Artillery Strike Effectiveness in the Virtual Battlefield

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Abstract
Artillery plays an irreplaceable role in modern warfare. But due to its huge destructive power, artillery experiment is dangerous and costly. Computer simulation technology uses mathematical models to replace the missile systems and thus avoid accidents and save funds. Based on the trajectory equation model, this paper presents a computer simulation of the projectile distribution. Then, by the simulation results, this paper analyzes each factor’s influence on the fall point’s distribution, as well as the distribution type and distribution parameters. The damage probability and the average amount of ammunition are calculated to assess the strike effectiveness.

Keywords: projectile distribution, striking assessment, virtual battlefield

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1. Introduction
Computer virtual simulation technology has become widely applied and attracts lots of attention. Virtual battlefield is a typical application of virtual reality technology in the military field, and it represents the development trend of modern military technology [1-3]. In peacetime, the actual strike opportunities are limited, the military exercise are restricted by material and other factors. But the virtual battlefield technology can overcome these limitations, playing an important role in tactical exercises, system demonstration, and military training [4-5].

Artillery, a conventional weapon which has a wide fire covering and huge destructive power, plays an irreplaceable role in modern warfare. Thus, many researchers are trying to apply the virtual simulation technology into the artillery firing simulation. Reference [6] put forward a method to construct simulated object’s hierarchy in the artillery simulation battlefield based on the geometric modeling engine and physical modeling engine. Reference [7] and [8] discuss how to apply the collision effects generating technology and the terrain rendering method into artillery simulation training system, in order to enhance the visual effects. This article considers how to apply virtual technology into quantitative description of artillery’s strike effectiveness and proposes to advise the gunners and commanders. In this paper, data error will be introduced in the exterior trajectory equation, and how to assess the strike effectiveness will also be discussed.

2. Exterior Trajectory Equations
Due to the presence of air resistance, gravity and Coriolis force, the projectile will change its velocity, direction, and flight attitude. At the same time, other factors like different weather conditions, wind and earth curvature will also impact the movement of the projectile. The projectile’s equation group is shown in Equation (1).
In Equation (1), \( v_x, v_y, v_z \) is the x-axis, y-axis and z-axis components of the speed respectively. \( x \) is the range and \( y \) is the height. \( t \) is the flight time, \( w_x \) is the wind component parallel to the shooting plane and \( w_z \) represents the wind speed component perpendicular to the plane of the shooting. \( c \) is the shape factor. \( H(y) = (20000 - y)/(20000 + y) \), \( F(v) = 4.737 \times 10^{-4} C_{\text{conN}}(v) \), \( C_{\text{conN}}(v) \) is the factor without resistance. \( R \) is the radius of the Earth. \( g_a \) is the gravitational constant, \( \Omega = 7.292 \times 10^{-5} \text{rad/s} \) is the Earth's rotation angular velocity. \( \alpha \) is direction angle under the condition that direction of north is zero, clockwise rotation is positive, \( \lambda \) represents latitude. \( r = (x^2 + y^2 + z^2)^{0.5} \), \( v = [(v_x - w_x)^2 + v_y^2 + (v_z - w_z)^2]^{0.5} \).

\[
\begin{aligned}
\frac{dv_x}{dt} &= -cH(y)F(v_x)(v_x - w_x) - g_a \frac{R^2}{r^3} x \\
\frac{dv_y}{dt} &= -cH(y)F(v_y)(v_y - w_y) - g_a \frac{R^2}{r^3} y \\
\frac{dv_z}{dt} &= -cH(y)(v_z - w_z) - g_a \frac{R^2}{r^3} z \\
\frac{dx}{dt} &= (v_x - w_x)dt + \frac{x}{R} y^{-1} \\
\frac{dy}{dt} &= v_y \\
\frac{dz}{dt} &= v_z - w_z
\end{aligned}
\] (1)

