Study on the Linearly Range of S-Shaped MEMS Planar Micro-spring

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Abstract
For the lack of formula of linear range for S-shape d MEMS planar micro-spring, this paper establishes physical and mathematical model and analysis the material stress-strain angle. The formula is deduced by calculating the strain and rotation of the basic unit of micro-spring tension. Compared with the results of the tests on micro-spring which produced by UV-LIGA process, the formula results is in about 10% higher, providing theoretical guidance for the design and application of S-shaped MEMS planar micro-spring in engineering practice.

Keywords: linear range, micro-spring, tensile test

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1. Introduction
MEMS micro-spring, as a typical micro-structure, is widely used in MEMS sensors, micro-gyro, micro-actuator and so on, and manly are used for the transfer of forces and energy. Therefore, the performance of the micro-spring such as elastic coefficient must be considered as an important factor in the micro-spring design. For the limitation of the process, most of the micro-springs are planar structure[1].

At present, many achievements have been made in the research of performance of planar micro-spring. On the research of fuse micro-springs which are almost metal micro-spring based on LIGA and UV-LIGA process, L.B. Zheng, H. Li and G. He have deduced the formula of Z-shaped [2], L-shaped [3] and W-shaped [4] planar micro-spring elastic coefficient which are optimized by Z.L. Wu [5] by using mechanical energy methods. And have conducted deep study on the relationship between the coefficient and the structure of MEMS planar micro-spring [6]. P.F. Wu, etc. have deduced O-shaped micro-spring coefficient [7]. L.J. Niu has deduced the vertical elastic coefficient of S-shaped MEMS planar micro-spring [8]. On the research of other planar micro-spring, W.L. Lu and Y.M. Hwang have designed and processed a planar cylindrical micro-spring [9] for micro generator based on silicon process. And C. Wang has designed and processed a silicon planar micro-spring with low stiffness in low-g and deduced the elastic coefficient [10] for micro-actuator.

The tensile linearity which determines the MEMS mechanism can work safe and reliable is an important parameter, and it should be considered in micro-spring design. But there were no research reports of the micro-spring linear range. By analyzing the material stress-strain, this paper calculates the deformation of micro-springs basic unit [11] and the effect of deformation angle to the linear range, and gets the formula of linear range of S-shaped planar micro-spring. Compared with the results of the tests on micro-spring which are produced by UV-LIGA process and stretched on small force stretch device, the formula results is higher about 10%, providing theoretical guidance for the design and application of S-shaped MEMS planar micro-spring for engineering practice.

2. Formula Deduction of Micro-spring Linear range
Through the method of material nonlinear analysis, this paper deduces the linear range of S-shaped planar micro-spring. The S-shaped micro-spring, produced by UV-LIGA process, is shown in Figure 1.

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The S-shaped planar micro-spring consists of n section of the same basic unit which has the same structure. Select any one of the basic unit to analysis and establish physical and mathematical model, it can be further divided according to the method of material nonlinear analysis.

The divided part of basic unit and the size parameter is in Figure 2. The basic unit is divided into 5 parts. H is the micro-spring beam-width, and D is the beam-depth. R represents the central axis radius of I and IV parts, a is force section, and L is the length of the beam. After defining strain up negative, down is positive, Establishing cantilever physical model for each part and calculating the strain of each part, and the result is \( \delta_1 \), \( \delta_2 \) is the calculation result of displacement which is brought by the effect of the deformation angle of each part under the function of bending moment and stress. The sum of \( \delta_1 \) and \( \delta_2 \) is the formula of linear range of S-shaped planar micro-spring.

Because the maximum stress cross-section of micro-spring, get form structural analysis, is in the level cross section of the I and IV parts, the micro-spring needs a strength check to get the \( F_{\text{max}} \), the maximum load within the micro-spring elastic range under the condition of material yield point \( \sigma_s \).

