Adaptive Integrated Navigation Filtering Based on Accelerometer Calibration

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Abstract
In this paper, a novel GPS (Global Positioning System) and DR (Dead Reckoning) system which was based on the accelerometer and gyroscope integrated system was designed and implemented. In this system, the odometer used in traditional DR system was replaced by a MEMS tri-axis accelerometer in order to decrease the cost and the volume of the system. The system was integrated by the Kalman filter and a new mathematical model was introduced. In order to reasonably use the GPS information, an adaptive algorithm based on single measurement system which could estimate the measurement noise covariance was obtained. On the purpose of reducing the effect of the accumulated error caused by drift and bias of accelerometer, the accelerometer was calibrated online when GPS performed well. In this way, the integrated system could not only obtain the high-precision positioning in real time, but also perform stably in practice.

Keywords: GPS; DR; dynamic calibration algorithm; adaptive Kalman filtering.

1. Introduction
The global positioning system (GPS) could provide users with all-weather, real-time and high-accuracy positioning information. GPS is widely used in the vehicle navigation and positioning system because of its high stability and low cost [1]. However, the GPS signal is easily blocked when the vehicle runs into underpass or is sheltered by trees and tall buildings, and the receiver could not obtain accurate positioning information. In the other hand, the dead reckoning (DR) system has the features that it is not affected by the circumstance but the error is accumulated in the using process because of the drift and bias of the inertial sensors. Therefore, considering both the advantages and disadvantages of the two systems, the GPS and DR systems are integrated together in order to use their complementary features rationally to acquire the stable and reliable positioning system.

The traditional DR system is based on the gyroscope and the odometer [2]. However, in the condition that the odometer is destroyed and the cost of replacing is high, the DR system will operate abnormally. In order to overcome this problem, the DR system which is based on the gyroscope and accelerometer is proposed. But the drift and bias of the accelerometer will result in great accumulated error of the DR inertial system, so the accurate calibration of accelerometer is essential. The existed calibration methods of accelerometer are usually off-line and the results are static value. But in practice, the bias of accelerometer will change due to various temperature and load voltage. Therefore, in this paper, a novel approach of dynamic calibration of accelerometer is proposed to solve the above problems.

GPS/DR integrated navigation system usually adopts Kalman filtering as information fusion method [3]. In the Standard Kalman filtering, the statistical characteristics of process noise and measurement noise are known and fixed. But in practical applications, it is hard to obtain the accurate measurement noise statistics, and at the same time the characteristic of measurement noise may change in the working process. In order to obtain high precision results of the integrated system, an adaptive estimation approach of measurement noise covariance is needed. In this field, A.P. Sage and G.W. Husa proposed a weighting method of adaptively updating the measurement noise covariance [4]; Mehr put forward an approximation method based on the new information sequences [5]; Yang Yuanxi presented a robust adaptive filtering algorithm which could adaptively distribute weights between observation information and the model information [6]. However, the above-mentioned methods might bring in the influence of...
process noise, which would result in great impact when the system model is inaccurate. Hai Zhang [7] put forward a measurement noise estimation method based on different measurement systems; and Yongna Chang [8] presented a novel dynamical algorithm based on one-point observation information. However, the two methods could only estimate the specific variables which have at least two different measurement systems. To solve these problems, this paper proposed an adaptive estimation method based on the single measurement system which efficiently isolated the disturbance of process noise and could estimate every measurement variable in the filtering system. As a result, the measurement noise covariance could be estimated accurately and improve the precision of the adaptive Kalman filtering.

The paper is organized as follows. Section 2.1 presents the global design of GPS/DR integrated system. In Section 2.2 the mathematical model of the integrated system is introduced and in Section 2.3 the adaptive estimation of measurement noise covariance is put forward. Section 2.4 proposes a novel approach of dynamical calibration of accelerometer and explains how to calculate the parameters in practice. Section 3 simulates the integrated navigation system using the method. Finally, Section 4 provides the conclusion.

2. Research Method
2.1 The global design of GPS/DR integrated navigation system

The traditional DR system is based on the gyroscope and the odometer. However, in the condition that the odometer is destroyed and the cost of replacing is high, the DR system will operate abnormally. In order to overcome this problem, the DR system which is based on the gyroscope and accelerometer is designed and implemented. In this system, the odometer used in traditional DR system was replaced by a MEMS tri-axis accelerometer in order to decrease the cost and the volume of the system.

![Flow chart of GPS/DR integrated system](image-url)

Figure 1. Flow chart of GPS/DR integrated system
When the GPS receiver receives signals in good condition, the DR system as fault monitoring system assists GPS to realize the integrated system positioning. However, when the GPS output data are complete but inaccurate, it will adopt the adaptively estimating measurement noise method to make the best of the GPS data through adjusting the weights between measurement and model information. In addition, when there is no output of GPS receiver, the inertial recursion of DR system would be selected to achieve navigation calculation. The global design makes the integrated navigation system complete and robust enough to overcome every sudden circumstance. The flow chart is shown in Figure 1.

