Software Reliability Modeling with Testing-Effort Function and Imperfect Debugging

Zhao Qian1,2, Zheng Jun1, Li Jing2
1Computer department in Beijing Institution of Technology, Beijing 100081
2Beijing Army of 63961, Beijing 100012
*corresponding author, e-mail: zqzp1998@sina.com*, zhengjun@bit.edu.cn

Abstract

Considering testing effort and imperfect debugging in reliability modeling process may further improve the fitting and prediction results of software reliability growth models (SRGMs). For describing the S-shaped varying trend of the testing-effort increasing rate more accurately, this paper first proposes a inflected S-shaped testing effort function (IS-TEF). Then this new TEF is incorporated into the inflected S-shaped NHPP SRGMs for obtaining a new NHPP SRGMs which consider S-shaped TEF (IS-TEFM-IS). We further discuss this new NHPP SRGM with two imperfect-debugging assumptions to propose two new NHPP SRGMs, i.e. IS-TEFM-IS-ID1 and IS-TEFM-IS-ID2. Finally these three new NHPP SRGMs and several comparison NHPP SRGMs are applied into two real failure data-sets respectively for investigating the fitting power of the IS-TEFM-IS, IS-TEFM-IS-ID1 and IS-TEFM-IS-ID2. The experimental results show that the inflected S-shaped NHPP SRGMs considering IS-TEF and imperfect indeed can yield the best accurate estimation results than the other comparison SRGMs.

Keywords: testing effort; inflected S-shaped; software reliability growth model; reliability modeling; imperfect debugging

Copyright © 2012 Universitas Ahmad Dahlan. All rights reserved.

1. Introduction

Software reliability which can be defined as the probability of failure-free software operation for a special period of time in a special usage environment [1], has become one of the most important customer-oriented characters of software quality [2]. As the main approaches for software reliability estimation and prediction, many time-related software reliability growth models (SRGMs for short) have been proposed and applied into the development process of various types of safety-critical software [3]. Several previous studies suggested that the estimation or prediction accuracy of SRGMs can be further improved by considering the influence of some testing process factors (such as testing effort) on reliability into the modeling process [4-6].

Testing effort indicates the consumed resources during testing phase and can be measured as human power, the number of test cases [7-8]. Actually, the testing-effort index is a more suitable and direct exposure indicator for software reliability modeling than calendar time [8-10]. The researchers first proposed many various testing effort functions (TEFs) to describe the cumulated consumption amount of testing effort during software testing phase, such as exponential, rayleigh, weibull [11], exponentiated Weibull [12], logistic [4,8]. Then many TEF-based SRGMs are proposed by incorporating these above TEFs into the Non-homogeneous Poisson process (NHPP) SRGMs to illustrate the relationship among the calendar time, the amount of TE, and the detected faults.

Due to the integrated effects of software structure and learning factor on testing, the growth curve of the amount of TE may exhibit a S-shaped varying trend which is difficult or not suitable to describe by the exponential, Rayleigh or Weibull-type curves. The inflected S-shaped function is a representative S-shaped function which is flexible and applicable in many cases and has been applied into modeling software reliability. Thus in this paper, we first will present a S-shaped TEF, i.e. the inflected S-shaped TEF (IS-TEF). Then we will show how to incorporate the IS-TEF into the inflected S-shaped NHPP SRGMs to obtain a new NHPP SRGM. Moreover, we will further investigate this new NHPP SRGM under two imperfect-debugging assumptions. Finally a case study on two real failure data sets is presented and discussed.
2. Inflected S-shaped TEF (IS-TEF)

If we use the inflected S-shaped function to describe TEF, then the cumulative testing-effort expenditure consumed in \((0,t)\) can be depicted as follows:

\[
W(t) = W_{\text{max}} \frac{1 - e^{-bt}}{1 + \psi e^{-bt}}
\]

(1)

where \(b\) is the testing effort consumption rate and \(\psi\) is a constant parameter.

