The Research of Chaos-based M-ary Spreading Sequences

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Abstract
This paper is devoted to the generation and evaluation of the Chaos-based M-ary spreading sequences on communications systems. Sequences obtained by repeating a truncated and multi-ary quantized chaotic series are compared with classical m-sequences by means of the autocorrelation and cross-correlation properties and power-spectral features. Anti-noise performance of binary sequences and chaotic-based M-ary spreading sequences has been compared in the case of the same single-frequency interferences. Studies have shown that spectral features and anti-noise performance of chaotic-based M-ary spreading sequences which have great researching value are better than binary sequences.

Keywords: Chaotic sequence; M-ary; Spread spectrum sequence

1. Introduction
Spread Spectrum technology which has been applied in all respects has many unique advantages and greatly improves the anti-noise performance [1-3]. It has become the important technology of today’s radio communication [4-5]. Binary spreading sequences (such as m-sequences) with good autocorrelation property are common spreading sequences [6]. But in today’s increasingly harsh electromagnetic environment, the binary spreading sequences because of less available addresses and poor confidentiality are unable to meet the diverse needs of modern communications. Chaotic series with randomness, uncertainty and non-cyclical features are very suitable for synchronization and as spreading sequences in secure communications [7-10]. Chaotic series as binary spreading sequences have been studied [11-12]. But transformed into M-ary spreading sequences, chaotic series have better features than binary sequences [13-18]. Therefore, this paper proposed the chaos-based M-ary spreading sequences (CMSS) and generation method. The autocorrelation and cross-correlation features as well as anti-noise performance of CMSS were studied.

2. The generation method of chaos-based M-ary spreading sequences
2.1. Chaotic system
The cause of chaotic phenomenon is that the sequence copies the previous state of motion in some rules and produces unpredictable random effects. Chaos, a nonlinear and random movement, is a long-term behavior that is sensitive to initial conditions. In a power system, small changes of chaos system in initial conditions can led to a huge and long-term chain reaction.

Chaotic systems have the following features:
(1) Ergodicity. Chaotic motion is ergodic in the range of its provisions, which means that in limited areas, chaotic system traverses all states according to its own laws without repeat.
(2) Randomness. Chaotic system can be represented by certain equation. System status shows unpredictable randomness in the case without outside influence.
(3) Certainty. When the initial value is determined, the chaotic series generated are certain.
(4) Sensitive dependence on initial value. The final state of the system will make a huge difference as long as the initial conditions are slightly different or a slight disturbance.
(5) Nonperiodic. The chaotic series have no cycles.
(6) Wide power spectrum. The chaotic series have wide power spectrum as white noise.
2.2. The chaotic series

Chaotic systems can be divided into time continuous systems represented by differential equations and time-discrete systems represented by the equation of state. The sequences generated by the latter ones are chaotic spreading sequences we need.

A discrete-time dynamic system can be defined as in Eq. (1)

\[ X_{k+1} = f(X_k), \quad k=0,1,2,\ldots \]  

(1)

In Eq. (1), \(0 < X_k < 1\), \(X_k\) is the state of discrete-time dynamic system. \(f(X_k)\) maps the current state \(X_k\) to the next state \(X_{k+1}\). Thus a sequence \(\{X_0, X_1, X_2, \ldots\}\) called a trajectory is gotten.

With the maturity of the nonlinearity and chaos theory, chaotic systems have been widely used, especially in the communications. Chaotic series that have the features similar to random sequences are able to be described by mathematical equations accurately. It also has the controllability to certain signals and strong anti-noise ability and anti-intercept ability. In addition, chaotic series which can provide a huge number of unrelated, random and certain sequences that are easy to produce and regenerate because of the sensitivity to the initial phase can be used for synchronization secure communication and as spreading sequences.

2.3. The construction of the M-ary spreading sequences

At present, the chaotic mappings used to generate the spreading sequences have Tent-Mapping, Logistic-Mapping Chebyshev-Mapping, etc. Logistic-Mapping, one of the most simple one-dimensional nonlinear difference equations, has been extensively researched. In this paper, we use a modified Logistic-Mapping.

