The Performance of Synchronization Algorithm in Real-time OFDM-PON System

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Abstract
In OFDM-PON system, the synchronization signal is very important. We use synchronization signal to get a whole OFDM signal. Due to the real-time system, we need to get the fast change signal. So get a true signal is very important and very hard. That is to say, we need to design an excellent algorithm to get finish this demand. This is another point of this project. In the paper, the author designed a new algorithm to solve the problems. we use some simulations to compare the performances of different algorithms to get the further understanding. In the experiment, we use more detail algorithms to estimate if it is useful in real-time OFDM-PON system. When the SNR is 10, we can get the synchronization signal correctly.

Key words: OFDM, real-time system, algorithm, channel estimation

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1. Introduction
Orthogonal frequency-division multiplexing (OFDM) is a bandwidth-efficient signaling scheme for wideband digital communications. The main difference between traditional frequency-division multiplexing (FDM) and OFDM is that in OFDM, the spectra of individual carriers overlap. Nevertheless, the OFDM carries exhibit orthogonality on a symbol interval if they are spaced in frequency exactly at the reciprocal of the symbol interval. Both modulation and multiplexing are achieved digitally using an inverse fast Fourier transform (IFFT), and demodulation and demultiplexing are performed by a fast Fourier transform (FFT). With the development of modern digital signal processing technology, OFDM has become practical to implement and has been proposed as an efficient modulation scheme for applications ranging from modems, digital audio broadcast, to next-generation high-speed wireless data communications.

One of the principal advantages of OFDM is that it effectively converts a frequency-selective fading channel into a set of parallel flat-fading channels. Both the intersymbol interference (ISI) and intercarrier interference (ICI) can be eliminated completely by inserting between symbols a small time interval known as a guard interval. The length of the guard interval is made equal to or greater than the delay spread of the channel. If the symbol signal waveform is extended periodically into the guard interval (cyclic prefix), orthogonality of the carrier is maintained over the symbols do not overlap due to the guard interval. Hence at the receiver there is no need to perform channel equalization, and the complexity of the receiver is quite low [1].

One of the major disadvantages of OFDM is that it is quite sensitive to frequency offset and phase noise. This is because the spectrum of an individual OFDM subcarrier has a “sinc” form, and each carrier has significant sidelobes over a frequency range which includes many other subcarriers [2].

2. Channel estimation
OFDM is widely used in audio broadcasting, digital video broadcasting, and broadband local area networks [3]. In OFDM systems, the input high-rate data stream is divided into many low-rate streams that are transmitted in parallel, thereby increasing the
symbol duration and reducing ISI. OFDM receivers have two main types, in general, one is differential detection, and the other one is coherent detection.

Differential detection can avoid estimating channels and simplify the structures of the receivers. The reason is that channel information is covered in neighboring symbols. But at the same time, it has a big disadvantage that it makes noise has 3db increasing, and it has no chance to use multi-level modulation which has a high spectrum efficiency. Coherent detection can use multi-level modulation, but receivers must have the knowledge of channel status instantaneously. That is to say, system receivers must have a channel estimation to get the information on transmission instantaneously.

In general, there are two different types of channel parameter estimator: i) blind and ii) non-blind. Blind channel estimation techniques [4-5], try to estimate the channel without any knowledge of transmitted data. They are attractive because of the possible saving in training overhead, however they are effective only when a large amount of data can be collected. However, its algorithm is complex, convergent slowly. It needs large number of data to estimate channel. This is clearly a disadvantage in the case of wireless systems because of the time-varying nature of the channel.