According to Eq.1, we have:

a) \(W(t)\) is a non-negative and non-decreasing function of time \(t\);

b) The testing-effort consumption rate at time \(t\) is:

\[
w(t) = \frac{dW(t)}{dt} = \frac{W_{\text{max}} b \exp(-bt)(1 + \psi)}{(1 + \psi \exp(-bt))^2}
\]

(2)

\(w(t)\) is a function which first increasing and then decreasing with \(t\) and reaches its maximum value at

\[
t_{\text{max}} = \ln \psi / b
\]

(3)

3. Inflected S-shaped NHPP software reliability modeling with the IS-TEF

Although most existing NHPP SRGMs belong to the exponential-type, the inflected S-shaped NHPP SRGM are proposed to describe the S-shaped growth curve of the cumulative number of detected faults for obtaining a better description of the software fault removal and detection process. In this section, we will show how to integrate the IS-TEF into the inflected S-shaped NHPP SRGMs.

The general assumptions of the Inflected S-shaped NHPP SRGM with IS-TEF are shown as:

1) The occurrence and removal process of software faults follows the NHPP;

2) The software failures occur at random times caused by the remaining faults in the system;

3) The mean number of faults detected in the time interval \((t, t+\Delta t)\) to the current testing-effort expenditures is proportional to the mean number of remaining faults in the system;

4) The corresponding fault detection rate function \(r(t)\) is related to the consumption amount of testing-effort \(W(t)\), and can be written as \(r(t) = \frac{r}{1 + ke^{-rW(t)}}\);

5) The consumption amount of testing-effort \(W(t)\) is described by IS-TEF;

6) When a fault is detected, it is immediately removed and doesn’t introduce a new fault. In other words, the number of the total faults in the software system is a constant \(N\).

According to the above assumptions, the inflected NHPP SRGM with IS-TEF can be shown as follows:

\[
\frac{dm(t)}{dt} \times \frac{1}{w(t)} = \frac{r}{1 + ke^{-rW(t)}} \times [N - m(t)]
\]

(4)

where \(m(t)\) is the mean value function of the expected number of faults detected in time interval \((0,t)\). Taking the Eq.2 into the Eq.4, and solving the Eq.4 under the boundary condition \(m(0)=0\) and \(W(0)=0\), then the Inflected S-shaped NHPP SRGM with IS-TEF (IS-TEFM-IS) is shown as follows:

\[
m(t) = \frac{N(1 - \exp(-rW(t)))}{1 + k \exp(-rW(t))}
\]

(5)

where \(W(t) = W_{\text{max}} \frac{1 - e^{-bt}}{1 + \psi e^{-bt}}\).
4. Inflected S-shaped NHPP software reliability modeling with the IS-TEF and imperfect debugging

Most traditional NHPP SRGMs, such as Goel-Okumoto Model [1], assume that once a failure occurs, the fault causing this failure is immediately removed and no new faults are introduced, namely the perfect debugging phenomenon. However, due to the complexity of software systems and debugging activities, in reality, the faults removal process may not be perfect and may generate another new fault, i.e. imperfect debugging. Thus the imperfect debugging problems should be considered in reliability modeling process for interpreting the debugging process more accurately and actually. Since Goel first incorporated the imperfect debugging into reliability modeling, many studies have been proposed for solving the imperfect debugging issues [9,13]. Thus we will incorporate the inflected S-shaped TEF and imperfect debugging into the inflected S-shaped NHPP SRGM to provide an intuitively and actual description of the imperfect debugging process.

The general assumptions of the inflected S-shaped NHPP SRGM with the IS-TEF and imperfect debugging are shown as follows:

1. The occurrence and removal process of software faults follows the NHPP;
2. The software failures occur at random times caused by the remaining faults in the system;
3. The mean number of faults detected in the time interval \((t, t+\Delta t)\) to the current testing-effort expenditures is proportional to the mean number of remaining faults in the system;
4. The corresponding fault detection rate function \(r(t)\) is related to the consumption amount of testing-effort \(W(t)\), and can be written as \(r(t) = \frac{r}{1+ke^{-W(t)}}\);
5. The consumption amount of testing-effort \(W(t)\) is described by IS-TEF;
6. When a fault is detected, it is immediately removed. However when removing or fixing a detected fault, a new fault may be generated with a certain probability \(\beta\). In other words, the number of the total faults in the software system is not a constant and may be a function of \(t\), i.e. \(N(t)\), which can be generally regarded as the sum of the initial total faults \(N\) and the introduced faults.

