Minor Capacity Analysis of N Major Traffic Streams at Intersection

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Abstract

Based on the gap acceptance theory, this paper analyzed running state of the intersection streams about the main traffic priority controls. Theories were used in this paper, such as gap theory, the probability theory and the calculation methods of minor capacity under two vehicle streams. The calculating formula of minor capacity for n main vehicle streams was derived under the conditions of M3 distribution by the main stream headway at the uncontrolled intersection.

Keywords: n major traffic streams, intersection, minor stream, capacity

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1. Introduction

For the road, the intersection is the basic point of the road network. Diverse traffic movement behaviors caused by traffic steering such as the traffic conflict, crossing, diversion, have made intersection capacity analysis very complicated. Intersection is not only the traffic accident-prone location, but also the traffic flow bottleneck location of road network. The highway intersection capacity affects the entire capacity of the road network. The study of intersection traffic capacity has great significance to the intersection program, design, alterations and management, and other work.

In general, an unsignalized intersection control mode usually adopts stop sign and give-way sign mode, collectively referred to as the main road priority control. This control method has been piloted and adopted in our country. At present, many scholars have made some in-depth study on the theoretical capacity of minor road [1-6]. Based on the gap theory and probability theory, the minor road capacity calculation method was given. This kind of method is for one major traffic flow. And the theoretical model of this minor road traffic was derived when the main traffic headway time was obeying the M3 distribution [5]. In the traffic engineering theory and practice, China has cited much foreign experience. To the entire intersection, analysis method of conflict traffic flow-critical gap method is consistent with the actual situation in China. Possible traffic capacity of each traffic flow through traffic intersection is interrelated with not only the sum size of individual traffic flow, but also the critical gap through traffic flow.

Many scholars have made some in-depth research on the minor capacity calculation of major traffic priority control; most of researches are on the one major lane. But study on n shares of major traffic is relatively small, because any model is not suitable for all circumstances. Moreover, conditions differ from one country to another in road, traffic, and vehicle. In the use of theoretical research and calculation methods, it must be combined with the status quo of China’s intersections, road conditions, traffic conditions and vehicle condition; then get the parameters for critical gap and attendant, ultimately work out minor road traffic capacity of the different zero-signalized intersection.

2. Traffic State Analysis on Major Traffic Priority Control Intersection

The intersection has a plurality of flows and directions. The running state of the vehicle is shown in Figure 1.

According to the rules of the traffic, different direction flows have different priority levels at the major traffic priority control intersection. Low level of traffic stream gives way to the high levels of stream in correct order.
Various motion modes finite levels at intersection are provided as follows:
- Priority 1 traffic has absolute priority and it has no need to give way to other traffic;
- Priority 2 traffic must give way to priority 1 traffic;
- Priority 3 traffic must give way to priority 1 and 2 traffic;
- Priority 4 traffic must give way to priority 1, 2 and 3 of the traffic streams.

Vehicle flow movement at unsignalized intersection is quite complicated, for each flow is faced with the conflict of different direction of movements. The size of the conflict traffic directly affect the flow capacity. Lower rank streams at intersection faces the conflicts with other different direction flows. All the flows of conflict are known as the conflicting traffic flow and the vehicle sum of the conflicts traffic flow is called conflict traffic. Therefore, the size of the conflict traffic will directly affect traffic capacity of a flow.

The priority class and conflict traffic flow are shown in Table 1.