Non-blind channel estimation has two types as well. One of the channel estimations is based on training sequence by sending training block-type pilot. In block-type pilot based channel estimation, OFDM channel estimation symbols are transmitted periodically, in which all sub-carriers are used as pilots. If the channel is constant during the block, there will be no channel estimation error since the pilots are sent at all carriers. The estimation can be performed by using either LS or MMSE [6-7]. Since the decision feedback equalizer has to assume that the decisions are correct, the fast fading channel will cause the complete loss of estimated channel parameters. Therefore, as the channel fading becomes faster, there happens to be a compromise between the estimation error due to the interpolation and the error due to loss of channel tracking. For fast fading channels, the comb-type based channel estimation performs much better. This method is often used in burst mode transmission systems. Another type is pilot-aided channel estimation which is the other approach in which sending training sequence consisting of known data symbols is transmitted at the beginning of a session or multiplexed into the user data stream at a later stage, and the initial estimation of the channel parameters is performed using the received pilot signal. Then receivers use many kinds of techniques like interpolation or filtering to finish sub-carriers channel estimations. This method is widely used in continuous transmission communications systems.

As previously discussed, the multicarrier modulation is also sensitive to frequency offset and phase noise. Frequency offset and phase noise cause loss of orthogonality among subcarriers, and they consequently introduce inter-carrier interference (ICI). The effect of phase noise has been analyzed in many papers [8-10]. Though it is impossible to estimate random phase noise, frequency offset estimation can be achieved by using pilot signals. It has been shown that the pilot-aided channel estimation is the optimum way to estimate the channel when signal-to-noise ratio (SNR) is sufficient high [11]. As pilot-aided methods can cause loss of bandwidth efficiency, non-pilot-aided frequency offset estimation has been used. The cyclic prefix (CP) based method, is quite attractive among non-pilot-aided approaches due to its simplicity. Nevertheless, the accuracy of the CP-based method could not be guaranteed for multipath fading channels. The method of solving this problem is to consider the channel impulse response (CIR) length.

By studying many literatures, in engineering, the method based on non-blind by using coherent detections is widely used. Channel estimations based on it mainly focuses on three points.
- The distribution and selection of pilot symbols
- The information in the pilot symbols' locations.
- Recovery the information in data locations by using known channel information.

System description is shown in Figure 1. The OFDM system based on pilot channel estimation is given in Figure 1. The binary information is first grouped and mapped according to the modulation in “signal mapper”. After inserting pilots either to all sub-carriers with a specific period or uniformly between the information data sequence, IDFT block is used to transform the data sequence of length $N X(k)$ into time domain signal $x(n)$ with the following equation:
where $N$ is the DFT length. Following IDFT block, guard time, which is chosen to be larger than the expected delay spread, is inserted to prevent inter-symbol interference. This guard time includes the cyclically extended part of OFDM symbol in order to eliminate inter-carrier interference (ICI). The resultant OFDM symbol is given as follows:

$$\begin{align*}
    x_f(n) &= \begin{cases} 
        x(N+n), n = -N_g, -N_g + 1, \ldots, -1 \\
        x(n), n = 0, 1, \ldots, N - 1 
    \end{cases} 
\end{align*}$$

(2)

where $N_g$ is the length of the guard interval.

The transmitted signal $x_f(n)$ will pass through the frequency selective varying channel with additive noise. The received signal is given by:

$$y_f(n) = x_f(n) \ast h(n) + w(n)$$

(3)

$$h(n) = \sum_{i=0}^{r-1} h_i e^{-j 2\pi m_i n} \delta(\lambda - \tau_i), 0 \leq n \leq N - 1$$

(4)

Where $r$ is the total number of propagation paths, $h_i$ is the complex impulse response of the $i^{th}$ path, $f_i$ is the $i^{th}$ path Doppler frequency shift, $T$ is the sample period. At the receiver, after passing to discrete domain through A/D and low pass filter, guard time is removed:

$$y(n) = y_f(n + N_g), n = 0, 1, \ldots, N - 1$$

(5)

Then $y(n)$ is sent to DFT block for the following operation:

$$Y(k) = DFT\{y(n)\} = \frac{1}{N} \sum_{n=0}^{N-1} y(n) e^{-j 2\pi n k / N}, k = 0, 1, \ldots, N - 1$$

(6)

