Study on the Energy-Regeneration-based Velocity Control of the Hydraulic-Hybrid Vehicle

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Abstract
This paper simplifies the energy regeneration based vehicle velocity system of the hydraulic hybrid bus into a process in which the extension rod of the hydraulic cylinder drives the secondary-element variable delivery pump/motor to change its displacement. This process enables braking of the vehicle and also allows recovery of energy. The stability, energy efficiency and other characteristics of the system are studied based on analysis of mathematical models of the vehicle velocity control. The relevant controller is designed to study effects of the controller on system characteristics. The vehicle velocity control module of the energy regeneration system is stable and able to recovery the inertia energy generated in vehicle braking. After the controller intended to improve response speed is added, system response becomes quicker but energy recovery rate declines.

Key Words: Hydraulic Hybrid, Energy Regeneration, Mathematical Models, Pole Assignment

1. Introduction
Relative to motor vehicles that rely solely on conventional internal combustion engines or electric motors, hybrid vehicles have many distinctive advantages. They inherit the high specific energy and high specific power of petrol engines, fills electric vehicles' shortcoming in running distance and carries forward the high energy efficiency and low emissions of electric vehicles to markedly improve the fuel efficiency and emission performance of the vehicle, combining strengths of both vehicle types [1-4].

The energy regeneration technology of the hydraulic hybrid vehicle means that the hydraulic accumulator is used as an energy storage element, the reversible hydraulic variable deliver pump/motor is used to convert inertia energy generated during vehicle braking into hydraulic energy stored in the accumulator to recover braking energy, and, when the vehicle starts up, accelerates or climbs, the hydraulic variable deliver pump/motor works as a motor that converts hydraulic energy stored in the accumulator to mechanical energy to drive the vehicle. Japan, the US and some European countries have developed various hydraulic energy storage systems and tested them on buses that need to start up and stop frequently, proving that such systems cut fuel consumption and CO2 emissions by 30%-50% [5-8].

The structural diagram of the hydraulic energy regeneration system studied in this paper is shown in Figure. 1.

The energy recovery process of the system is as follows: When the vehicle brakes, the directional control valve 6 shifts to the low-order function. Then the variable delivery pump/motor works as a pump, the wheel inertia drives the variable delivery pump/motor to rotate and the variable delivery pump/motor pumps oil into the accumulator and convert mechanical energy to hydraulic energy. Though the fixed delivery pump rotates now, as unloading valve 3 is open, the engine run idly and the lost energy is low.

The energy release process of the system is as follows: When the vehicle starts up or accelerates, the directional control valve 6 shifts to the low-order function. Then the variable delivery pump/motor works as a motor and the torque required for vehicle startup is satisfied by changing the displacement of hydraulic motor via changing its sloping cam plate's angle of inclination. If the accumulator pressure is large enough to start the bus, the bus will be started up solely by the accumulator. If the accumulator pressure is insufficient to start the bus, the fixed delivery pump is used to increase the accumulator pressure to a level sufficient to start the bus. Then the bus is solely started up by the accumulator. In this way, the engine always works in a good condition to save energy and reduce emissions.
The stability, energy efficiency and other characteristics of the system are studied based on analysis of mathematical models of the vehicle velocity control. The relevant controller is designed to study effects of the controller on system characteristics which is rarely seen in other documents.

![Figure 1. Energy Regeneration System of Series Hydraulic Hybrid Bus](image)

### 2. Transfer Functions for the Energy Regeneration System

#### 2.1. Transfer function of the electro-hydraulic servo valve

\[
G_f(s) = \frac{Q_f(s)}{I(s)} = \frac{K_f}{s^2 + \frac{2\xi\omega_f}{\omega_f^2} + 1}
\]

Of which: \(Q_f\): output flow of electro-hydraulic servo valve, m³/s; \(I\): input current of valve, A; \(\omega_f\): natural frequency of valve, rad/s; \(\xi\): damping ratio of valve; \(K_f\): flow gain of valve.