<table>
<thead>
<tr>
<th>travel direction</th>
<th>grade</th>
<th>mark</th>
<th>Conflict traffic</th>
</tr>
</thead>
<tbody>
<tr>
<td>The major road</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>turn right</td>
<td>1</td>
<td>3</td>
<td>none</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9</td>
<td>none</td>
</tr>
<tr>
<td>The major road</td>
<td>1</td>
<td>2</td>
<td>none</td>
</tr>
<tr>
<td>straight</td>
<td></td>
<td>8</td>
<td>none</td>
</tr>
<tr>
<td>The main road</td>
<td>2</td>
<td>1</td>
<td>q2+q3</td>
</tr>
<tr>
<td>turn left</td>
<td></td>
<td>7</td>
<td>q2+q3</td>
</tr>
<tr>
<td>The minor</td>
<td>2</td>
<td>6</td>
<td>q2+0.5×q3</td>
</tr>
<tr>
<td>traffic turn right</td>
<td>12</td>
<td></td>
<td>q2+0.5×q3</td>
</tr>
<tr>
<td>The minor</td>
<td>3</td>
<td>5</td>
<td>q2+0.5×q3+q1+q4+q7</td>
</tr>
<tr>
<td>traffic straight</td>
<td></td>
<td>11</td>
<td>q2+0.5×q3+q1+q4+q7</td>
</tr>
<tr>
<td>The minor</td>
<td>4</td>
<td>4</td>
<td>q2+0.5×q3+q1+q4+q7+q12</td>
</tr>
<tr>
<td>traffic turn left</td>
<td>10</td>
<td></td>
<td>q2+0.5×q3+q1+q4+q7+q12</td>
</tr>
</tbody>
</table>

According to foreign experience, the major traffic right-turn flow might have influence on other related flow. For example, if the main road right-turn traffic does not use turn signal or use it in time, the right-turn traffic traveling on the minor road will be affected. So a weighted coefficient of 0.5 is added in the table [7]. In addition, the above conflict traffic should also base on the actual situation at the intersection. Various factors should be considered. In different conditions, the different situations should be corrected [8].
3. Minor Traffic Capacity Analysis in N Major Traffic Flow

In order to make it clear, the author will use the capacity analysis method of two shares traffic [7]. Below are n shares of major traffic capacity analysis based on the two shares of traffic. N shares conflict traffic diagram is shown in Figure 2.

![Figure 2. Schematic diagram of traffic conflict](image)

3.1. Gap Theory

Critical gap and following time are two main parameters of the gap theory. Critical gap refers to minimum clearance which minor traffic driver can accept in the major traffic flow. It is generally credited as \( t_c \). Under normal circumstances, the driver would usually accept a time interval greater than the critical clearance rather than a less than critical gap [8].

In a long interval, there will be more than one car across the main road from a minor road. Following time refers to the time interval between two adjacent minor road vehicle which uses a the main road headway time continuously go through the intersection, usually expressed by \( t_f \). The critical gap and following time have a great influence on the intersection traffic capacity calculation. The critical gap is related with the minor lane traffic flow, motorcycle type, imports Road parking as well as the major traffic speed. It cannot be measured directly. But it is larger than the largest refused clearance and less than acceptable gap, so its distribution and parameter values can be estimated with a variety of methods, using direct observation of the accept gap values and refused clearance values.

3.2. Minor Flow Capacity Analysis Method

According to the gap theory, minor flow capacity is restricted by the headway distribution rule of major flow, critical gap and following time and other factors. Among them, the flowing headway distribution of major flow is one of the main factors. Set the probability density of major road headway time be \( f(t) \), when the major road headway time is \( t \), the number of minor vehicle which can pass through is \( g(t) \), and the major traffic volume is \( q(\text{veh/h}) \), then minor road capacity:

\[
C = \int_0^\infty f(t)g(t)dt \\
(1)
\]

The discrete formula (1), then:

\[
C = q \sum_{x=1}^n xP(x) \\
(2)
\]

According to the gap theory, let critical gap and following time when minor road flow across the major road traffic flow are respectively \( t_c \) and \( t_f \), and let each of them be an fixed value. When the major road headway time \( t \) satisfies \( t_c < t < t_c + t_f \), a minor road vehicles can pass; when \( t_c + t_f < t < t_c + 2t_f \), allowing two cars from the minor road to pass; when \( t_c + (n-1)t_f < t < t_c + nt_f \), it allows \( n \) vehicles from the minor road to pass. \( g(t) \) for piecewise function:
According to the formula (1) and the formula (3), or the formula (2) and the formula (3), when the distribution form of the main road headway time is known, minor traffic capacity can be worked out.

