An Algorithm for Quantitatively Calculating I/Q Gain and Phase Mismatch

Gang Yang\(^1\), Jianzeng Li\(^1\), Jin Liu\(^2\)
\(^1\)School of Information Engineering, Communication University of China, Beijing, China
\(^2\)ECDAV, Communication University of China, Beijing, China
e-mail: gangy@cuc.edu.cn*, jzli@cuc.edu.cn, benliujin@cuc.edu.cn

Abstract

The in-phase and quadrature modulator (IQ modulator) is a key component in modern wireless transmitters. It provides a convenient method for modulating data bits or symbols onto an RF carrier. It has become the architecture of choice for implementing transmitter signal chains for end applications such as cellular, WiMAX, and wireless point-to-point. However, there are several non-ideal aspects of analog IQ modulator, include IQ gain imbalance, imperfect quadrature, and/or LO leakage. These imperfections will result in image spectral and degrade MER of the modulator, which in turn degrades bit error rate (BER). Gain matching and phase compensation both affect the total amount of image rejection. Factory calibration can detect the IQ gain and phase mismatch in different frequency and store the correction coefficients in non-volatile memory. The key issue is how to get the exact mismatch. In this paper, a mathematical model of IQ modulator was established and an analytic solution is obtained about how to get the exact mismatch parameters.

Keywords: gain imbalance, IQ modulator, modulator imperfections, phase mismatch

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1. Introduction

In a traditional radio transmit (Tx) architecture, a baseband signal is modulated to an intermediate frequency (IF) and then modulated again to the transmit radio frequency (RF). The in-phase and quadrature modulator (IQ modulator) is a new method in modern wireless transmitters and widely used in communication, broadcasting, radar etc. It provides a convenient method for modulating data bits or symbols onto an RF carrier. This paper describes a typical zero-IF or direct-conversion transmitter. It also discusses several non-ideal aspects of analog IQ modulator, include IQ gain imbalance, imperfect quadrature, and/or LO leakage. These imperfections will result in image spectral and degrade MER of the modulator, which in turn degrades bit error rate (BER). Factory calibration can detect the IQ gain and phase mismatch and store the correction coefficients in nonvolatile memory. The method how to get exact mismatches is the key issue in factory calibration. We recently proposed an algorithm which can quantitatively calculate the exact mismatches of an IQ modulator. This can improve the accuracy and efficiency of the factory calibration and transmitter performance.
2. Theoretical Derivation

The ideal analog IQ modulator was shown in Figure 1. A signal bit stream is split into two parallel bit streams in the DSP or FPGA. The two streams are applied to a pair of digital-to-analog converters (DAC). The DAC outputs drive to low-pass filters whose primary role is to remove Nyquist images. The outputs of these filters then drive the baseband inputs of the IQ modulator. The LO signal is split into two signals, equal in amplitude but with a phase difference of exactly 90°. These two quadrature signals drive the inputs of the two mixers. The outputs of these two multipliers are added together to provide the IQ modulator’s output.

![IQ Modulator Diagram](image)

Figure 1. A ideal analog IQ modulator

The output of the IQ modulator is described as

\[
RF = I_b \times \omega_r + Q_b \times \omega_o
\]

\[
= I_b \times \cos(\omega_b t) + Q_b \times \sin(\omega_b t)
\]

(1)

An I/Q modulator is split into digital and analog parts. Before DAC is the digital part and after DAC is the analog part. In the digital part, the modulation is realized by DSP or FPGA, which will not introduce any IQ imbalance. However, it is widely known that during the analog modulation process, gain and phase mismatches of IQ signals have a direct impact on sideband suppression performance. This results in degraded error vector magnitude (EVM) at the receiver, which in turn degrades the bit error rate (BER).

Let IQ gain imbalance is \( \Delta A \) and phase error is \( \Delta \phi \). The non-ideal IQ modulator model is:

\[
RF = I_b \times \cos(\omega_b t) + \Delta A \times Q_b \times \sin(\omega_b t + \Delta \phi)
\]

(2)

In order to illustrate the imperfection of the IQ modulator, usually the baseband signal is a sine wave. Let the baseband signal is a complex sine wave:

\[
I_b = \cos(\omega_b t)
\]

\[
Q_b = \sin(\omega_b t)
\]

(3)

If there are IQ gain imbalance or phase error, the output spectrum of the IQ modulator is:

\[
SS_{im} = A_{2im} - A_{1im}
\]

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The undesired component image signal results from gain and phase imbalances between the I and Q signal paths along with LO quadrature imbalance. SSlog is defined as image rejection, and \(SS_{\text{log}} = A_{2\text{log}} - A_{1\text{log}}\).