Relative to the controlled hydraulic system, the electro-hydraulic servo valve has a very high natural frequency, so it can be approximated to be a proportional component:

\[
G(s) = \frac{Q_f(s)}{I(s)} = K_f
\]

#### 2.2. Functional relationship between flow, pressure and displacement of hydraulic cylinder

\[
Q_s(s) = A_y Y(s) s + C_y P_s(s) + \frac{V_s}{4B_e} P_s(s)
\]

Of which \(Q_s\): flow into high-pressure chamber of hydraulic cylinder, m³/s; \(A_y\): effective action area of hydraulic cylinder, m²; \(Y\): piston displacement of hydraulic cylinder, m; \(P_s\): pressure difference between high and low pressure chambers of hydraulic cylinder, Pa; \(V_s\): total volume of two cylinder chambers, m³; \(B_e\): effective elastic modulus of oil volume, Pa; \(C_t\): total leak factor of hydraulic cylinder, m⁵/(N·s).

#### 2.3. Transfer function of pressure and displacement of hydraulic cylinder

\[
A_y P_s(s) = m_y Y(s) s^2 + B_y Y(s) s + k Y(s) + F_s(s)
\]

Of which: \(m_y\): piston mass of variable oil cylinder, kg; \(B_y\): damping factor of variable oil cylinder, N(m·s)^{-1}; \(k\): equivalent spring stiffness of variable oil cylinder, N/m; \(F_s\): force acting between piston and sloping cam plate, N.

The hydraulic cylinder is installed inside the shell of the variable delivery pump/motor. Both of its chambers have a small volume and thus the flow changes arising from volume compression is ignorable. In addition, the drag load of the variable delivery pump/motor has a very low frequency. Therefore, the hydraulic cylinder can be simplified into a proportional integral component.
2.4. Transfer function of variable delivery pump/motor

\[ G_m(s) = \frac{D_m(s)}{Y(s)} = \frac{D_{\text{max}}}{Y_{\text{max}}} \]  

Of which: \( D_m \): displacement of variable delivery pump/motor, m\(^3\)/rad; \( D_{\text{max}} \): Maximum displacement of variable delivery pump/motor, m\(^3\)/rad; \( Y_{\text{max}} \): maximum displacement of cylinder piston, m.

2.5. Analysis of Vehicle Running Resistance

Forces acting upon the bus when it is moving are as follows [9-10]:

\[ F = F_r + F_w + F_{F_{a}} + F_{F_{b}} \]  

Of which \( F \): running resistance of bus; \( F_r \): rolling resistance of bus; \( F_w \): aerodynamic resistance of bus, \( F_{F_{a}} \): grade resistance of bus; \( F_{F_{b}} \): acceleration resistance of gravity, \( m/s^2 \); \( \theta \): climbing angle of moving bus; \( f_{a} \): rolling resistance coefficient of bus; \( A \): frontal area of bus, \( m^2 \); \( C_d \): aerodynamic resistance coefficient; \( u \): velocity of bus, \( m/s \); \( \rho \): air density, \( kg/m^3 \).

2.6. Torque equilibrium equation of variable delivery pump/motor

\[ P_{j} D_{m}(s) = J_{2} \omega_{e}(s) s + B_{2} \omega_{e}(s) + M_{L}(s) \]  

Of which: \( P_{j} \): pump discharge pressure, Pa; \( J_{2} \): moment of inertia of variable delivery pump/motor and moment of inertia of vehicle mass at the output shaft of the variable delivery pump/motor, \( kg \cdot m^2 \); \( B_{2} \): rotational damping factor of variable delivery pump/motor, \( N\cdot s/m \); \( \omega_{e} \): output angular velocity of variable delivery pump/motor, rad/s; \( M_{L} \): load torque of variable delivery pump/motor, N·m.