2.2 Mathematical model of GPS/DR system
2.2.1 System state equation

In this paper, DR system is based on the combination of gyroscopes and three-axis accelerometer. In the GPS/DR integrated system, the system state vector $X$ can be defined as,

$$X = [N \ E \ \psi \ v \ a \ \dot{\psi} \ B \ D]^T \quad (1)$$

Where $N$、$E$ denote the estimation of north position and east position; $\psi$、$v$、$a$ denote the heading angle, velocity, acceleration and heading angle rate; $B$、$D$ respectively denote the bias of gyroscope and accelerometer.

The state equation of system is described as,

$$\begin{align*}
N(k) &= N(k-1) + T \cdot v(k) \cdot \cos(\psi(k)) + w_1(k) \\
E(k) &= E(k-1) + T \cdot v(k) \cdot \sin(\psi(k)) + w_2(k) \\
\psi(k) &= \psi(k-1) + T \cdot \dot{\psi}(k) + w_3(k) \\
v(k) &= v(k-1) + T \cdot a(k) + w_4(k) \\
a(k) &= a(k-1) - D(k) + w_5(k) \\
\dot{\psi}(k) &= \dot{\psi}(k-1) + B(k) + w_6(k) \\
B(k) &= B(k-1) + w_7(k) \\
D(k) &= D(k-1) + w_8(k)
\end{align*} \quad (2)$$

where $T$ denotes the filtering cycle, and is taken as 1 second in system.

According to the motion characteristics of the vehicle, nonlinear item exists in the state transition equation. Therefore, the Extended Kalman Filter (EKF) is used to linearize the state equation. The linearized equation is as follows,

$$X(k+1) = \Phi(k+1,k)X(k) + W(k) \quad (3)$$

$$\Phi(k+1,k) = \begin{bmatrix}
1 & 0 & -T \cdot v(k) \cdot \sin(\psi(k)) & T \cdot \cos(\psi(k)) & 0 & 0 \\
1 & T \cdot v(k) \cdot \cos(\psi(k)) & T \cdot \sin(\psi(k)) & 0 & 0 \\
1 & 0 & 0 & T & 0 \\
1 & 0 & 0 & 0 & T \\
1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0
\end{bmatrix} \quad (4)$$

The system noise $W$ is defined as,
\[ W = \left[ w_N \ w_E \ w_{\psi} \ w_v \ w_a \ w_{\psi} \ w_B \ w_{D} \right]^T \]  

(5)

2.2.2 System measurement equation

The system measurement vector can be defined as:

\[ Z = \left[ N_{\text{gps}} \ E_{\text{gps}} \ \psi_{\text{gps}} \ v_{\text{gps}} \ a_{\text{acc}} \ \psi_{\text{gyro}} \right]^T \]  

(6)

Where \( N_{\text{gps}} \), \( E_{\text{gps}} \) denote the GPS output of north position and east position; \( \psi_{\text{gps}} \), \( v_{\text{gps}} \) denote the GPS output of heading angle and velocity; \( a_{\text{acc}} \) denotes the output of the accelerometer and \( \psi_{\text{gyro}} \) denotes the output of the gyroscope.

Then, the system measurement equation is obtained as follows,

\[ Z(k) = H(k)X(k) + V(k) \]  

(7)

Where \( Z(k) \) denotes measurement vector, \( H(k) \) is measurement matrix, and \( V(k) \) is the measurement noise vector.

\[ H = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \]  

(8)

\[ V = \left[ v_N \ v_E \ v_{\psi} \ v_v \ v_a \ v_{\psi} \right]^T \]  

(9)

2.3 The measurement noise covariance estimation

2.3.1 The description of the algorithm

Through analyzing the measurement equation, some convertible relation is found between state and measurement variables. Because that the measurement noises are uncorrelated in different time, the difference sequence at \( k+1 \) contains \( V(k) \) and \( V(k+1) \).