Integral initial condition is when \( t = 0, v_0 = v_0 \cos \theta_0, v_0 = v_0 \sin \theta_0, v_y = 0, p = p_0, x = y = z = 0, p_0 \) is the measured value of surface pressure, \( \theta_0 \) is the firing angle and \( v_0 \) is the initial velocity. Equation (1) is variable coefficient differential equations. It has no analytical solution, and thus we can only do numerical integration. To the variable coefficient differential equations as follow, \( \frac{dy}{dx} = f(x, y) \) \( a \leq x \leq b \)

\( y(a) = \eta \) (2)

We usually choose the fourth-order Runge-Kutta method. Though this method needs a lot of calculation, it has high accuracy. The method is:

\[
\begin{align*}
    k_1 &= hf(x_n, y_n) \\
    k_2 &= hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}) \\
    k_3 &= hf(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}) \\
    k_4 &= hf(x_n + h, y_n + k_3) \\
    y_{n+1} &= y_n + \frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_4}{6} + O(h^5)
\end{align*}
\] (3)

Here \( h \) is the step depended on the accuracy requirements.

3. Causes and Types of Projectiles Distribution
Theoretically, for a certain type of artillery, when its firing angle is determined, the flight trajectory will also be determined. However, even with the same firing conditions, launching a
set of shells always get a bunch of trajectories. This phenomenon is called the projectile distribution.

Figure 1. The diagram of the fall points

Figure 2. (a) is the range distribution histogram, (b) is the non-parameter dictation of range

Figure 3 (a) is the bias of z-axis distribution histogram, (b) is the non-parameter distribution of the bias of z-axis
The projectile distribution includes the range distribution and the bias of z-axis distribution. The cause for the projectile distribution is the unavoidable errors, which include the system errors and random errors. The system errors can be corrected by some certain methods, but the random errors is unknown before firing, so they cannot be corrected. The errors include the firing angle fluctuation ($\sigma_{\theta_x}$ and $\sigma_{\theta_z}$), the deviation of initial velocity($\sigma_v$), the temperature deviation($\sigma_t$), the pressure deviation($\sigma_p$), the humidity deviation($\sigma_hu$), the fluctuation of the wind speed($\sigma_{w_x}$ and $\sigma_{w_z}$).

So we do a simulation with all factors. For a 37mm antiaircraft artillery, let $\sigma_{\theta_x}=0.18^\circ$, $\sigma_{\theta_z}=0.18^\circ$, $\sigma_v=5m/s$, $\sigma_t=1^\circ C$, $\sigma_p=10000Pa$, $\sigma_hu=5\%$, $\sigma_{w_x}=3m/s$, $\sigma_{w_z}=3m/s$, we can draw the fall points diagram of the range distribution diagram and the bias of z-axis distribution in Figure. 1. Respectively does normal distribution test on range and the bias of z-axis. The results have been shown in Figure 2 and Figure 3. We can find the curve (in Figure 2(b) and Figure 3(b)) is well linear, and is approximately normal distribution. For other types of artilleries, the same conclusions can be drawn. It means that the distribution of range and the bias of z-axis is approximately fit the Normal distribution.

4. Assessment of the Fire Effectivness

The firing efficiency represents the achieved effectiveness when an artillerist complete the required firing task. Through assessment of the firing effectiveness, we can not only demonstrate the weapons’ index, but also find ways to improve firing effectiveness. Usually, we use the following tow indexes to assess firing effectiveness:

(i) Damage probability-the probability of damage the target when firing out the required artilleries.

(ii) Average ammunition consumption-when there is no limit on the number of shells, the required ammunition consumption to complete the task.

4.1. Calculation of the Damage Probability

Conditional damage probability set the target center as the origin O, the x-axis as the fire direction, the z-axis perpendicular to the x-axis and points to the right Cartesian coordinate system {O-x, z}. C(x, z) is projectiles distribution center. It means x is the distance error of fire data and z is the direction error of fire data.

