Research on Passivity based Controller of Three Phase Voltage Source PWM Rectifier

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Abstract

Euler-Lagrange (EL) model of voltage source PWM rectifier is set up based on its model in synchronous dq coordinates. Passivity based controller is designed on the basis of passivity and EL model of voltage source PWM rectifier. Three switching function are educed by passivity based controller. A switching function is only realized in engineering consequently. Voltage source PWM rectifier using passivity based controller has many advantages, such as simpler structure, low total harmonic distortion, and good disturbance rejection performance. Passivity based control law is proved feasible by simulink simulation.

Keywords: PWM rectifier, Passivity based controller, EL model, switching function

1. Introduction

Three phase voltage source PWM rectifier has many advantages, such as sinusoidal current control, unity power factor at the AC side, DC output voltage control at the DC side, and reversible power flow between both sides etc.. Many nonlinear control strategies are introduced due to nonlinear characters of PWM rectifiers\textsuperscript{[1]}. The control strategy of exact feedback linearization can transform nonlinear system into linear one\textsuperscript{[2]}\textsuperscript{[3]}, then design method of linear system is used and the responses for PWM rectifier become faster than those in a conventional cascade control structure. Deficiencies of exact feedback linearization are complicated feedback control law, singularity, and poor robustness. Passivity based control theory is a essence control one of energy\textsuperscript{[4]}. Therefore passivity based control theory is used in PWM rectifier\textsuperscript{[5]}\textsuperscript{[6]}\textsuperscript{[7]}. In order to obtain good performances, passivity based controller of three phase voltage source PWM rectifier is researched in this paper.

2. EL Model of Voltage Source PWM Rectifier

The power circuit of three phase voltage source PWM rectifier is shown in Figure 1. \(u_a\), \(u_b\), and \(u_c\) are the sinusoidal phase voltages of three phase balanced voltage source and \(i_a\), \(i_b\), and \(i_c\) are phase current in Figure 1. \(S_a\), \(S_b\), and \(S_c\) are the switching function of PWM rectifier. \(u_{DC}\) is the DC output voltage, \(R \) and \(L \) mean resistance and inductance of filter reactor respectively, \(C \) is the DC side capacitor, \(R_L \) is the DC side load, \(u_{ra}\), \(u_{rb}\) and \(u_{rc}\) are the input voltages of PWM rectifier, and \(i_L \) is load current. From Figure 1, the model of three phase voltage source PWM rectifier in abc coordinates can be expressed as \[1\].

\[
\begin{align*}
\frac{d i_a}{dt} & = u_a - R i_a - u_{ra} \\
\frac{d i_b}{dt} & = u_b - R i_b - u_{rb} \\
\frac{d i_c}{dt} & = u_c - R i_c - u_{rc} \\
C \frac{du_{ab}}{dt} & = S_a i_a + S_b i_b + S_c i_c - \frac{u_{DC}}{R_L}
\end{align*}
\]
\[
\begin{align*}
  u_{ra} &= u_{dc} \left( S_a - \frac{1}{3} \sum_{j=\alpha,\beta,\gamma} S_j \right), \\
  u_{rb} &= u_{dc} \left( S_b - \frac{1}{3} \sum_{j=\alpha,\beta,\gamma} S_j \right), \\
  u_{rc} &= u_{dc} \left( S_c - \frac{1}{3} \sum_{j=\alpha,\beta,\gamma} S_j \right).
\end{align*}
\]

where 

\[
\frac{L_i}{3} \frac{di_a}{dt} + R_i i_a - \omega L_i i_q + S_a u_{dc} = u_a \\
\frac{L_i}{3} \frac{di_q}{dt} + R_i i_q + \omega L_i i_a + S_q u_{dc} = u_q \\
\frac{2}{3C} \frac{du_{dc}}{dt} - S_i i_a - S_q i_q + \frac{2}{3R}\, u_{dc} = 0
\]

Figure 1. Power circuit of three phase voltage source PWM rectifier

As existing time-varying sinusoidal voltage in abc coordinates, so we can transform equation (1) to two phase synchronous rotating dq coordinates as follow.