According to the above assumptions, the modeling framework of the inflected S-shaped NHPP SRGM with IS-TEF and imperfect debugging is shown as follows:

\[
\frac{dm(t)}{dt} \times \frac{1}{W(t)} = r(t)\left[N(t) - m(t)\right]
\]

(6)

Many expression forms of \(N(t)\) have been proposed for describing the imperfect debugging process. The following two expression forms of \(N(t)\) are commonly used in the research and practice related to reliability modelling.

Form 1: The introduced faults is proportional to the current time \(t\) or testing-effort \(W(t)\), that is

\[
N(t) = N(1 + \beta W(t))
\]

(7)

where \(\beta\) is the fault introduction rate.

Form 2: The introduced faults is proportional to the current detected faults \(m(t)\), that is

\[
N(t) = N(1 + \beta m(t))
\]

(8)

(1) Inflected S-shaped NHPP SRGM with IS-TEF and the form 1 of imperfect debugging

According to the assumption (4) and the Equation (7), the Equation (6) is rewritten as

\[
\frac{dm(t)}{dt} \times \frac{1}{W(t)} = \frac{r}{1+ke^{-W(t)}} \times \left[N(1+\beta W(t)) - m(t)\right]
\]

(9)

Solving the Eq.9 under the boundary condition \(m(0)=0\) and \(W(0)=0\), then the inflected S-shaped NHPP SRGM (i.e. IS-TEF-IS-ID1) with IS-TEF and the form 1 of imperfect debugging is shown as follows:
where the $W(t)$ is shown in Eq.1.

(2) Inflected S-shaped NHPP SRGM with IS-TEF and the form 2 of imperfect debugging
According to the assumption (4) and the Equation (8), the Equation (6) is rewritten as

$$
\frac{dm(t)}{dt} \times \frac{1}{w(t)} = \frac{r}{1+ke^{-rW(t)}} \times [N + \beta m(t) - m(t)]
$$

(11)

Solving the Eq.11 under the boundary condition $m(0)=0$ and $W(0)=0$, then the inflected S-shaped NHPP SRGM (i.e. IS-TEF-IS-ID2) with IS-TEF and the form 2 of imperfect debugging is shown as follows:

$$
m(t) = \frac{N}{1-\beta} \left[ 1 - \left( \frac{k + \exp(rW(t))}{k + 1} \right)^{\beta-1} \right]
$$

(12)

where the $W(t)$ is shown in Eq.1.

5. Case Study
In this section, we will propose a case study on two real failure data-sets for investigating the effectiveness and applicability of the proposed IS-TEFM-IS, IS-TEFM-IS-ID1 and IS-TEFM-IS-ID2 in terms of fitting results.

1) Two real failure data-sets
Two real failure data-sets concluding the testing-effort data are selected which are popular and frequently used as the benchmark for comparison in reliability modeling field. The first data-set (DS1) was collected by Ohba [14]. During 19 weeks, 328 faults were detected with 47.65 CPU hours consumed. The second data-set (DS2) was collected by Wood [15]. During 20 weeks, 100 faults were detected with 10000 CPU hours consumed.

2) TEFs or NHPP SRGMs for comparison
We also select seven various existing NHPP SRGMs as the comparison SRGMs in Table 1.