\[ M_{L} = M_{L1} + M_{L2} \]  

Of which: \( M_{L1} \): sum of inertia load torque, frictional load torque and grade resistance torque, N·m. This paper mainly studies the braking performance of the regenerative braking system on a level and straight road and the rolling friction is very small at the time of braking. Thus this term is ignorable; \( M_{L2} \): aerodynamic resistance torque, N·m.

\[ M_{L2} = F_{w} R \theta = (\rho C_d A R / 2i)(\frac{d\theta(t)}{dt})^2 = K_{m}(\frac{d\theta(t)}{dt})^2 \]  

Of which, \( K_{m} = \rho C_d A R / 2i \).

2.7. Rotational speed sensor

\[ G_{s}(s) = \frac{U_{s}(s)}{n(s)} = K_{n} \]  

Of which: \( U_{s} \): feedback voltage signal of sensor, V; \( n \): rotational speed of variable delivery pump/motor, rad/s; \( K_{n} \): Rotational speed sensor gain.

2.8. Transfer Function of servo amplifier

\[ G_{a}(s) = \frac{\Delta I(s)}{U_{s}(s)} = K_{a} \]  

Of which: \( \Delta I \): output current of servo amplifier, A; \( K_{a} \): gain of servo amplifier.

3. Vehicle Velocity Control Model for the Energy Regeneration System

According to the foregoing analysis, the vehicle velocity control model for the energy regeneration system of the hydraulic hybrid vehicle is illustrated in Figure. 2. Open-loop transfer function \( G_{v}(s) \):

\[ G_{v}(s) = \frac{Y(s)}{Q_{v}(s)} = \frac{1}{A_{v} s} \]
Study on the Energy-Regeneration-based Velocity Control of the Hydraulic- .... (SONG Yunpu)

Figure 2. Vehicle Velocity Control Model for the Energy Regeneration System

\[ G_k(s) = \frac{K_v K_s D_{\text{max}} P_R}{A_y \gamma_{\text{max}} J_2 s^2 + A_y \gamma_{\text{max}} (B + K_m) s} \]  

(13)

Closed-loop transfer function \( G_b(s) \):

\[ G_b(s) = \frac{K_v K_s K_f D_{\text{max}} P_R}{A_y \gamma_{\text{max}} J_2 s^2 + A_y \gamma_{\text{max}} (B + K_m) s + K_c K_a K_f D_{\text{max}} P_R} \]  

(14)

Parameters in the model are selected, calculated and provided in Table 1.

Table 1. Parameters in the Vehicle Velocity Control Model for the Energy Regeneration System

<table>
<thead>
<tr>
<th>Notation</th>
<th>Variables</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_v )</td>
<td>Gain of servo amplifier</td>
<td>0.01</td>
</tr>
<tr>
<td>( K_f )</td>
<td>Flow gain of servo valve</td>
<td>( 4.16 \times 10^{-3} \text{m}^3/(\text{s} \cdot \text{A}) )</td>
</tr>
<tr>
<td>( A_y )</td>
<td>Effective action area of hydraulic cylinder</td>
<td>( 25 \times 10^{-6} \text{m}^2 )</td>
</tr>
<tr>
<td>( J_2 )</td>
<td>Total moment of inertia at the shaft of variable delivery pump/motor</td>
<td>0.1 Kg-m^2</td>
</tr>
<tr>
<td>( D_{\text{max}} )</td>
<td>Maximum displacement of variable delivery pump/motor</td>
<td>125 ml/r</td>
</tr>
<tr>
<td>( \gamma_{\text{max}} )</td>
<td>Maximum piston displacement of hydraulic cylinder</td>
<td>0.1 m</td>
</tr>
<tr>
<td>( B_2 )</td>
<td>Rotational damping factor of variable delivery pump/motor</td>
<td>0.2 N·m/(rad/s)</td>
</tr>
<tr>
<td>( P_0 )</td>
<td>Pressure of isobaric network</td>
<td>15 MPa</td>
</tr>
<tr>
<td>( \omega_r )</td>
<td>Natural frequency of servo valve</td>
<td>120 rad/s</td>
</tr>
<tr>
<td>( K_c )</td>
<td>Rotational speed sensor gain</td>
<td>0.191 V·s/rad</td>
</tr>
<tr>
<td>( \xi_e )</td>
<td>Damping ratio of servo valve</td>
<td>0.5</td>
</tr>
</tbody>
</table>