\[
\begin{align*}
  L \frac{di_a}{dt} + R_i i_a - \omega L_i i_q + S_a u_{dc} &= u_a \\
  L \frac{di_q}{dt} + R_i i_q + \omega L_i i_a + S_q u_{dc} &= u_q \\
  \frac{2}{3C} \frac{du_{dc}}{dt} - S_i i_a - S_q i_q + \frac{2}{3R}\, u_{dc} &= 0
\end{align*}
\]

where \(S_a, S_q\) are switching function in synchronous rotating dq coordinate respectively. \(u_a, u_q\) and \(i_a, i_q\) are voltage source, current in synchronous rotating dq coordinate respectively. \(\omega\) is angular frequency.

Equation (2) can be expressed as

\[
M \dot{x} + Jx + Rx = u
\]

(3)

where 

\[
M = \begin{bmatrix} L & 0 & 0 \\ 0 & L & 0 \\ 0 & 0 & 2C/3 \end{bmatrix}, \quad x = \begin{bmatrix} i_a \\ i_q \\ u_{dc} \end{bmatrix}, \quad J = \begin{bmatrix} 0 & -\omega L & S_a \\ \omega L & 0 & S_q \\ -S_a & -S_q & 0 \end{bmatrix}, \quad R = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & 2/3R \end{bmatrix}, \quad R = R^T
\]

Equation (3) is EL model of voltage source PWM rectifier in synchronous dq coordinates.
3. Passivity based Controller of Three Phase Voltage Source PWM Rectifier

3.1. Passivity

Let $V = x^T Mx/2$ denotes the storage function of PWM rectifier, on the basis of equation (3) and $x^T Jx = 0$, $V$ can be expressed as

$$
\dot{V} = x^T Mx = x^T (u - Jx - Rx) = x^T u - x^T Rx
$$

(4)

Let $y = x$ and $Q(x) = x^T Rx$. equation (4) can be written as

$$
\dot{V} = y^T u - Q(x)
$$

(5)

From equation (5), three phase voltage source PWM rectifier is strict passive for input $u$, output $y$ and supply rate[4].

3.2. Passivity Based Controller

3.2.1. Determining of Desired Point

In order to realize unity power factor and constant control of DC voltage, desired points of PWM rectifier are $u_{dc}$, $i_d$ and $i_q$. $u_{dc}$ is selected by $u_{dc} > \sqrt{3} U_m$ ($U_m$ is amplitude of AC phase voltage), $i_q = 0$. Power equation of AC side and DC side in stable state can be expressed as

$$
\frac{3}{2} u_{d} i_d - \frac{3}{2} R i_d^2 = \frac{u_{dc}^2}{R_L}
$$

(6)

From equation (6), $i_d^*$ is calculated by

$$
i_d^* = \frac{1}{2} \left( \frac{u_{d}}{R} - \sqrt{\frac{u_{d}^2}{R} - \frac{8 u_{dc}^2}{3 R L}} \right)
$$

(7)

3.2.2. Design of Passivity Based Controller

Let $x_e = x - x^*$ and error storage function $V_e = -x_e^T Mx_e/2$, equation (3) under considering damping injecting can be changed into

$$
Mx_e + R_a x_e = u - [Mx^* + J(x^* + x_e) + Rx^* - R_a x_e]
$$

(8)

Then passivity based controller is

$$
u = Mx^* + Jx + Rx^* - R_a x_e
$$

(9)

Passivity based controller (9) can realize $\dot{V}_e = -x_e^T (R + R_a) x_e < 0$ and convergence rate of error storage function depend on $R + R_a$ ($R_a$ is damping matrix).

3.3. Analysis of Passivity based Controller

Considering $Mx^* = 0$, equation(9) can be expanded as

$$
\begin{pmatrix}
u_d \\ u_q \\ 0
\end{pmatrix} =
\begin{pmatrix}
0 & -\omega L & S_q \\
\omega L & 0 & S_q \\
-\omega S & -S & 0
\end{pmatrix}
\begin{pmatrix}
i_d^* \\ i_q^* \\ u_{dc}
\end{pmatrix} +
\begin{pmatrix}
R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & 2/3R_L
\end{pmatrix}
\begin{pmatrix}
R_a & 0 & 0 \\ 0 & R_a & 0 \\ 0 & 0 & G_a
\end{pmatrix}
\begin{pmatrix}
i_d \\ i_q \\ u_{dc}
\end{pmatrix}
$$

(10)

where $G_a = 2/(3R_a)$.

