), a(θ_2),...,a(θ_K)] is so called array manifold matrix, whose (m, k) element represents the kth source AOA information to the mth sensor.

s(t) = [s_1(t), s_2(t),...,s_K(t)]' is the signal from K emitters.

Taking noise into account and discretizing the waveforms, the final version of the model takes the following form:

y(t) = A(θ)s(t) + n(t), t ∈ {1,...,T}

(2)

where θ = [θ_1,...,θ_K] is the vector of unknown emitter locations.

2.3. Problem Formulation

To cast this problem into a sparse representation problem, the basic steps are:

(i) Construct a known vector θ which is the expansion of vector θ considering all possible source locations. Let θ be filled with N vectors {θ_1,...,θ_N} which are the possible locations of unknown emitters.

(ii) Fill each column of A(θ) with each potential emitter location: A = [a(θ_1), a(θ_2),...,a(θ_N)].

Suppose the number of sensor arrays is M, the number of all possible emitters is N, the number of real unknown emitters is K. A is a M×N matrix. The relationship among M, N and
K is \( K < M < N \).

(iii) The formulation equals to

\[
y = \tilde{A}s + n
\]

\( \tilde{s} \) is the sparse vector to be recovered. Define the lp norm of the vector \( \tilde{s} \) as and when \( p = 0 \), the \( l_0 \) norm counts the number of non-zero entries in \( \tilde{s} \); here \( \| \tilde{s} \|_0 = K \), \( y \) is the observation samples, \( n \) is the noise vector.

(iv) Solve L1-denoise problem with quadratic constraints.

\[
\min \| \tilde{s} \|_1 \quad \text{subject to} \quad \| \tilde{A}s - y \|_2 \leq \varepsilon
\]  
\( \varepsilon \) is the variance of noise vector.

3. Recovery Algorithm

CS is the recently developed theory to solve the sparse signal recovery. Since SAS has a sparse structure, and the vector \( \theta \) is considered as a sparse vector, CS may be used to solve the problem in (4). However, it is not able to be used directly in solving our problem. Therefore, an extended CS is proposed in this paper.

First, we explain the basic theorem of CS and then show how it is applied.

CS theorem indicates that an N×1 discrete-time signal vector \( x \) can be recovered from an M×1 samples which is far fewer than N. To make this possible, the CS relies on two principles: sparsity and incoherence [17].

Sparsity expresses the idea that the number of freedom degrees of a discrete time signal may be much smaller than its length. For example, we have a vector \( x \in \mathbb{R}^N \) which can be converted into an orthogonal basis \( \Psi = [\phi_1, \phi_2, \ldots, \phi_N] \) as follows: \( x = \psi \alpha \), \( \alpha \) is the coefficient sequence gotten from \( x = \langle x, \phi \rangle \). By K-sparse we mean that only \( K \leq N \) of the expansion coefficients \( \alpha \) representing \( x = \psi \alpha \) are nonzero. Compressible means that the entries of \( \alpha \), when sorted from the largest to smallest, decay rapidly to zero. Such a signal is well approximated using a K-term representation.

Incoherent means the coherence between the measurement matrix \( \psi \) and the sensing matrix \( \Phi \) which is a M×N matrix should be as large as possible. The sensing matrix is used to convert the original signal to fewer samples by using the transform

\[
y = \Phi x = \Phi \Psi \alpha
\]

The definition of coherence is

\[
\mu(\Phi, \Psi) = \sqrt{\max_{\psi, \phi} \left| \langle \psi, \phi \rangle \right|^2}
\]

\( \mu(\Phi, \Psi) \) denotes the absolute value of the inner product.

Since \( M < N \), recovery of the signal \( x \) from the measurements \( y \) is ill-posed; however the additional assumption of signal sparsity in the basis \( \Psi \) makes recovery both possible and practical.

The signal can be recovered by solving the convex program below

\[
\alpha = \arg \min ||\alpha||_1 \quad \text{s.t.} \quad y = \Phi \Psi \alpha
\]

And, \( M \) should obey \( M \geq C.\mu^2(\Phi, \Psi).K.\log N \), where \( C \) is a small constant, \( K \) is the sparsity, \( N \) is the length of the original signal.

So CS provides a signal sampling method when the signal is sparse in a known basis. However, in our problem (3)(4), the basis is not known. We extend the CS into a general framework called Extended CS (ECS). ECS supposes that signal is already known that it is sparse, but we don’t know in which basis it is sparse. Define the M×N matrix \( A = \Phi \Psi \), (5) can be written as

\[
y = \Phi x = \Phi \Psi \alpha = A \alpha
\]
Where \( \alpha \) is the sparse signal, \( \Phi \Psi \) is not separately known, but their product is known as a union. Our goal is to solve for the sparse signal with the smallest \( \ell_1 \) norm that agrees with the measurement \( y \) [18], [19].

\[
\alpha = \arg \min ||\alpha||_1 \quad \text{s.t.} \quad y = \Phi \Psi \alpha
\]  

We call (9) extended CS and apply it to AOA sparse estimation problem. The solution becomes to

\[
s = \arg \min ||s||_1 \quad \text{s.t.} \quad ||y-A\alpha||_2 \leq n
\]

4. Numerical Experiments

In this section, several experimental results are shown by comparing four AOA estimation approaches: BF, MVDR, MUSIC and SAS.

In Fig.1, the SNR is -5dB and the distance between two sources are 20°, all the methods are able to solve the two sources. In Fig.2, the SNR is still -5dB, but the distance between two sources are close to 10° separation, the BF method begins to merge the two peaks while the other three methods are able to solve two sources. In Figure 3, when the two sources are close to 4° separation, SNR is -1dB, BF, MVDR and MUSIC all merge except SAS.

\[
\text{Figure 1. Spatial spectra for BF, MVDR, MUSIC and SAS. AOAs: 57° and 77°. SNR=-5dB}
\]

\[
\text{Figure 2. Spatial spectra for BF, MVDR, MUSIC and SAS. AOAs: 60° and 70°. SNR=-5dB}
\]

\[
\text{Figure 3. Spatial spectra for BF, MVDR, MUSIC and SAS. AOAs: 73° and 77°. SNR=-1dB}
\]

\[
\text{Figure 4. Spatial spectra for BF, MVDR, MUSIC and SAS. AOAs: 73° and 77°. SNR=6dB}
\]
number of sensors is 15, SNR is -6dB, all of the methods failed, (b) change the number of sensors to 30, all of the methods are able to solve the sources. From above five experiments, the following conclusions can be gotten: first, SAS method is most robustness to noise. Second, BF is subtlest to the separation between sources. Third, by increasing the number of sensors, all the methods would improve.

5. Conclusion
In this paper, the SAS approach is proposed for the AOA estimation. In this approach, a sparse space for AOA sensing is constructed. Four approaches are compared and the results demonstrate that the SAS outperforms the other three approaches in terms of robustness and spatial resolution.

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References