3.3. Minor Road Traffic Capacity Model When the Major Road Traffic Flow Obeys M3 Distribution

Previous studies have indicated that: Binary distribution is a relatively good headway time distribution. M3 distribution of Cowan is a good dichotomy headway distribution[9], and Cowan proposed that M3 model is a good acceptance gap headway distribution models [10]. M3 is more suitable for single lane where overtaking is not allowed, and in the intersection, for security, geometric constraints and vehicles intersection and other factors, the vehicle is generally not overtaking. So the M3 distribution to describe vehicle arrival distribution at the intersection is more suitable for deriving minor roads traffic capacity model when n shares of the major road traffic flow is obeying M3 distribution. Assuming that a portion of the major road vehicles are results team traveling, the minimum headway time of n shares flow is for $t_{mi}$. When another portion of the vehicles running freely, the headway is greater than $t_{mi}$, and a percentage of free vehicle in the i share major traffic flow is $\alpha_i$. When headway time of n shares major road traffic flow obeying M3 distribution, probability distribution of the i Share major traffic is for $F_i(t)$:

$$F_i(t) = \begin{cases} 1 - \alpha_i e^{-\lambda_i (t - t_{mi})}, & t \geq t_{mi} \\ 0, & t < t_{mi} \end{cases}$$  (4)

Traffic flow density function in lane i:

$$D_i(t) = q(1 - F_i(t))$$  (5)

So:

$$D_i(t) = q \alpha_i e^{-\lambda_i (t - t_{mi})}$$  (6)

So traffic density distribution function in lane i:

$$G_i(t) = 1 - \frac{q \alpha_i e^{-\lambda_i (t - t_{mi})}}{\lambda_i}$$  (7)

Assuming that the headway time distribution in each major lane is independent, therefore the density distributions function of n lanes:

$$1 - H(t) = (1 - G_1(t))(1 - G_2(t)) \cdots (1 - G_n(t))$$  (8)

Then the density distribution function

$$H(t) = 1 - \frac{q \alpha_1 \alpha_2 \cdots \alpha_n e^{\lambda_1 t} \cdots e^{\lambda_n t}}{\lambda_1 \lambda_2 \cdots \lambda_n} t \geq t_{mi}$$  (9)

n traffic flow density function

$$D(t) = (\lambda_1 + \lambda_2 + \cdots + \lambda_n) \frac{q \alpha_1 \alpha_2 \cdots \alpha_n e^{\lambda_1 t} \cdots e^{\lambda_n t}}{\lambda_1 \lambda_2 \cdots \lambda_n}$$  (10)

Set $\lambda = \lambda_1 + \lambda_2 + \cdots + \lambda_n$, then formula (10) can be written as
\[
D(t) = \frac{\lambda q_{1} \cdots q_{n} \alpha_{1} \cdots \alpha_{n}}{\lambda_{2} \cdots \lambda_{n}} e^{-\lambda_{1}t} e^{-\lambda_{2}t} \cdots e^{-\lambda_{n}t} \tag{11}
\]

Wherein, \( q_{i} \) is a free traffic probability in lane \( i \); \( \lambda_{i} \) is an traffic attenuation coefficient in lane \( i \), it is a constant value. The relationship between these parameters

\[
\lambda_{i} = \frac{\alpha_{i} q_{i}}{(1 - t_{m} q_{i})} \tag{12}
\]

If each flow minimum headway is equal. Set \( tm1=tm2=tmn=\ldots=tm \), then formula (11) into

\[
D(t) = \frac{\lambda q_{1} \cdots q_{n} \alpha_{1} \cdots \alpha_{n}}{\lambda_{2} \cdots \lambda_{n}} e^{-\lambda_{1}t} \tag{13}
\]

Substituting (13) into (4), probability distribution of \( n \) major lanes lead to

\[
F(t) = 1 - \frac{\lambda q_{1} \cdots q_{n} \alpha_{1} \cdots \alpha_{n}}{\lambda_{2} \cdots \lambda_{n}} e^{-\lambda_{1}t} \tag{14}
\]

Here \( q = q_{1} + q_{2} + \ldots + q_{n} \).