<table>
<thead>
<tr>
<th>No.</th>
<th>Model Name (Abbreviation)</th>
<th>Mean Value Function ($m(t)$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Goel-Okumoto (GO) [3]</td>
<td>$N\left[1-\exp(-\eta)\right]$</td>
</tr>
<tr>
<td>2</td>
<td>Inflected S-Shaped (IS) [3]</td>
<td>$N\left(1-e^{-\eta}\right)/(1+\psi e^{-\eta})$</td>
</tr>
<tr>
<td>3</td>
<td>Generalized GO (GGO) [3]</td>
<td>$N\left(1-\exp(-r\eta)\right)$</td>
</tr>
<tr>
<td>4</td>
<td>Weibull-TEF-based Model (WE-TEFM) [11]</td>
<td>$N\left(1-\exp\left(-rW_{\text{max}}\left(1-\exp\left(-bt''\right)\right)\right)\right)$</td>
</tr>
<tr>
<td>5</td>
<td>Logistic-TEF-based Model (LO-TEFM) [8]</td>
<td>$N\left(1-\exp\left(-r\frac{W_{\text{max}}}{1+A\exp(-ar)}-W_{\text{max}}\right)\right)$</td>
</tr>
<tr>
<td>6</td>
<td>ExpWeibull-TEF-based Model (EW-TEFM) [12]</td>
<td>$N\left(1-\exp\left(-rW_{\text{max}}\left(1-\exp\left(-bt''\right)\right)\right)^a\right)$</td>
</tr>
<tr>
<td>7</td>
<td>ExpWeibull-TEF-based Inflected S-Shaped Model (EW-TEFM-IS) [9]</td>
<td>$\frac{N\left(1-\exp\left(-rW_{\text{max}}\left(1-\exp\left(-bt''\right)\right)\right)^a\right)}{1+(1/k-1)\exp\left(-rW_{\text{max}}\left(1-\exp\left(-bt''\right)\right)\right)^a}$</td>
</tr>
</tbody>
</table>

3) Comparison criteria
Generally, the SRGMs can be compared in terms of its ability to fit the observed failure data. Two criteria are selected for comparing the descriptive power of SRGMs:

The MSE is defined as Equation 13:
MSE = \frac{1}{n} \sum_{i=1}^{n} (m(t_i) - m_i)^2 \quad (13)

and the R-square is defined as Equation 14:

R = 1 - \frac{\sum_{i=1}^{n} (m(t_i) - m_i)^2}{\sum_{i=1}^{n} (m_i - m_{ave})^2} \quad (14)

where \( m_i \) is the observed number of faults at \( t_i \), \( m(t_i) \) is the estimated value of the number of faults at \( t_i \), \( m_{ave} \) is the average value of \( m_i \). The smaller the value of MSE is, the better the SRGM fits. The value of R-Square is limited in \([0,1]\). The larger the value of R-Square, the better the SRGM fits.

4) Fitting results analysis of IS-TEFM-IS

The estimated parameters and the fitting results in terms of MSE and R-square of the comparison SRGMs for each failure data-set are shown in Table 2 and Table 3. Then we can find that: For DS1 and DS2, the proposed IS-TEFM-IS yields the best fitting results, that is, have the smallest values of MSE and the largest values of R-square. Thus, it indicates that the IS-TEFM-IS has a significantly pretty fitting power as well as is applicable for various data-sets.

<table>
<thead>
<tr>
<th>Model</th>
<th>Estimated Parameters</th>
<th>MSE</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>GO</td>
<td>( N=760.53, r=0.0323 )</td>
<td>139.82</td>
<td>0.986</td>
</tr>
<tr>
<td>IS</td>
<td>( N=374.39, r=0.179, \psi =3.01 )</td>
<td>127.31</td>
<td>0.988</td>
</tr>
<tr>
<td>GGO</td>
<td>( N=427.83, r=0.0361, c=1.28 )</td>
<td>102.12</td>
<td>0.990</td>
</tr>
<tr>
<td>WE-TEFM</td>
<td>( N=565.1, r=0.01966 )</td>
<td>116.79</td>
<td>0.989</td>
</tr>
<tr>
<td>LO-TEFM</td>
<td>( N=395.6, r=0.04164 )</td>
<td>114.05</td>
<td>0.989</td>
</tr>
<tr>
<td>EW-TEFM</td>
<td>( N=575.7, r=0.01916 )</td>
<td>113.68</td>
<td>0.999</td>
</tr>
<tr>
<td>EW-TEFM-IS</td>
<td>( N=398.4, r=0.05642, k=0.4074 )</td>
<td>94.63</td>
<td>0.991</td>
</tr>
<tr>
<td>IS-TEFM-IS</td>
<td>( N=387.1, r=0.06109, k=1.634 )</td>
<td>80.58</td>
<td>0.992</td>
</tr>
</tbody>
</table>

5) Fitting results analysis of IS-TEFM-IS-ID1 and IS-TEFM-IS-ID2

Due to the limited space, we only compare the fitting results of the IS-TEFM-IS-ID1 and IS-TEFM-IS-ID2 with several SRGMs for DS1. The estimated parameters and the fitting results in terms of MSE and R-square of the comparison SRGMs for DS1 are shown in Table 4. From the results of Table 4, it can be observed that: The fitting results of the proposed IS-TEFM-IS, IS-TEFM-IS-ID1 and IS-TEFM-IS-ID2 are all the best compared with all the other comparison SRGMs for DS1. It indicates that incorporating IS-TEF and imperfect debugging...
into the inflected S-shaped NHPP SRGMs indeed can yield a novel fitting power for accurately describing failure behaviors as we expect.