For the Maximum Tensile Stress Theory [12], \( \sigma_s \) is given as bellow:

\[
\sigma_L + \sigma_W \leq \sigma_s \quad (1)
\]

Where \( \sigma_L \) is the tensile stress, \( \sigma_W \) is bending stress, \( I \) is structure moment of inertia.

\[
\sigma_L = \frac{F}{S} = \frac{F}{DH} \quad (2)
\]

\[
\sigma_W = \frac{M_{\text{max}}y_{\text{max}}}{I} = \frac{6F(l + R)}{DH^2} \quad (3)
\]

\[
I = \frac{DH^3}{12}
\]

Substituting Eq.(2) and Eq.(3) into Eq.(1), the maximum load \( F_{\text{max}} \) within the micro-spring elastic range can be derived as:

\[
F_{\text{max}} = \frac{\sigma_s DH^2}{6(l + R) + H} \quad (4)
\]
(1) Analysis of part I. The deflection of part I, \( W_I \), can be obtained under the affect of maximum load \( F_{\text{max}} \) if I is regarded as ordinary cantilever:
\[
W_I = \frac{F l^3}{3EI} = \frac{2\sigma_D H^2}{3(l + R)HE} \tag{5}
\]

(2) Analysis of part II. With the effect of \( F_{\text{max}} \) and the clockwise torque \( M = F_{\text{max}} L \) passed from part I at the same time, the torque distribution is shown as in Figure 3.

\[\text{Figure 3. Torque distribution on part II}\]

The deflection of part II, \( W_{II} \), is derived by analyzing the effect of \( F_{\text{max}} \) and \( M \) separately and the principle of superposition. In order to simplify the calculation, part II is divided into upper part and lower part and the physical model are also established separately [13].

The deflection \( W_{II} \) under the effect of \( F_{\text{max}} \) of can be obtained as
\[
W_{II} = W_{IIu} + W_{IIl} = \frac{2F(R)^3}{3EI} \tag{6}
\]

Where \( W_{IIu} = \frac{F(R)^3}{3EI} \) is the upper part deflection and \( W_{IIl} = \frac{F(R)^3}{3EI} \) is the lower.

The deflection \( W_{II} \) under the effect of \( M \) of can be obtained as
\[
W_{II} = W_{IIu} + W_{IIl} = -\frac{F(R)^3}{2EI} \tag{7}
\]

Where \( W_{IIu} = -\frac{F(l + R)(R)^2}{2EI} \) is the deflection of upper part and \( W_{IIl} = \frac{F l(R)^2}{2EI} \) is the lower.

For the principle of superposition, the deflection of part II can be obtained as below.
\[
W_{II} = W_{IIu} + W_{IIl} = \frac{2F(R)^3}{3EI} - \frac{F(R)^3}{2EI} = \frac{F(R)^3}{6EI} \tag{8}
\]

(3) Analysis of part III. With the effect of \( F_{\text{max}} \) and the counter-clockwise torque \( M = F_{\text{max}} L \) at the same time, the deflection of part III can be obtained as
\[
W_{III} = W_{IIIu} + W_{IIIl} = \frac{2F l^3}{3EI} \tag{9}
\]
(4) Part IV and part II are symmetric structure. So, the deflection of part IV is derived as

$$W_{IV} = \frac{F(R)^3}{6EI}$$  \hspace{1cm} (10)

(5) Part V is under the force of $F_{max}$ and the counter-clockwise torque $M = F_{max}L$, and the deflection of Part V can be obtained as below.

$$W_V = W_{VF} + W_{VM} = -\frac{FL^3}{6EI}$$  \hspace{1cm} (11)

Therefore, the $\delta_1$ is obtained as

$$\delta_1 = W_1 + W_{II} + W_{III} + W_{IV} + W_V$$  \hspace{1cm} (12)

(6) Each deformation angle of the part, due to the strain, is calculated for the effect of displacement of force section $a$ in the direction of $F$.