By the formula (14), a certain range of the headway time probability can be obtained:

\[
P(t_{i} \leq t < t_{i} + t_{f}) = F(t_{i}) - F(t_{i})
\]

\[
= 1 - \frac{\lambda q_{1} \cdots q_{n} \alpha_{1} \cdots \alpha_{n}}{\lambda_{2} \cdots \lambda_{n}} e^{-\lambda_{1}(t_{i} - t_{m})} - (1 - \frac{\lambda q_{1} \cdots q_{n} \alpha_{1} \cdots \alpha_{n}}{\lambda_{2} \cdots \lambda_{n}} e^{-\lambda_{1}(t_{i} - t_{m})})
\]

\[
= \frac{\lambda q_{1} \cdots q_{n} \alpha_{1} \cdots \alpha_{n}}{\lambda_{2} \cdots \lambda_{n}} e^{-\lambda_{1}(t_{i} - t_{m})} - \frac{\lambda q_{1} \cdots q_{n} \alpha_{1} \cdots \alpha_{n}}{\lambda_{2} \cdots \lambda_{n}} e^{-\lambda_{1}(t_{i} - t_{m})} \tag{15}
\]

From formula (2) and the formula (15), the probability of one vehicle passing major road can be obtained:

\[
P(1) = P(t_{i} \leq t < t_{i} + t_{f}) = \frac{\lambda q_{1} \cdots q_{n} \alpha_{1} \cdots \alpha_{n}}{\lambda_{2} \cdots \lambda_{n}} (e^{-\lambda_{1}(t_{i} - t_{m})} - e^{-\lambda_{1}(t_{i} + t_{f} - t_{m})}) \tag{16}
\]

Similarly, the probability of allowing two vehicles, three vehicles... \( n \) vehicles from minor road to pass is \( P(2) \), \( P(3) \)...\( P(n) \). So, the average total number of vehicles from minor road to pass the major road is:

\[
Q = \sum_{i=1}^{n} xP(x) = P(1) + 2P(2) + \ldots + nP(n)
\]

\[
= \frac{\lambda q_{1} \cdots q_{n} \alpha_{1} \cdots \alpha_{n}}{\lambda_{2} \cdots \lambda_{n}} (e^{-\lambda_{1}(t_{i} - t_{m})} - e^{-\lambda_{1}(t_{i} + t_{f} - t_{m})}) + 2 \cdot \frac{\lambda q_{1} \cdots q_{n} \alpha_{1} \cdots \alpha_{n}}{\lambda_{2} \cdots \lambda_{n}} (e^{-\lambda_{1}(t_{i} + t_{f} - t_{m})} - e^{-\lambda_{1}(t_{i} + 2t_{f} - t_{m})})
\]

\[
+ \ldots + n(\frac{\lambda q_{1} \cdots q_{n} \alpha_{1} \cdots \alpha_{n}}{\lambda_{2} \cdots \lambda_{n}} (e^{-\lambda_{1}(t_{i} + nt_{f} - t_{m})} - e^{-\lambda_{1}(t_{i} + (n+1)t_{f} - t_{m})}))
\]

\[
= \frac{\lambda q_{1} \cdots q_{n} \alpha_{1} \cdots \alpha_{n}}{\lambda_{2} \cdots \lambda_{n}} (e^{-\lambda_{1}(t_{i} - t_{m})} - e^{-\lambda_{1}(t_{i} + nt_{f} - t_{m})}) + 2(e^{-\lambda_{1}(t_{i} + t_{f} - t_{m})} - e^{-\lambda_{1}(t_{i} + 2t_{f} - t_{m})})
\]

\[
+ \ldots + n(e^{-\lambda_{1}(t_{i} + nt_{f} - t_{m})} - e^{-\lambda_{1}(t_{i} + (n+1)\frac{t_{f}}{2} - t_{m}))})
\]