Assuming there is no II, so the relation of the resulting $Y(k)$ to $H(k)$=DFT($h(n)$), $I(k)$ that is ICI because of Doppler frequency and $W(k)$=DFT($w(n)$), with the following equation:

$$Y(k) = X(k)H(k) + I(k) + W(k), k = 0, 1, \ldots, N - 1$$

(7)

Where
Following DFT block, the pilot signals are extracted and the estimated channel $H_{\text{e}(k)}$ for the data sub-channels is obtained in channel estimation block. Then the transmitted data is estimated by:

$$X_{e} = \frac{Y(k)}{H_{e}(k)}, \quad k = 0, 1, \ldots, N - 1$$

In the following channel estimation block, we can get the whole channel estimation by using interpolation and filter. At last, the binary information data is obtained by demultiplicating estimated information.

3. The selection of pilot signals

In general, the distance between neighboring pilots is the closer the better with the goal of getting the channel status change in time and frequency domains. At the same time, pilots can take the bandwidth. That is to say, we need to make neighboring pilots' distribution dispersed in order to improve the transmission speed. So we need to consider these two questions. With the goal to adapt the change of the channel, the lowest frequency is decided by Nyquist sample theorem [13]. There are three kinds of pilot distributions: block-type, comb-type, two-dimension.

Assuming that in OFDM system, the maximum Doppler shift is $f_{D_{\text{max}}}$, the maximal multi-path delay is $\tau_{\text{max}}$, and frequency interval between each sub-carrier is $S$. Symbol period with CP in OFDM system is $\frac{\Delta f}{f_{c}}$. $T_{s}$ is the sample period, $G$ is the length of CP. If the channel width is $f_{D_{\text{max}}} T_{s}$ and $\tau_{\text{max}}$ $N_{r}$, according to sample theorem, $T_{s}$ and $T_{F}$ must be obey these two equations:

$$f_{D_{\text{max}}} T_{s} N_{r} \leq \frac{1}{2}, \quad \tau_{\text{max}} \Delta f T_{F} \leq \frac{1}{2}$$

![Figure 2. Pilot application](image)

- The block-type pilot: Pilots are inserted periodically in time domain, and they are used each sub-carriers in frequency domain. It does not need to interpolate in frequency domain. We just need to estimate in the head once, and then the following symbols use the estimation. This method is mainly used in slow fading channels.
- The comb-type pilot: Pilots are distributed in frequency domain in the same interval, and they are continuous in time domain. We just need to use a small number of pilots to
estimate each OFDM symbol. The estimation is continuous in each symbol. This distribution has a strong tracking ability in fast fading channel. It is used in fast fading channel. Pilots are discrete in frequency domain. We can use interpolation to get transmission function in sub-carriers.

- The two-dimension-type pilot: Pilots are mesh distributed, and estimator must save many OFDM symbols before estimating channel. In this way, the use ratio of spectrum is high, but we cannot get the instantaneous information.

4. Some classical estimation algorithms

4.1 The minimum mean square error algorithm

LS algorithm is easy to be interfered by white Gaussian noise and ICI. So its accuracy is limited. However, the accuracy of MMSE is greatly improved than LS because MMSE considers the noise as a fact.

Assuming that channel estimation's error is:

\[ e = H - \hat{H} \]  

So mean square error (MSE) of the channel is

\[ E\left( e^2 \right) = E\left( \left( H - \hat{H} \right)^2 \right) = E\left( \left( h - \hat{h} \right)\left( h - \hat{h} \right)^H \right) \]  

Under the premise of signal independent of noise covariance, we can get the optimal estimation when channel transmission function is mean-square-error.

\[ \hat{R}_{HH} = R_{HH} + \sigma^2 (X^H X)^{-1} \hat{H}_{LS} \]  

Minimum mean-square-error (MMSE) estimator is an optimal filter which meets the requirement of MSE. It has a better estimated accuracy, when MSE and LS have the same value, MMSE estimator's SNR has a larger gain about 10-15 dB. MMSE estimator has a big disadvantage, as well. Its algorithm is complex, and with the increase in the number of points, the algorithm complex rate is increasing by exponential. At the same time, we need to know the second order statistics of the channel as prior information. Its disadvantage restricts its application.