Applying Eq 3 into Eq 2, the RF output is calculated by:

\[
RF = \cos(\omega t) \times \cos(\omega t) - \triangle A \sin(\omega t) \times \sin(\omega t + \triangle \phi)
\]  

Using trigonometric formulas:

\[
RF = \frac{1}{2} \left[ \cos(\omega t - \omega t) + \cos(\omega t + \omega t) \right] - \\
\frac{\triangle A}{2} \left[ \cos(\omega t - \omega t - \triangle \phi) - \cos(\omega t + \omega t + \triangle \phi) \right] \\
= \frac{1}{2} \left[ \cos(\omega t - \omega t) + \cos(\omega t + \omega t) \right] - \\
\frac{\triangle A}{2} \left[ \cos(\omega t - \omega t) \times \cos(-\triangle \phi) - \cos(\omega t + \omega t) \times \sin(-\triangle \phi) \right] + \\
\frac{\triangle A}{2} \left[ \cos(\omega t + \omega t) \times \cos(\triangle \phi) - \sin(\omega t + \omega t) \times \sin(\triangle \phi) \right]
\]

\[
= \frac{1}{2} \left[ 1 + \frac{\triangle A}{2} \cos(\triangle \phi) \right] \times \cos(\omega t - \omega t) - \frac{\triangle A}{2} \sin(\triangle \phi) \times \sin(\omega t + \omega t) + \\
\frac{1}{2} \left[ 1 - \frac{\triangle A}{2} \cos(\triangle \phi) \right] \times \cos(\omega t - \omega t) - \frac{\triangle A}{2} \sin(\triangle \phi) \times \sin(\omega t + \omega t)
\]

Applying the Eq. (6)

\[
a \times \sin(x) - b \times \cos(x) = \sqrt{a^2 + b^2} \cos(x - \psi), \quad \tan(\psi) = a / b
\]

Hence,

\[
RF = \sqrt{\left(\frac{1}{2} + \frac{\triangle A}{2} \cos(\triangle \phi)\right)^2 + \left(\frac{\triangle A}{2} \sin(\triangle \phi)\right)^2} \times \cos(\omega t + \omega t - \psi_1) + \\
\sqrt{\left(\frac{1}{2} - \frac{\triangle A}{2} \cos(\triangle \phi)\right)^2 + \left(\frac{\triangle A}{2} \sin(\triangle \phi)\right)^2} \times \cos(\omega t - \omega t - \psi_2)
\]

\[
= \frac{1}{2} \sqrt{1 + \triangle A^2 + 2 \triangle A \cos(\triangle \phi) \times \cos(\omega t + \omega t - \psi_1)} + \\
\frac{1}{2} \sqrt{1 + \triangle A^2 - 2 \triangle A \cos(\triangle \phi) \times \cos(\omega t - \omega t - \psi_2)}
\]

In the Eq.(7), \(\cos(\omega t + \omega t - \psi_1)\) is the desired signal A1 and \(\cos(\omega t - \omega t + \psi_2)\) is the image signal A2.

\[
SS = f(\triangle A, \triangle \phi) = \frac{1}{2} \sqrt{1 + \triangle A^2 - 2 \triangle A \cos(\triangle \phi)} - \\
\frac{1}{2} \sqrt{1 + \triangle A^2 + 2 \triangle A \cos(\triangle \phi)}
\]

\[
= \left(\frac{1 + \triangle A^2 + 2 \triangle A \cos(\triangle \phi)}{1 + \triangle A^2 - 2 \triangle A \cos(\triangle \phi)}\right)^{\frac{1}{2}}
\]

\[
SS_{\text{log}} = 20 \times \log_{10} \left(\frac{1 + \triangle A^2 - 2 \triangle A \cos(\triangle \phi)}{1 + \triangle A^2 + 2 \triangle A \cos(\triangle \phi)}\right)^{\frac{1}{2}}
\]
Image rejection ratio $SS_{\text{bg}}$ is the binary function of $(\Delta A, \Delta \phi)$. If $\Delta A$ is equal 1, then there is no IQ gain imbalance; if $\Delta \phi$ is equal 0, then there is no phase error. If $\Delta A$ is equal 1 and $\Delta \phi$ is equal 0, it is an ideal modulator and $SS_{\text{bg}}$ is infinitely great.

3. System Simulation Results

In the following analysis, the range of $\Delta A$ is set by $[1, 1.15]$ ($[0, 1.2\text{dB}]$) and the range of $\Delta \phi$ is set by $[0, 1^\circ]$. Based on Eq.9 analysis (let $\Delta \phi = 0$), Figure2 reflects the relationship between sideband suppression and IQ amplitude mismatch. Based on Eq. 9 (let $\Delta A = 1$), Figure 3 shows a relationship between sideband suppression and IQ phase error.