3.1. Stability Analysis

According to the closed-loop transfer function \( G_b(s) \) for the vehicle velocity control model of the energy regeneration system, the characteristic curve and the response curve as shown in Figures 3-6 can be generated in MATLAB/Simulink.

It is obvious from the characteristic curve that all poles of the system are located in the left half-plane of plane \( s \). The system is stable and has a certain stability allowance. The rise time of the system’s unit-step response is at 3.04s and the stable point is reached at 8s, representing a low response.

3.2. Stability Error or Anti-interference Analysis

In a steady closed-loop control system, with externally applied input signals and after a certain period of time (adjustment time) \( t_e \), the transient response component is attenuated to an ignorable degree, the output signal \( c(t) \) tends to be a steady-state component \( c_{ss}(t) \) and the error signal \( e(t) \) tends to be a steady-state value \( e_{ss}(t) \). We define steady-state error as a steady-state component of the error signal, or \( e_{ss}(t) = \lim_{t \to \infty} e(t) \), so the final value theorem of Laplace transform is used to solve for the steady-state error \( e_{ss}(t) = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s) \).

For convenience of analysis, steady-state errors of the system are divided between given steady-state error and disturbance steady-state error. The former represents an steady-
state error arising from given input signals, reflecting the accuracy of system; the latter
represents an error arising from disturbance signals, reflecting the anti-interference capacity of
system, or system rigidity.

As the model is simplified to a standard two-order system, parameters are substituted to
transform the transfer function as follows:

\[ G_p(s) = \frac{1}{0.0966(s + 0.723)(s + 14.3)} \]  

(15)

The open-loop transfer function is transformed as follows:

\[ G_i(s) = \frac{0.6879}{s(0.0664s + 1)} \]  

(16)

It is obvious that the open-loop gain \( K = 0.8139 \), time constant \( T = 0.0483 \) and the system
is an \( I \)-type system. The given steady-state error of the system is:

\[ e_{ss} = \lim_{s \to 0} \frac{sR(s)}{0.6879 + \lim s} \]  

(17)

As the system is an \( I \)-type system, for the step input signal \( r(t) = R \cdot 1(t) \), the steady-state error is 0; for the ramp input signal \( r(t) = R \cdot t \), its value is directly proportional to the input signal's rate of slope \( R \), \( e_{ss} = R/0.6879 \); for the acceleration input signal \( r(t) = R \cdot t^2 \), the steady-state error is \( \infty \).

In MATLAB/Simulink, an interference signal is added at the output shaft of the variable
delivery pump/motor for the vehicle velocity control model created for the energy regeneration
system. Then the simulation result is shown in the response chart in Figure 7.

According to the simulation results, the system is affected by one interference signal at
15s and resumes stability at 21s. It is obvious that the system is resistant to interference, but its
response time is long.
3.3. Energy Efficiency Analysis

The power simulation curve of the variable delivery pump/motor is shown in Figure 8. When the vehicle brakes, the variable delivery pump/motor works as a pump, the variable delivery pump/motor works as a pump and the area in the energy recovery power diagram where power is less than zero is the working range in which the variable delivery pump/motor recovers the vehicle inertia energy. The area formed by this part and the time axis represents the energy recovered by the variable delivery pump/motor of the system. At the early stage of braking, recovery power increases quickly and then decline gradually to zero.