Because the force on the part II and part IV equals to the size and the torque of equal size and opposite in direction, the superposition of the resulting deformation angle $\theta$ is 0. The displacement of part II and part IV, due to the deformation angle is obtained as

$$W_{III} = \frac{4FL^2R}{EI} + \frac{FR^2l}{EI}$$  \hspace{1cm} (13)

Respectively, the displacement determined by the force $F$ and torque $M$ equals in size and opposite in direction, it is 0 for the principle of superposition, and the same to the deformation angle. Thus, the whole displacement in the direction of part III is derived as

$$W_{III} = 0$$  \hspace{1cm} (14)

For the deformation angle on part I and part V equals in size and opposite in direction, it is 0 after superposition. And the displacement is derived as

$$W_{OV} = -\frac{FL^3}{2EI}$$  \hspace{1cm} (15)

And the final result that the displacement of force section $a$ in the direction of $F$ comes

$$\delta_2 = W_{III} + W_{III} + W_{OV} = \frac{FL}{EI}(4IR + 2R^2 - \frac{1}{2}l^2)$$  \hspace{1cm} (16)

Of the role of the maximum load $F_{max}$, the total deformation of the basic unit of S-shaped planar micro-spring is obtained as below.

$$\delta = \delta_1 + \delta_2 = \frac{12\sigma_s}{[6(l + R)H]} \left( \frac{1}{3}l^3 + \frac{1}{3}R^3 + 4l^2R + 2R^2l \right)$$  \hspace{1cm} (17)

Therefore, the formula of linear range of $n$ section of S-shaped planar micro-spring is derived as below.

$$\delta_n = \frac{12n\sigma_s}{[6(l + R)H]} \left( \frac{1}{3}l^3 + \frac{1}{3}R^3 + 4l^2R + 2R^2l \right)$$  \hspace{1cm} (18)
3. Verification of S-Shaped Planar Micro-spring Linear range

We use the micro-springs produced by UV-LIGA process for tests and select three micro-springs from them and numbered 01, 02 and 03.

![Micro-spring measurement](image)

![Stress and strain curves of micro-spring](image)

### Table 1. The structural parameters micro-springs

<table>
<thead>
<tr>
<th>NO.</th>
<th>L(µm)</th>
<th>R(µm)</th>
<th>H(µm)</th>
<th>D(µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>528.570</td>
<td>111.245</td>
<td>65.025</td>
<td>207.7</td>
</tr>
<tr>
<td>02</td>
<td>573.384</td>
<td>110.478</td>
<td>71.520</td>
<td>196.2</td>
</tr>
<tr>
<td>03</td>
<td>573.645</td>
<td>114.300</td>
<td>66.540</td>
<td>192.7</td>
</tr>
</tbody>
</table>

![Stress and strain curves of micro-spring](image)
The parameters of beam-width $H$, length of the beam $L$, beam-depth $D$ and the central axis radius of part II $R$ of micro-springs are measured and shown in Figure 4. Measurement results are derived by using Multi-point measurement method and taking the average value, and shown in Table 1. The micro-spring deformation and the tension curves are obtained by using Small force and small displacement test equipment. The curves are shown in Figure 5.

The measured linear range of micro-spring can be derived from the stress and strain curves. And the calculated linear range of micro-spring is obtained by taking measurement data into formula. The comparative results are shown in Table 2.

The results shows that there is A difference of 10% between Experimental results and calculation results and the formula of linear range of S-shaped planar micro-spring derived by Maximum Tensile Stress Theory is credible. The formula can reflect the micro-spring linear range in engineering practice reasonable.

4. Conclusion

By analyzing the material stress-strain, we get the formula of linear range of S-shaped planar micro-spring. Compared with the results of the tests on micro-spring which are produced by UV-LIGA process and stretched on small force stretch device, the formula results is higher about 10%, providing theoretical guidance for the design and application of S-shaped MEMS planar micro-spring in engineering practice.

The analysis method is also applicable to other forms of planar micro-spring. Effects to the micro-spring by the parameters of micro-spring can be derived during the process of formula derivation and conducive to the optimization of the micro-spring design.

References