Equation(10) can be expressed as
$$ \begin{align*}
    u_d &= -\omega L i_d + S_d u_{dc} + R i_m - R_i (i_d - i_d^*) \\
    u_q &= -\omega L i_q + S_q u_{dc} - R i_q \\
    0 &= -S_q i_q - S_d i_d + 2u_{dc}^* / (3R_L) - G_s (u_{dc} - u_{dc}^*)
\end{align*} $$ (11)

From equation (11), $i_d, i_q$ and $u_{dc}$ are state variables, $S_d$ and $S_q$ are control input (switching function). Then three program to calculate switching function $S_d$ and $S_q$ as follow.

3.3.1. $S_d$ and $S_q$ calculated by first and second formula in (11)

$S_d$ and $S_q$ calculated by first and second formula in (11) can be written as

$$ \begin{align*}
    S_d &= \frac{u_d + \omega L i_d - R i_d^* + R \Delta i_d}{u_{dc}} \\
    S_q &= \frac{u_q + R i_q - \omega L i_q}{u_{dc}}
\end{align*} $$ (12)

where $\Delta i_d = i_d - i_d^*$.

Under considering $u_q = 0$ the following equation is obtained by replacing (12) in (2).

$$ \begin{align*}
    \frac{L}{R + R_i} \frac{di_d}{dt} + i_d &= i_d^* \\
    \frac{L}{R + R_i} \frac{di_q}{dt} + i_q &= 0 \\
    Cu_{dc} \frac{du_{dc}}{dt} &= \frac{3}{2} u_d i_d - \frac{3}{2} R i_d^2 + \frac{3}{2} R i_d (i_d - i_d^*) + \frac{3}{2} R i_q^2 - \frac{u_{dc}^2}{R_L}
\end{align*} $$ (13)

If $R_i$ is chosen bigger, $i_q$ track desired value $i_q^*$ and $i_d$ track desired value $0$ fast. Then

$$ \begin{align*}
    \frac{L}{R + R_i} \frac{di_d}{dt} + i_d &= i_d^* \\
    \frac{L}{R + R_i} \frac{di_q}{dt} + i_q &= 0 \\
    Cu_{dc} \frac{du_{dc}}{dt} &= \frac{3}{2} u_d i_d^* - \frac{3}{2} R i_d^2 - \frac{u_{dc}^2}{R_L}
\end{align*} $$ (14)

Power balance of AC side and DC side is satisfied from third formula in (14) and $u_{dc}$ track desired value $u_{dc}^*$ fast. Therefore passivity based controller depend on equation (12) can be obtain dynamic and static state decoupling of $i_q$ and $i_d$ so that the performances of PWM rectifiers are became better.

3.3.2. $S_d$ and $S_q$ Calculated by First and third Formula in (11)

$S_d$ and $S_q$ calculated by first and third formula in (11) can be written as

$$ \begin{align*}
    S_d &= \frac{u_d + \omega L i_d - R i_d^* + R \Delta i_d}{u_{dc}} \\
    S_q &= \frac{3R_i i_d + 2u_{dc} u_{dc}^*}{3R_L u_{dc} i_d}
\end{align*} $$ (15)
where \( p_i = i_i (\text{R}_i^2 - \text{u}_q - \text{oL}_i \text{R}_i \Delta \text{a}_i) - \text{G}_i \text{u}_d \Delta \text{u}_d \). \( \Delta \text{u}_d = \text{u}_d - \text{u}_d^* \).

Switching function (15) is not realized in engineering because of \( \text{i}_q^* = 0 \).