![Figure 2](image1.png)

**Figure 2. Sideband suppression VS I/Q gain imbalance**

![Figure 3](image2.png)

**Figure 3. Sideband suppression VS I/Q phase error**

In real systems, I/Q gain imbalance and phase error exist at the same time, and combined effect the sideband suppression. Figure 4 shows a plot that can be used to relate sideband suppression to I/Q gain mismatch and phase error mismatch. From the plot, it can be noted that a phase error of $1^\circ$, coupled with an I/Q gain mismatch of 0.5dB, results in $-30.34\text{dB}$ of sideband suppression. It is notable in this example that improving the phase error has no effect on the sideband suppression unless the gain mismatch is also improved.

![Figure 4](image3.png)

**Figure 4. Sideband suppression VS I/Q gain and phase imbalance**
4. Algorithm and Implementation

In-factory algorithms can reduce the image signal. By using a directional coupler and a power splitter, it is quite simple to add an auxiliary output to the transmitter that can be used during factory calibration. A spectrum analyzer is connected to the auxiliary RF output. Usually, the DACs have the ability to adjust I/Q gain and phase compensation. For example: the AD9122 of Analog Device has digital gain and phase adjustment function for sideband suppression. AD9912’s resolution of the gain compensation is 1/32768, and it of the phase compensation is approximately $3.5^\circ/1024$ or $0.00342^\circ$. So the key issue is what is the exact I/Q gain and phase imbalance $(\triangle A, \triangle \phi)$. Based on the mathematical model, we propose an algorithm which can quantitatively calculate the exact mismatches of an IQ modulator.

The mismatch of I/Q gain and phase is $(\triangle A, \triangle \phi)$, and we can calculate the parameters by following procedures.

Step 1: Without any component, we detect the image rejection ratio $SS_1 = f(\triangle A_1, \triangle \phi_1)$.

Step 2: We adjust the I/Q gain parameter, then detect $SS_2 = f(\triangle A_2, \triangle \phi_2)$, and

$\triangle A_2 = \triangle A_1 + \Delta A$.

Step 3: We adjust the phase component parameter, then detect $SS_3 = f(\triangle A_3, \triangle \phi_3)$ and

$\triangle \phi_3 = \triangle \phi_2 + \Delta \phi$.

According to the above steps, we can get a Equation set:

\[
\begin{align*}
SS_1 &= f(\triangle A_1, \triangle \phi_1) \quad (10.1) \\
SS_2 &= f(\triangle A_2, \triangle \phi_2) \quad (10.2) \\
SS_3 &= f(\triangle A_3, \triangle \phi_3) \quad (10.3) \\
\triangle A_2 &= \triangle A_1 + \Delta A \quad (10.4) \\
\triangle \phi_3 &= \triangle \phi_2 + \Delta \phi \quad (10.5)
\end{align*}
\]

where $SS_1, SS_2, SS_3, \Delta A, \Delta \phi$ are known.

Through the Equations (10.1), (10.2), (10.3), (10.4) or (10.1), (10.3), (10.5), the mismatching parameters can be solved. Equation set can be transformed into a cubic equation with one unknown. It can be solved by using Shengjin Equations. The detailed derivation is not provided in this paper. The key parameters I/Q gain and phase imbalance $(\triangle A, \triangle \phi)$ can be solved from the Equation set 10.

According to the I/Q gain and phase imbalance parameters $(\triangle A_1, \triangle \phi_1)$, exact compensation can be done by setting the DAC’s registers. This algorithm degrades the image signal and also accelerates the speed and accuracy of system debugging.

There are some measurement errors because using spectrum analyzer. To minimize the spectrum analyzer measurement error, a series of $\Delta A$ or and $\Delta \phi$ are selected and multiple sets $(\triangle A_i, \triangle \phi_i)$ can be solved. The measurement errors can be reduced by averaging the multiple sets $(\triangle A_i, \triangle \phi_i)$.

5. Conclusion

This paper analyzes the causes of image signal, and establishes mathematical model, according to the function of image rejection and gain and phase imbalance in the relationship.

On this basis, proposed an algorithm to calculate the mismatch parameters $(\triangle A_1, \triangle \phi_1)$ by measuring the amplitude of image signal using spectrum analyzer. Meanwhile, in order to overcome the spectrum analyzer’s measurement errors, proposed a method by using arithmetic average to achieve the purpose of reducing measurement errors. This paper has a strong theoretical guidance of image rejection, and the method improve the accuracy and remove the subjectivity of the debugging.
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References