\[
= \frac{\lambda q_{1} \cdots q_{n} \alpha_{1} \cdots \alpha_{n}}{\lambda_{2} \cdots \lambda_{n}} (e^{-\lambda_{1}(t_{i} - t_{m})} + e^{-\lambda_{1}(t_{i} + t_{f} - t_{m})} + e^{-\lambda_{1}(t_{i} + 2t_{f} - t_{m})} + \ldots + e^{-\lambda_{1}(t_{i} + (n\frac{t_{f}}{2} - t_{m})}) - ne^{-\lambda_{1}(t_{i} + nt_{f} - t_{m}))}
\]

\[
= \frac{\lambda q_{1} \cdots q_{n} \alpha_{1} \cdots \alpha_{n}}{\lambda_{2} \cdots \lambda_{n}} (e^{-\lambda_{1}(t_{i} - t_{m})} \frac{1 - e^{-\lambda_{1}(t_{f})}}{1 - e^{-\lambda_{1}(t_{f})}} - ne^{-\lambda_{1}(t_{i} + nt_{f} - t_{m}))}) \tag{17}
\]
Then n take the limit:

$$\lim_{n \to \infty} Q = \frac{\lambda q_1 q_2 \cdots q_n \alpha_1 \alpha_2 \cdots \alpha_n e^{-\lambda (1-t_s)}}{q \lambda \alpha_2 \cdots \alpha_n (1-e^{-\lambda t_s})}$$

(18)

And since i major lane traffic capacity in one hour, that is the traffic rate. The traffic rate is qi (veh/h), and headway time on the major road is qi=q+q2+…+qn in one hour, so minor road capacity is:

$$C = (q_1 + q_2 + \cdots + q_n) \lim_{n \to \infty} = \frac{\lambda q_1 \cdots q_n \alpha_1 \alpha_2 \cdots \alpha_n e^{-\lambda (1-t_s)}}{\lambda \lambda \cdots \lambda (1-e^{-\lambda t_s})}$$

(19)

Substituting formula (12) into formula (19) leads to

$$C = \frac{\lambda (1-t_s q_1) (1-t_s q_2) \cdots (1-t_s q_n) e^{-\lambda (1-t_s)}}{1-e^{-\lambda t_s}}$$

(20)

### 3.4. Capacity Analysis of Major Traffic Priority Control Intersection

At the major traffic priority control intersection, “n” major traffic streams can take the priority to pass the conflict zone of the intersection, which requires that the major traffic should not be too big but have a certain gap for a vehicle passing through. If major traffic is too big, vehicles on minor road can not pass through the intersection. Then signal lamps are additionally needed to distribute travel time.

It needs a big enough gap for minor traffic to find the major traffic to pass through. The capacity will be influenced by factors, like the time headway distribution and the critical gap of major traffic. So the capacity of intersection should be the major traffic volume plus the traffic capacity of minor traffic. And the maximum of the traffic capacity of intersection should be the traffic capacity of major roads, which is calculated by the traffic capacity of road.

In the situation with n traffic streams, the key point is to do researches on the traffic capacity of minor traffic. In reality, there are a variety of traffic streams with multiple directions at the intersection, as shown in Figure 1; the traffic capacity of intersection can be calculated by the critical gap. As figure 1 show, among the ranks of traffic streams, the first rank takes the priority to pass through. Among n major traffic streams the traffic capacity of minor traffic can be calculated by the model formula (20).

### 4 Conclusion

In reality, the capacity of the major road priority control is not big. It is important to focus on the capacity of minor traffic at intersection. This paper uses the achievements of the two strands of vehicle streams as reference [7]. Assuming that n major traffic streams obey to the distribution of M3 and deducing the formula of the minor traffic capacity of n major traffic streams. For the sake of model's briefly, each time headway takes the same value as is formed model (20). The minor road capacity has a great association with the rate of each major traffic stream, and headway being the main factor of minor traffic capacity. The larger the n is, the smaller the gap will be and it will become difficult for the vehicles on minor road to pass through the major road. But in reality the n will not be great. The derived mathematical model in this paper is well suited to China’s calculation of the traffic capacity of intersection on n major traffic streams, and provides a basis for China’s projects and evaluation on road, highway network and intersection.

### Reference