3.3.3 \( \text{S}_d \) and \( \text{S}_q \) calculated by second and third formula in (11)

\( \text{S}_d \) and \( \text{S}_q \) calculated by second and third formula in (11) can be written as

\[
\begin{align*}
\text{S}_d &= \frac{3\text{R}_i \text{p}_2 + 2\text{u}_i^* \text{u}_d}{3\text{R}_i \text{i}_q^* \text{u}_d} \\
\text{S}_q &= \frac{\text{u}_q + \text{R}_i \text{i}_q - \text{oL}_i \text{a}_i}{\text{u}_d} \\
\end{align*}
\]

(16)

where \( \text{p}_2 = \text{oL}_i \text{i}_q - \text{R}_i \text{i}_q^2 - \text{G}_i \text{u}_d \Delta \text{u}_d \).

Under considering \( \text{u}_q = 0 \) the following equation is obtained by replacing (16) in (2)

\[
\begin{align*}
\text{L}_i \frac{\text{d} \text{i}_q}{\text{d} \text{t}} + \text{R}_i \text{i}_q^2 - \text{R}_q \text{i}_q^2 + \frac{2}{3} \text{u}_\text{dc} \left( \text{u}_\text{dc} - \text{u}_\text{dc}^* \right) & = \text{u}_i \text{i}_q \\
\frac{\text{L}}{\text{R} + \text{R}_q} \frac{\text{d} \text{i}_q}{\text{d} \text{t}} + \text{i}_q & = 0 \\
\frac{\text{R}_i \text{R}_q^2}{\text{R}_q + \text{R}_c} \frac{\text{d} \text{u}_\text{dc}}{\text{d} \text{t}} + \text{u}_\text{dc} & = \text{u}_\text{dc}^* \\
\end{align*}
\]

(17)

From mathematical sense and equation (17), \( \text{i}_q \) track desired value 0 and \( \text{u}_\text{dc} \) track desired value \( \text{u}_\text{dc}^* \) fast. From power or energy transmission, ascending of \( \text{u}_\text{dc} \) imply one of electric field energy of capacitor. Then power of AC side must be supplied fast. Power ascending of AC side depend on one of \( \text{i}_q \) and then ascending of \( \text{u}_\text{dc} \) depend on one of \( \text{i}_q \). In other words, \( \text{u}_\text{dc} \) is limited by \( \text{i}_q \) which is seen from the first formula in (17). Therefore equation (16) can be not implemented.

From the above, passivity based controller depend on equation (12) is only feasible.

4. Simulation of Three Phase Voltage Source PWM Rectifier with Passivity based Controller

4.1. Passivity based control system

Passivity based control system of PWM rectifier is shown refer equation (7) and (12) in Figure 2. Mabc/dq is synchronous rotating transformation matrix in Figure 2.

Figure 2. Passivity based control system of PWM rectifier
4.2. Simulation of Passivity based Control System

Simulink model is set up according to Figure 2. Simulation parameters are as follows: phase voltage $U_p=220V$, source frequency $f=50Hz$, $L=16mH$, $R=0.3\Omega$, $C=2200\mu F$, $RL=100\Omega$, $u^{*}\text{DC}=600V$, $Ra=10$, modulation frequency 5kHz.

Simulation results of PWM rectifier under $R_L=100\Omega$ is shown in Figure 3. Operation states under load disturbance are shown in Figure 4. The change of $R_L$ from 100 $\Omega$ to 10 $\Omega$ at point A, from 50 $\Omega$ to 100 $\Omega$ at point B, from 100 $\Omega$ to 200 $\Omega$ at point C, and from 200 $\Omega$ to 100 $\Omega$ in Figure 4.

From Figure 3, passivity based controller depend on equation (12) can realize sinusoidal current control, unity power factor at AC side, and DC output voltage control at the DC side. Good disturbance rejection performances of PWM rectifier are shown in Figure 4.

Figure 3. Operation states of PWM rectifier with $R_a=10$ and $R_L=100\Omega$
5. Conclusions

Passivity based controller depend on equation(12) is only realizable by analyzing equation(11). PWM rectifier has good performances based on (12). id and iq are better controlled from(13) or(14), while uDC is not. Control of uDC is researched further.

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References