4.2 The linear minimum mean square error algorithm

In MMSE algorithm, we need to calculate the matrix inversion. If we estimate each channel by calculating the matrix inversion, the complex rate is high and it is hard to realize MMSE's superiority. An easy way to solve this problem is to use same pilots with same location. We can use average SNR to take the place of inverse pilots. So we get LMMSE estimator equation:

\[ \hat{H}_{LMMSE} = R_{HH} \left( R_{HH} + \frac{\beta}{SNR} \right)^{-1} \hat{H}_{LS} \]  

Where \( \beta = E\left( |x|^2 \right) E\left( \frac{1}{|x|^2} \right) \) is a constant decided by modulated signal's constellation. When the baseband code's shine upon 16QAM constellation, \( \beta = 17/9 \), and \( SNR = \frac{E\left( |x|^2 \right)}{\delta^2} \) which is an average value.

5. Improved algorithms based on MMSE

5.1 Filter separation algorithm

When the pilots' distribution is 2-dimensional, the optimal interpolation is 2-dimensional wiener filtering based on MMSE [13-14]. It is hard to realize this kind of filter due to its high complexity. So we can use two filters to make up for its disadvantage. These two filters are connected cascading. One is the frequency domain filter and the other is time domain filter. They use their own domain's information to filter signal. Their performance is the same to 2-
5.2 SVD algorithm

We can use singular vector to do low-order approximation. The channel auto covariance can be resolved [15]:

\[ R_{HH} = U \Lambda U^H, \Lambda = \text{diag}(\lambda_1, \lambda_2, ..., \lambda_\text{N-1}) \]

is a diagonal matrix that \( R_{HH} \)'s proper value \( \lambda_i \) is listed from small until big. So we can simplify MMSE algorithm:

\[
H_{\text{MMSE}} \approx U \Lambda U^H H_{LS}, \Delta = \Lambda (\Lambda + \frac{\beta}{\text{SNR}})^{-1}
\]

(14)

We can get the estimation in piloted place of sub-carrier:

\[
H_{\text{SVD}} \approx U \begin{pmatrix} \Delta^{-1} & 0 \\ 0 & 0 \end{pmatrix} U^H H_{LS}
\]

(15)

\( M \) is equal to the length of CP. As the signal spectrum is focused on low frequency mainly, we need to put \( M \geq L \), \( L \) is the sample number when delay is maximal.

5.3 Windowed DFT algorithm

We can use windowed DFT algorithm in time domain of senders and receivers or frequency domain of receivers. The frequency domain data windowing is used to reduce the aliasing errors for the interpolation case and get better noise filtering performance for the non-interpolation case. The time domain MMSE weighting is also used to suppress the channel noise for both cases [16]. In senders’ time domain, windowed DFT algorithm can make power outside the band lower, and make frequency spectrum’s border cliffy [17]. In receivers’ time domain, windowed DFT algorithm can lower the power of ICI, and improve the rate of useful signal and noise. A windowed DFT based pilot-symbol-aided channel estimator that can eliminate in the multipath fading channels with the non-sample-spaced path time delays.

Among Rectangle window, Hamming window and Hanning window, the Hanning windowed DFT based channel estimator had the best performance when compared with the more complex MMSE channel estimator. The performance of the Hanning windowed DFT based channel estimator has a very small degradation.

6. Approximate algorithms

6.1 Interpolation algorithm

- Linear interpolation algorithm
  
  It is simple and easy to realize the algorithm. But, the great disadvantage of it is that it is not accuracy to get the channel estimation in the fast-fading channels when pilots distributed separately.

- Second order interpolation algorithm
  
  Its error is smaller than linear interpolation algorithm, because the channel information in one point is a sum of three neighboring points.

- Cubic spline interpolation algorithm
  
  It is a high order interpolation and it can decrease the noise threshold of OFDM systems. But, the structure of its cubic multinomial is complex and calculated quantity is high.