4. Controller Design and System Simulation

The foregoing analysis shows that the system response speed is slow, so a PID controller is designed and added after the differential value between input and feedback signals of the vehicle velocity control model for the energy regeneration system [11-12], as shown in Figure 9.

Parameters of the PID controller are determined using the pole assignment method first. If the open-loop transfer function $G_k(s)$ of the system is expressed as follows:

$$G_k(s) = \frac{1}{as^2 + bs} \quad (18)$$

Assume the transfer function of the PID controller is $G_{PID}(s)$:

$$G_{PID}(s) = \frac{K_i + K_p s + K_d s^2}{s} \quad (19)$$

The pen-loop transfer function $G_{K1}(s)$ after PID controller is added is:

$$G_{K1}(s) = \frac{K_i + K_p s + K_d s^2}{as^2 + bs^2} \quad (20)$$

The closed-loop transfer function $G_{B1}(s)$ after PID controller is added is:

$$G_{B1}(s) = \frac{K_i + K_p s + K_d s^2}{as^3 + (b + K_d) s^2 + K_p s + K_i} \quad (21)$$
Assume that the closed-loop transfer function of the system is pole-assigned to consist of an inertia component \( \frac{1}{s^2 + 2\xi\omega_n s + \omega_n^2} \) of closed-loop non-dominant poles and a two-order oscillation component formed by a pair of dominant poles. Then the state equation after pole assignment is:

\[
a(s^2 + 2\xi\omega_n s + \omega_n^2)(s + p) = a(s^2 + (2\xi\omega_n + p)s^2 + (\omega_n^2 + 2\xi\omega_n p)s + p\omega_n^2)
\]

Then the following system of equations is generated:

\[
\begin{align*}
2\xi\omega_n + p &= \frac{b + K_p}{a} \\
\omega_n^2 + 2\xi\omega_n p &= \frac{K_p}{a} \\
p\omega_n^2 &= \frac{K_p}{a}
\end{align*}
\]

Preset the maximum overshoot \( M_p \) to be reached for pole assignment at 5% and adjusting time at 3s.

The formula for calculating the maximum overshoot is:

\[
M_p = \exp\left(-\frac{\xi\pi}{\sqrt{1-\xi^2}}\right)
\]

If a 2% tolerance is adopted, adjusting time \( t_a \) approximately equals four times the time constant of the system, expressed as follows:

\[
t_a = \frac{4}{\xi\omega_n}
\]

Calculation results are shown in Table 2.

<table>
<thead>
<tr>
<th>Design requirement</th>
<th>Result of calculation</th>
<th>PID parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. overshoot ( M_p )</td>
<td>0.05</td>
<td>damping ratio</td>
</tr>
<tr>
<td>Adjustment time ( t_a )</td>
<td>3s</td>
<td>natural frequency(rad/s)</td>
</tr>
<tr>
<td>Tolerance</td>
<td>2%</td>
<td>( K_d )</td>
</tr>
</tbody>
</table>

Table 2. Parameters Calculated Using the Pole Assignment Method

![Figure 10. Step Response Diagram of System with PID Control](image1)

![Figure 11. Power-Time Diagram of Variable Delivery Pump/Motor](image2)
PID parameters are substituted into the simulation model in Figure. 9 to generate simulation curves as shown in Figure. 10 and Figure 11. The figures show that, with the PID controller added, the vehicle braking time is shorter, the response speed is higher, the vehicle velocity stabilizes within 4s, the maximum overshoot $M_p$ of the system is controlled within 20% but the energy recovery efficiency declines.

5. Conclusion
The vehicle velocity control module of the energy regeneration system of the hydraulic hybrid vehicle that consists of the hydraulic accumulator and the reversible variable delivery pump/motor is stable and able to recovery the inertia energy generated in vehicle braking. After the PID controller intended to improve response speed is added, system response becomes quicker but energy recovery rate declines. Therefore, it should be used according to needs or a different controller should be developed.

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References