Table 4. The Estimated Parameters and Fitting results of the comparison SRGMs for DS1

<table>
<thead>
<tr>
<th>Model</th>
<th>Estimated Parameters</th>
<th>MSE</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>GO</td>
<td>N=760.53, r=0.0323</td>
<td>139.82</td>
<td>0.986</td>
</tr>
<tr>
<td>IS</td>
<td>N=374.39, r=0.179, Ψ =3.01</td>
<td>127.31</td>
<td>0.988</td>
</tr>
<tr>
<td>GGO</td>
<td>N=427.83, r=0.0361, c=1.28</td>
<td>102.12</td>
<td>0.990</td>
</tr>
<tr>
<td>WE-TEFM</td>
<td>N=565.1, r=0.01966</td>
<td>116.79</td>
<td>0.989</td>
</tr>
<tr>
<td>LO-TEFM</td>
<td>N=395.6, r=0.04164</td>
<td>114.05</td>
<td>0.989</td>
</tr>
<tr>
<td>EW-TEFM</td>
<td>N=575.7, r=0.01916</td>
<td>113.68</td>
<td>0.989</td>
</tr>
<tr>
<td>EW-TEFM-IS</td>
<td>N=398.4, r=0.05642, k=0.4074</td>
<td>94.63</td>
<td>0.991</td>
</tr>
<tr>
<td>IS-TEFM-IS</td>
<td>N=387.1, r=0.06109, k=1.634</td>
<td>80.58</td>
<td>0.992</td>
</tr>
<tr>
<td>IS-TEFM-IS-ID1</td>
<td>N=387.1, r=0.06109, β = 2.388*10^-10, k= 1.634</td>
<td>80.58</td>
<td>0.992</td>
</tr>
<tr>
<td>IS-TEFM-IS-ID2</td>
<td>N=387.1, r=0.06109, β = 3.279*10^-7, k= 1.634</td>
<td>80.58</td>
<td>0.992</td>
</tr>
</tbody>
</table>

The estimated values of the fault introduction rate (i.e. β) in Table 4 are all close but not equal to zero. It indicates that the fault removal process of DS1 may not be a completely perfect debugging process. As a result, we should consider the effect of imperfect debugging in the modeling process.

It should be noted that the fitting results of IS-TEFM-IS-ID1 & 2 are nearly the same with the corresponding NHPP SRGMs without imperfect debugging i.e. IS-TEFM-IS. It means that the influence of imperfect debugging on improving fitting power is less significant for the NHPP SRGMs. Huang proposed an explanation for this phenomenon that the imperfect debugging should be taken into account when the software product is reaching the mature stage [4]. Certainly, it is just a primary conclusion which should be validated by the further experiments and discussions in our future study.

6. Conclusions

This paper first presents a S-shaped TEF, i.e. the inflected S-shaped TEF (IS-TEF) for describing the S-shaped growth trend of the testing-effort consumption more accurately. Then this IS-TEF is incorporated into the inflected S-shaped NHPP SRGM to obtain a new NHPP SRGM (i.e. IS-TEFM-IS). Moreover, we further investigate this new NHPP SRGM under two imperfect-debugging assumptions to propose two new NHPP SRGMs, i.e. IS-TEFM-IS-ID1 and IS-TEFM-IS-ID2. Finally, a case study is presented for validating the fitting power of the IS-TEFM-IS, IS-TEFM-IS-ID1 and IS-TEFM-IS-ID2 by applying it and several comparison NHPP SRGMs into two real failure data-sets. The experimental results show that the IS-TEFM-IS, IS-TEFM-IS-ID1 and IS-TEFM-IS-ID2 yields the most accurate fitting results compared with the other comparison SRGMs.

References