Fire N shells under the same condition and assume that every shell is independent with each other. Then, under the conditions of fire date of (x, z), the conditional damage probability of at least one shell hit the target is:

$$R_{N}(x, z) = 1 - [1 - R_1(x, z)]^N$$

(4)

$R_1$ is the damage probability when fire only one shell. $R_N$ is the damage probability when fire N shells. When $R_1(x, z)$ is very small

$$R_N(x, z) = 1 - e^{-NR_1(x, z)}$$

(5)

Set up firing data errors with no system error, which means distribution center of error gets through the Target Center. The total damage probability $R$

$$R = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi(x, z) R_N(x, z) dxdz$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi(x, z) R_N(x, z) dxdz$$

(6)

$\phi(x, z)$ is normal distribution’s density function, the expression is

$$\phi(x, z) = \frac{e^{-\frac{1}{2} \left( \frac{x^2}{B_x} + \frac{z^2}{B_z} \right)}}{\sqrt{\pi B_x B_z}}$$

(7)
Here $B_i$ is the standard variance of the range distribution and $B_j$ is the standard variance of the bias of z-axis distribution.

In the actual artillery firing, firing data error is limited. Thus, the limits of integration can be a finite value $X_i$, $Z_i$ (like $X_i=nB_i$, $Z_i=mB_j$, $n\geq 5$, $m\geq 5$). If we divide it into $n$ and $m$ segments respectively, then the integral domain is divided into $nm$ small squares. Let $x_i$ and $z_j$ be the center point of small squares (i.e. $i$), and make distribution density of $(x_i,z_j)$ represent the distribution density of the small square, then

$$R = 4 \int_0^h \int_0^l \varphi(x,z)R_{N_i}(x,z)dxdz$$
$$= 4 \sum_{i=1}^n \sum_{j=1}^m \varphi(x_i,z_j)R_{N_i}(x_i,z_j)dx_i dz_j$$

(8)

where $x_i=(2i-1)X_i/2n$, $z_j=(2j-1)Z_j/2m$, $\Delta x=X_i/n$, $\Delta z=Z_j/m$, $(i=1,2,3,...; j=1,2,3,..., \infty)$

### 4.2. Calculation of the Average Ammunition Consumption

To damage the target, the damage probability needs to meet the required level $R_N$. Then we may need to fire more than one shell. For this situation, we need to calculate the average ammunition consumption. The minimum $N$ ($N$ is a positive integer) which meets Equation (9) is called the average ammunition consumption.

$$R_{N_i}(x,z) = 1 - [1 - R_{N_i}(x,z)]^N \geq R_N$$

(9)

For example, the distance between artillery and target $D_{pm}=18km$, average range error $E_p=105m$, the bias of z-axis error $E_z=126m$, distance distribution error $R_p=96m$, direction distribution error $R_z=134m$, the target's area $S=2l\times2l$, and $l_p=200m$, $l_z=300m$. The required damage probability $R_N=0.9$. Use Equation (9), we can compute the average ammunition consumption $N=6$.

### 4.3. Comparison of Distribution Error in Different Firing Angles

When $D_{pm}$ is shorter than the maximum range of a artillery, there exist high and low firing angles ($\theta_h$ and $\theta_l$) can be chosen. But two firing angles will lead to different projectiles distribution error. For the general, this paper will study five types of artillery. That's, 37mm antiaircraft artillery, 160mm mortar, 122mm howitzer, 152mm gun-howitzer and 155mm self-propelled howitzer represented with T1, T2, T3, T4, T5, respectively. The relation between the distribution errors and the firing angles is listed in Table 1 with the parameters $\theta_h=0.18^\circ$, $\sigma_d=5m/s$, $\alpha=1^\circ$, $\sigma_p=10000Pa$, $\sigma_{\theta_h}=5\%$, $\sigma_{dx}=3m/s$, $\sigma_{\alpha_z}=3m/s$.

![Table 1. Distribution error in different firing angle](image)

From the data in Table 1, we can find that the distribution error, when $D_{pm}$ is determined, the high firing angle’s distribution error is greater than the low firing angle’s.