Therefore, when constructing the pseudo measurement sequence, the value at \( k \) should be set as \( \hat{X}(k | k-1) \). After one-step state transfer, the state estimation at \( k+1 \) is obtained as \( \hat{X}(k+1 | k-1) \). The pseudo measurement sequence is structured as follows,

\[ Z_p(k+1) = H(k+1)\Phi(k+1,k)\hat{X}(k | k-1) \]  

(10)

Similarly to the condition of two measurement system, the measurement noise covariance estimation of the real measurement system could be calculated as described,

\[ \text{Var}[V_i(k)] = \frac{E\left\{ [\Delta Z_p(k) - \Delta Z_r(k)]^T [\Delta Z_p(k) - \Delta Z_r(k)] \right\}}{2} \]  

(11)

Where \( \Delta Z_p(k) = Z_p(k) - Z_p(k-1) \), \( \Delta Z_r(k) = Z_r(k) - Z_r(k-1) \). The proof process is represented in [9].
2.3.2 The realization of the adaptive filtering algorithm

The adaptive Kalman filtering formula is expressed as,

\[
\hat{X}(k+1|k) = \hat{X}(k+1|k) + K(k+1)[Z(k+1) - H(k+1)\hat{X}(k+1|k)]
\]

\[
\hat{X}(k+1|k) = \Phi(k+1,k)\hat{X}(k|k)
\]

\[
K(k+1) = P(k+1|k)H^T(k+1)[H(k+1)P(k+1|k)H^T(k+1) + R_{k+1}]^{-1}
\]

\[
P(k+1|k+1) = P(k+1|k) - P(k+1|k)H^T(k+1)[H(k+1)P(k+1|k)H^T(k+1) + R_{k+1}]^{-1} \cdot H(k+1)P(k+1|k)
\]

In practical calculation, the statistics could not be directly obtained. Therefore, some approximate methods need to be used.

2.3.2.1 The selection of the data window size.

Whether the estimation is accurate or not, the crucial factor is the window size of data which is chosen to calculate the statistics. For the sequence which is identically distributed, a long window will increase the reliability of the statistic results. But as to the sequence whose distribution characteristic changes along the time, the long window size includes so much historical information so that the statistical result could not track the rapidly varying data in real time.

In practice the measurement noise fluctuates in some scope in normal situation but sometimes may change a lot by sudden disturbance. Therefore, it is considered that adaptively adjusting the window size instead of a fixed value to improve the accuracy under various circumstances.

\[
M = \begin{cases} 
  a & \varepsilon \leq \Delta_0 \\
  a - \frac{b}{f(\varepsilon)} & \varepsilon > \Delta_0 
\end{cases}
\]

\[
\varepsilon = R(k) - R(k-1)
\]

Where \( M \) denotes the size of data window, and \( a \) denotes a constant which depends on accuracy level, \( \Delta_0 \) is a difference threshold.

2.3.2.2 The weighted sliding-window statistical calculation.

Through the continuous adaptive sliding-window sample, the approximate estimation of the statistics is obtained. However, instead of changing so rapidly, the statistics vary more slowly in real systems. To overcome this problem, an exponential weighting method is proposed. The calculation equation of \( R \) is described as,

\[
\hat{R}(k) = \begin{cases} 
  \frac{1}{M+1} & \text{for } 0 < b < 1 \\
  \sum_{i=k-M+1}^{k} [C(i) - \frac{1}{M} \sum_{i=k-M+1}^{k} C(i)]^2 / 2 & k > M \\
  (1-d_k)\hat{R}(k-1) + d_k\hat{R}(k)
\end{cases}
\]

where \( b \) is named as the forgetting factor.
2.3.2.3 The outlier elimination.

Because the local-window estimation is based on a small sample, the existence of outliers would result in great errors. In order to make sure the estimation accuracy, the criterion of the outliers is designed as,

\[
C(i) - \frac{1}{M} \sum_{i=k-M+1}^{k} C(i) \mid > \alpha \sqrt{\frac{1}{M-1} \sum_{i=k-M+1}^{k} [C(i) - E(k)]^2} \quad k - M + 1 \leq i \leq k
\]

where \(i\) denotes the current time, \(M\) presents the window size and \(\alpha\) is defined as eliminating factor, which is normally set between 2.5 to 5. When \(C(i)\) matches the equation (16), it would be treated as outlier to be eliminated.

2.4 Accelerometer Calibration in the DR inertial recursive

In the process of inertial recursive, the error of inertial devices is accumulated and will cause the positioning result far from the true value in long term. The major error of accelerometer and gyroscope is from bias and random drift error, and the random drift error could be basically eliminated through smooth method. However, although the datasheet of the sensor would give the reference value of bias, its true value will change due to the difference of temperature, load voltage and vibration. Therefore, a rational dynamic calibration approach is needed to estimate bias and correct the output effectively.

The basic calibration idea for gyroscope and accelerometer is similar. Here only the accelerometer calibration method is illustrated as an example. Due to the lack of odometer’s data, the velocity information of inertial system would be calculated entirely by the integral of acceleration. If is installed in strict accordance with the provision that ensuring one of the axial coinciding to the forward direction of the vehicle, the three-axial accelerometer would be used as one-axial. The sky-axial accelerometer is used to monitor if the road surface is flat or pitched.