The expression of Logistic-Mapping is in Eq. (2)

\[ X_{k+1} = rX_k (1 - X_k), \quad 0 < X_k < 1 \]

(2)

In Eq. (2), \(1 \leq r \leq 4\), \(r\) is called the fractal parameter. The system works in chaotic state when \(3.5699 < r \leq 4\). Two trajectories of different initial values are shown in Fig. (1). It proves that the chaotic series are aperiodic, do not converge, and unrelated.

![Figure 1. The two trajectories of close initial values (r=4)](image)

Characteristics of the chaotic series can be quantitatively analyzed by methods of probability statistical because of the randomness of chaotic. The probability density function of chaotic series generated by Logistic-Mapping is in Eq. (3)

\[
\rho(x) = \begin{cases} 
\frac{1}{\pi \sqrt{x(1-x)}} & 0 < x < 1 \\
0 & x \leq 0, \quad x \geq 1 
\end{cases}
\]

(3)

The mean of the sequences is
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\[ \bar{x} = E\{x\} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=0}^{N-1} x_i = \int_{0}^{1} x \rho(x) dx = 0.5 \]  
(4)

Autocorrelation function is

\[ R(m) = \lim_{N \to \infty} \frac{1}{N} \sum_{i=0}^{N-1} (x_i-x)(x_{i+m}-x) = \int_{0}^{1} x f''(x) \rho(x) dx x^2 = \begin{cases} 0.125 & \text{if } \tau = 0 \\ 0 & \text{if } \tau \neq 0 \end{cases} \]  
(5)

Cross-correlation function is (initial values \( x_1 \) and \( x_2 \) are mutually independent)

\[ R_{ab}(m) = \lim_{N \to \infty} \frac{1}{N} \sum_{i=0}^{N-1} (x_i-x)(x_{i+m}-x) = \int_{0}^{1} \int_{0}^{1} x_i f''(x_2) \rho(x_1) \rho(x_2) dx_1 dx_2 x^2 = 0 \]  
(6)

On this basis, Eq. (2) transforms to improved Logistic-mapping sequences of better performance with mean 0 as shown in Eq. (7).

\[ x_{k+1} = 1 - 2x_k^2 \quad -1 \leq x_k \leq 1 \]  
(7)

The probability density is

\[ \rho(x) = \begin{cases} \frac{1}{\pi \sqrt{1-x^2}} & -1 < x < 1 \\ 0 & \text{else} \end{cases} \]  
(8)

The chaotic series constructed through the above methods are real-valued sequences. In order to get the spreading sequences, multi-value quantization is needed. The probability density that produced by the improved Logistic-mapping sequences are not uniform. So, non-uniform quantization method is used. Correspond chaotic series whose interval are (-1, 1) to CMSS whose values are in \{0, 1, 2... q-1\} through the mapping of \( G(x_0) \).

Take the \( n \) values “\( a_0, a_1, a_2...a_{q-1} \)” from interval (-1, 1) by size and \( a_0 = -1, a_{q-1} = 1 \). If \( a_k \leq x_n \leq a_{k+1} \), then

\[ G(x_n) = k \quad (k=0,1,2,...,q-1) \]  
(9)

The values of CMSS are uniform. So

\[ \rho(G(x_n) = k) = \frac{1}{q} \]  
(10)

That is

\[ \int_{a_k}^{a_{k+1}} \rho(x) dx = \int_{a_k}^{a_{k+1}} \frac{1}{\pi \sqrt{1-x^2}} dx = \frac{1}{q} \]  
(11)

Then

\[ a_k = -\cos\left(\frac{k\pi}{q}\right) \]  
(12)

So, for any \( k \in \{0, 1, 2,...,q-1\} \)

\[ -\cos\left(\frac{k\pi}{q}\right) \leq x_n \leq -\cos\left(\frac{(k+1)\pi}{q}\right) \quad n = 0, 1, 2... \quad q-1 \]  
(13)
If $y_n = G(x_n)$, then, $y_n$ are the CMSS values in $\{0, 1, 2, \ldots, q-1\}$.