6.2 Polynomial fitting algorithm

It is well known that if the correlation function of channel response is known, we can get the MMSE estimation by using the singular value decomposition of the correlation matrix. However, in practice such knowledge is usually not available and the channel statistics may vary by time. We are trying to design an estimation scheme under the condition that the channel statistics are not known or not completely known.

One such scheme is proposed in [18] . The channel estimation algorithm based on polynomial approximation of the channel parameters and it does not need to know the channel statistics. The method exploits both the time and frequency correlation of the channel parameters. The estimator is robust and needs a little prior knowledge about the delay and
fading properties of the channel. It can even adjust itself to follow the variation of the channel statistics.

7. Simulation and Conclusion
7.1 Classical algorithms
Assuming that each channel is non-interval sampling, and multipath delay is [0.3, 2.5]. Figure 3, 4 are MSE and SER performance comparison of LS and MMSE estimation algorithm. We do this simulation and comparison under the assuming that all data are BPSK modulation. MMSE algorithm’s performance is superior to LS algorithm under small SNR. When SNR is increasing, both mean-square values are approaching, but MMSE’s BER is smaller than LS’s.

![Figure 3. MSE performance comparison of LS and MMSE estimation algorithms](image1)
![Figure 4. SER performance comparison of LS and MMSE estimation algorithms](image2)

7.2 Interpolation algorithms
Assuming that each is non-interval sampling. In order to find the performance of different interpolation algorithm, we do this simulation under two different time delay conditions: [0.3, 2.5], [0.5, 5.5]. Figure 5 and 6 are MSE performance comparison of different interpolation algorithms under small and great delays. We can conclude that the accuracy of these interpolations are decreasing from: Cubic spline interpolation, Gaussian interpolation, time-domain interpolation, linear interpolation. When channel frequency response is changing fast, all interpolations’ accuracy decrease fast.

![Figure 5. MSE performance comparison of different interpolation algorithms(small delay)](image3)
![Figure 6. MSE performance comparison of different interpolation algorithms(great delay)](image4)
7.3 Windowed DFT algorithms [19]

Assuming that the channel consists of 5 independent Rayleigh fading paths with an exponential power delay profile. We also assume that the channel has a Doppler frequency shift of 50Hz. The path time delays are \{0, 3.5T, 6.5T, 9.5T, 12.5T\}. Throughout, a 16QAM-OFDM system is considered with N = 1024 sub-carriers and a bandwidth of 5MHz operating in the 2.4GHz frequency band. Thus, the effective symbol length is 205us and the sample period is 0.2 us. In the simulation, we assume that the synchronization is perfect. Figure 7 and 8 are MSE andSER performance of the windowed DFT based channel estimations with different window functions (Rectangle, Hamming, Hanning).

![Figure 7. MSE performance of the windowed DFT based channel estimators with different window functions](image1)

![Figure 8. SER performance of the windowed DFT based channel estimators with different window functions](image2)

Among the three windowed DFT based channel estimators, the Hanning windowed DFT based channel estimator has the best performance. When compared with the more complex MMSE channel estimator, the performance of the Hanning windowed DFT based channel estimator has a very small degradation.

7.4 Conclusion [20-21]

We use Matlab to simulate the designed algorithm, suppose that the channel is mixed with Gaussian white noise like figure 9 shown. And figure 10 shows the biggest threshold is when the OFDM signal begins. Figure 11 and 12 is the best proving on FPGA.
board with the former and post simulations are the same.

![Figure 9. 512 bit OFDM Signal with Gaussian white noise](image1)

![Figure 10. The biggest value means the OFDM signal begins](image2)

In OFDM-PON system, the synchronization signal is very important. We use synchronization signal to get a whole OFDM signal. Due to the real-time system, we need to get the fast change signal. So get a true signal is very important and very hard. That is to say, we need to design an excellent algorithm to get finish this demand. This is another point of this project. In the paper, the author designed a new algorithm to solve the problems. When the SNR is 10, we can get the synchronization signal correctly.
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