When the vehicle is static, the bias could be estimated by the method of averaging as follows,

\[
\text{bias}_{\text{acc}} = \frac{\sum_{i=k-n}^{k} a_{\text{acc}}(i)}{n} \quad n = 30 \sim 50
\]

![Figure 2. Comparison of accelerated velocity](image-url)
Through the analysis of real data, it is found that the bias of accelerometer would change after long hours of work. Therefore, it is necessary to design a dynamic calibration approach which is both effective and easy to implement.

Hence, an accelerometer calibration method based on the least-square criteria is proposed. This method employs the measurement reliable GPS velocity difference to calculate the scale factor and the bias of the accelerometer, where the so-called reliable velocity difference means the difference of GPS velocity while the GPS receiver performs well and the vehicle continuously drives for 30 to 50 seconds. According to the comparison of the curves between GPS velocity difference and accelerometer output, it could be found that they have linear relation in some extent, as shown in Figure 2.

Through the validation of the linear relation, it could be assumed that the acceleration satisfies the following equation:

\[ a_{\text{real}} = K \cdot a_i - a_{\text{bias}} \]  

(18)

Where \( K \) denotes the scale factor of accelerometer, \( a_{\text{bias}} \) denotes the bias of accelerometer, and \( a_i \) denotes the output of accelerometer at time \( i \).

According to the least-square criteria, the following equation could be obtained:

\[ J = \sum_{i=1}^{m} \left[ v_{\text{GPS}} - (K \cdot a_i - a_{\text{bias}}) \right]^2 = \min \]  

(19)

If the proper parameters are calculated to make \( J \) least, the accurate estimation of scale factor and bias could be obtained.

Tab.1 is the operation results of calibration based on some reliable difference of GPS velocity. The left two columns are the calibration results when the vehicle is static, and the right two columns are the calibration results when the vehicle is moving. Through analyzing the data, the result of the approach is effective and accords to the characteristic of the actual accelerometer.

<table>
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<tr>
<th>Scale factor</th>
<th>bias</th>
<th>Scale factor</th>
<th>Bias</th>
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<td>0.153117</td>
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3. Results and Discussion
3.1 The simulation of the measurement noise covariance estimation
3.1.1 Simulation conditions.

In this simulation, the system errors are chosen as the state variables, and the real values of the measurement noise covariance are shown as black line in Figure 3. As the compared algorithms, Sage-Husa adaptive estimation, MAKF are selected to implement the experiment under the same situation. The yellow line shows the method proposed in the paper.

3.1.2 Analysis of results.

During the two periods from 1000s to 2200s and from 4300s to 5700s, the GPS signal fault results in badly impact on positioning accuracy of GPS, and causes the real value of
measurement noise covariance $R$ to be quite high. In the Figure 3, it could be found that the result using Sage-Husa method deviates the real value because of coupling process errors. In addition, the result of MAKF approach has better accuracy but of more limitations. The result of the single measurement system algorithm proposed in this paper presents satisfactory precision which has good robustness at the same time. Result of heading angle filtering and velocity filtering are shown as Figures 4 and 5 respectively.

![Figure 3. Comparison of different estimation methods](image)

![Figure 4. Result of heading angle filtering](image)

![Figure 5. Result of velocity filtering](image)

3.2. The simulation of the integrated navigation system
3.2.1 Simulation conditions.

The initial conditions are set as the real position and attitude of vehicle. In the experiment, the coordination transformed from latitude and longitude to WGS-84. GPS takes the differential positioning mode, and some parts of the output data have faults because of being
3.2.2 Analysis of results.

When GPS works well, its positioning accuracy is high, and it turns out that the curve of integrated system is quite close to the actual one. In this experiment, during 1800s to 1900s, GPS receiver has no output, so the DR inertial recursive is used to calculate position information. The results are accurate after using the calibration method.

The filtering results of heading angle and velocity are well. After calibration of the accelerometer, the integrated navigation system turns to be complete, and the positioning results has high precision. The following figures present the different results between without calibration and with calibration. Output of GPS positioning, three-axis Accelerometers and integrated system with calibration are shown as Figures 6, 7 and 8 respectively.

![Figure 6. Output of GPS positioning](image1)

![Figure 7. Output of three-axis Accelerometers](image2)

![Figure 8. Results of integrated system with calibration](image3)
4. Conclusion
This paper has introduced the design and implement of the GPS/DR integrated navigation system. On the purpose of reasonably using the GPS information, an adaptive algorithm based on single measurement system which could estimate the measurement noise covariance was obtained. In order to solve the problem of the GPS signal blocked when the car is operating, an algorithm used to calibrate the gyroscope and accelerometer dynamically is proposed. This algorithm is able to effectively solve the problem that the accumulated error is serious in DR system, and can provide accurate positioning result using dead reckoning when GPS is unavailable. According to the improvement upon, the integrated system can acquire an accurate and reliable positioning result of the vehicle in real time even the GPS signal is totally missed, the system has a high value in engineering application.

References