### 4.4. Comparisons of Different Types of Artillery’s Distribution Errors with the Same $D_{pm}$

In order to compare the different types of artillery’s strike effect on same target, we choose 122mm howitzers, 152mm gun-howitzers, and 130mm cannon to do simulations with the low firing angle. We assume $D_{pm}=10km$, $\theta_h=0.18^\circ$, $\sigma_{dx}=0.18^\circ$, $\sigma_d=5m/s$, $\alpha=1^\circ$, $\sigma_p=10000Pa$,
\[ \sigma_{\mu}=5\%, \ \sigma_{v_x}=3m/s \ \text{and} \ \sigma_{w_z}=3m/s. \] The results of the distribution errors of different types are shown in Table 2.

<table>
<thead>
<tr>
<th>Type (Firing angle)</th>
<th>Max Range(km)</th>
<th>B_d(m)</th>
<th>B_f(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>122mm howitzer ((\theta=24.65^\circ))</td>
<td>11.8</td>
<td>85.7</td>
<td>32.7</td>
</tr>
<tr>
<td>152mm gun howitzer ((\theta=11.92^\circ))</td>
<td>17.2</td>
<td>89.0</td>
<td>26.8</td>
</tr>
<tr>
<td>155mm American self-propelled howitzer ((\theta=8.30^\circ))</td>
<td>21.3</td>
<td>112.0</td>
<td>22.4</td>
</tr>
<tr>
<td>130mm cannon((\theta=4.54^\circ))</td>
<td>27.5</td>
<td>14.9</td>
<td>21.8</td>
</tr>
</tbody>
</table>

According to Table 2, we can find that with the same \(D_{pm}\), it is more advisable to choose artillery with long range because it will cause smaller distribution errors (except for the 155mm American self-propelled howitzer). Thus, when we care more about whether the maximum range can meet the \(D_{pm}\) rather than the type of artillery, we can divide 0-27.5km into six intervals. In each interval we use only one type of artillery, and in this way, the total distribution error can be minimized. As a result, firing efficiency can be improved. The strike interval of different types of artillery are shown in Table 3.

<table>
<thead>
<tr>
<th>Interval</th>
<th>Type of artillery</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-5Km</td>
<td>37mm antiaircraft artillery</td>
</tr>
<tr>
<td>5Km-8Km</td>
<td>160mm mortar</td>
</tr>
<tr>
<td>8Km-12Km</td>
<td>122mm howitzer</td>
</tr>
<tr>
<td>12Km-17Km</td>
<td>152mm gun howitzer</td>
</tr>
<tr>
<td>17Km-21Km</td>
<td>155mm self-propelled howitzer</td>
</tr>
<tr>
<td>21Km-27.5Km</td>
<td>130mm cannon</td>
</tr>
</tbody>
</table>

So, for the specified target with the given \(D_{pm}\), we can choose the most suitable type of artillery to fire and get the damage probability. An example is shown in Figure 4.

![Command Window](image)

**Figure 4.** The result with \(D_{pm}=20\text{km}\)

### 4.5. Relationship between the Lethal Radius of Artillery and the Damage Probability

The strike probability mentioned above has not taken the artillery’s lethal radius into consideration. They use the coverage probability to approximate to the strike probability. Yet, when we consider the lethal radius, the damage probability will be different. An example is shown in Figure 5, taking the lethal radius into consideration, the damage probability is 0.5339,
otherwise the probability is 0.4787. Thus, we can find when consider the lethal radius as a factor, the strike probability will be larger.

![Command Window](image)

Figure 5. Screenshot of results when taking the lethal radius as a factor with $D_{pm}=18000m$

5. Conclusion

This paper introduces different kinds of factors into the artillery trajectory equations, and through the simulation process, comes to the conclusion that range and bias of z-axis distribution is approximately normal distribution. Then, according to the conclusion above, we calculate the damage probability of artillery and the average ammunition consumption. Finally, we talk about how to choose the type of artillery in different situation, and provide suggestions for the gunner and commanders.

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References