3. Features of CMSS

3.1. Correlation features

The probability density function of CMSS is

$$
\rho(y) = \frac{1}{q} \quad (y = 0, 1, 2, \ldots, q-1) \quad (q \text{ is hexadecimal number}) \quad (14)
$$

Converted to the zero-symmetric collection, the range of CMSS will becomes to $(-\frac{(q-1)}{2}, \frac{(q-1)}{2})$ from $(0, q-1)$.

The mean of CMSS is

$$
\mu = \frac{1}{q} \int_{-\frac{q-1}{2}}^{\frac{q-1}{2}} y \rho(y) dy = \frac{1}{q} \int_{-\frac{q-1}{2}}^{\frac{q-1}{2}} y \times \frac{1}{q} dy = 0
$$

Autocorrelation function is

$$
R_{yy}(m) = \frac{1}{q^2} \int_{-\frac{q-1}{2}}^{\frac{q-1}{2}} y f^m(y) dy \cdot \mu = \frac{1}{q} \int_{-\frac{q-1}{2}}^{\frac{q-1}{2}} y f^m(y) dy = \begin{cases} 
(q-1)^2 / 12q & m = 0 \\
0 & m \neq 0
\end{cases} \quad (16)
$$

Cross-correlation function is

$$
R_{xy}(m) = \frac{1}{q^2} \int_{-\frac{q-1}{2}}^{\frac{q-1}{2}} \int_{-\frac{q-1}{2}}^{\frac{q-1}{2}} y_1 f^m(y_2) dy_1 dy_2 = 0 \quad (17)
$$

The results above are ideal situations. Figures 2-4 shows the contrast between m-sequences and CMSS at the length of 31, 63, and 127.

Figure 2. The comparison at the length of 31
As can be seen in Figures 2-4, the statuses of CMSS that closer to white noise are much more than binary m-sequence. And the available number of addresses of CMSS is more than that of binary m-sequences. The length of CMSS has been truncated in order to make a comparison with m-sequences in the actual simulation, which causes bad effect. But with the increases of sequence length, this effect will be gradually reduced. The cross-correlation of CMSS is relatively small, indicating that the interference between the address codes of each user is small and it is suitable for user-focused occasions.
3.2. Power spectral features

There are lots of statuses of CMSS. So, the power spectrum values of the sequences are wide and disperse in the frequency domain which makes a better spreading effect. At the same time, when the receiver despreads signals, the spectrum of noise has been extended as well. Then the energy of noise through the band-pass filter is reduced.

Fig. 5 shows the comparison of power spectrum between binary m-sequences and CMSS. In Fig. 5 (a) & (b) shows the comparison at the length of 7 and (c) & (d) at the length of 1023.

Figure 5. The comparison of power spectral

Calculated by simulation, the energy values of both 7-bit binary m-sequences and 7-bit CMSS within the filter passband are 49. The energy values of m-sequences and M-ary sequences are 1046529 and 1046500 at the length of 1023. So we can draw the conclusion that power spectral features of CMSS are better than that of binary m-sequences.

4. Anti-interference features

Single-frequency sine wave interference is a special kind of interference, which can be considered as narrowband interference of bandwidth $B_n = 0$. Assume that the system is linear and the receiver has established a synchronization of the useful signal. Then simulate the anti-noise performance with single frequency sine wave interferences added. After processed by receiving system, bit-error-rate (BER) curve is shown in Figure 6.
Figure 6. BER curves

Figure 6 (a), (b), (c) show bit error rate curves of 31-bit, 63-bit, 127-bit binary m-sequences and CMSS. Bit error rate of CMSS is lower than binary m-sequence in the same SNR. Therefore, the anti-noise performances of CMSS are superior to the binary m-sequences.
5. Conclusion

This paper proposes the generation method of Chaos-based M-ary spreading sequences. Theoretical calculations and simulation results show that the CMSS with randomness, certainty, nonperiodic, etc. are closer to white noise and have good security behaviors. There is smaller interference in a multi-user communication environment. CMSS used in spreading system have a lower bit error rate compared with binary m-sequences. Therefore, chaos-based M-ary spreading sequences have a good performance and further research is needed.

References