Study on Multi-objective Location of Distribution Center Based on Covering

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Abstract

Distribution center is an important part of modern logistics system. In the process of designing distribution center, running expense can be greatly reduced by selecting appropriate location of distribution center. In this paper, the covering method was used twice to choose the distribution center and minimize the total cost. At last, a case was given to test the model.

Keywords: Multi-objective, Location, Covering

1. Introduction

In the logistics network, distribution center serves as the link between the suppliers and the demand points, and is the essential part in the modern logistics system. In the competing environment, the scientific construction of distribution center is called for in order to reduce the circulation expenses and increase the efficiency. In addition, the distribution center is composed of numerous buildings and equipment, which results in tremendous waste once moved. Therefore, appropriate location selection of distribution center is imperative in terms of reducing the transporting expenses, which contributes to the decreased operation cost and increased economic efficiency.

The modeling and solution of the distribution center location selection have always been the focus of scientific study. Much research work has already been done in the field. The mathematic model research based on distribution center function and cost has been put into practice for over 60 years. The most frequently used models and methodologies can be generally divided into two categories: qualitative and quantitative. Qualitative method mainly uses multiple attribute methods strategy [1-3]. For example, Delphi Expert Consultation Method and Hierarchical Analysis Method etc. Quantitative methods mainly include consecutive method and discrete method. Consecutive model maintains any plane spot can be selected as the distribution center location, such as Gravity Method[4-5]; whereas discrete model identifies the optimal point[6-11] among limited number of alternatives as the location for distribution center. The discrete model is more often adopted.

The mathematic model in this essay is also based on the discrete case, which is to offer several location options, and determine the optimal one as the distribution center location. Covering is a commonly used method in overall resources arrangement. Covering is defined as the meeting of certain standard of time or distance between suppliers and demand points. Because the distribution center links the suppliers and the demand points, we use coverings twice to determine the location of distribution center. Through the covering model, the distribution center serves as the transit for the suppliers and demand points within certain area, which reduces the expenses greatly.

2. Research Method

2.1. Model Hypothesis

Any model is based on certain hypothesis. In order to describe the case adequately, the following hypothesis is made:
1) New distribution center needs to be set up within alternative area;
2) Only expenses on construction and transporting are calculated as cost;
3) Transporting cost covers the transportation between suppliers and distribution center and that between distribution center and demand points. The network of accesses enables multiple suppliers for one distribution center and one demand points served by many distribution centers;
4) The capacity of distribution center and suppliers and the demanding volume of the demand points are all limited;

The symbol in the model is defined as the following:

\[ S = \{ S_1, S_2, \cdots, S_m \} \], all suppliers;
\[ D = \{ D_1, D_2, \cdots, D_n \} \], all demand points;
\[ F = \{ F_1, F_2, \cdots, F_L \} \], distribution center alternatives available;
\[ r_k, (k = 1, 2, \cdots, L) \], the maximum capacity of each distribution center;
\[ s_i, (i = 1, 2, \cdots, m) \], the capacity of each supplier;
\[ d_j, (j = 1, 2, \cdots, n) \], the demanding volume of each demand point;

\( y^1_{ik} \) indicates the volume transported from supplier \( S_i \) to distribution center \( F_k \);
\( y^2_{jk} \) indicates the resources volume transported from distribution center \( F_k \) to demand point \( D_j \);
\( t^1_{ik} \) indicates the time took from supplier \( S_i \) to distribution center \( F_k \) under normal circumstance.
\( t^2_{jk} \) indicates time used from distribution center \( F_k \) to demanding point \( D_j \) under normal circumstance.

\[ M'_1 = \{ i | r^2_{ik} \leq T^1_k, i = 1, 2, \cdots, m \} \] is defined as the set of suppliers whose distance to the distribution center \( F_k \) is no further than \( T^1_k \). It is a covering. A matrix can also be defined as

\[ A = (a_{ik})_{m \times L} \], in which

\[ a_{ik} = \begin{cases} 0 & t^1_{ik} > T^1_k \\ 1 & t^1_{ik} \leq T^1_k \end{cases} \]

\[ M'_2 = \{ j | r^2_{jk} \leq T^2_k, j = 1, 2, \cdots, n \} \] is defined as the set of demand points whose distance to the distribution center \( F_k \) is no further than \( T^2_k \), a matrix can also be defined as

\[ B = (b_{jk})_{n \times L} \], in which

\[ b_{jk} = \begin{cases} 0 & t^2_{jk} > T^2_k \\ 1 & t^2_{jk} \leq T^2_k \end{cases} \]

\( c_k, k = 1, 2, \cdots, L \), indicates the cost of construction of the distribution center;
\( c^1_{ik} \) indicates the transportation expense per unit of resources from the supplier \( S_i \) to the distribution center \( F_k \), which is proportional to the distance or time, here the proportional coefficient is assumed as \( k \), then \( c^1_{ik} = k t^1_{ik} \);
\( c^2_{jk} \) indicates the transportation expenses per unit of resources from the distribution center \( F_k \) to the demand point \( D_j \), the proportional coefficient is assumed as \( k \), then \( c^2_{jk} = k t^2_{jk} \);
\( x_k = \begin{cases} 1, F \ is \ selected \ as \ the \ distribution \ center & \text{if} \ k = 1, 2, \cdots, L \\ 0, F \ is \ not \ selected \ as \ the \ distribution \ center \end{cases} \)

In which \( x_k \), \( y^1_{ik} \) and \( y^2_{jk} \) are variables.
2.2. Mathematical Model

\[
\begin{align*}
\text{min} & \quad \sum_{i=1}^{k} C_i x_i + \sum_{i=1}^{m} \sum_{j=1}^{n} y_{ik}^1 c_{ik} + \sum_{i=1}^{k} \sum_{j=1}^{n} y_{jk}^2 c_{jk} \\
\text{s.t.} & \quad \sum_{i=1}^{m} a_{ik} y_{ik}^1 \leq r_i + M (1-x_i) \quad k = 1, 2, \cdots, L \\
& \quad \sum_{j=1}^{n} b_{jk} y_{jk}^2 \leq r_j + M (1-x_j) \quad k = 1, 2, \cdots, L \\
& \quad \sum_{i=1}^{m} a_{ik} y_{ik}^1 \geq \sum_{j=1}^{n} b_{jk} y_{jk}^2 \quad k = 1, 2, \cdots, L \\
& \quad \sum_{k=1}^{t} a_{ik} y_{ik}^1 \leq s_i \quad i = 1, 2, \cdots, m \\
& \quad \sum_{k=1}^{t} b_{jk} y_{jk}^2 \geq d_j \quad j = 1, 2, \cdots, n
\end{align*}
\]

(1) Is objective function, which comprised two kinds of costs, namely distribution center construction cost and transportation cost.

(2) Indicates the resources provided by suppliers within the coverage area of distribution center can't exceed the capacity of the distribution center.

(3) Indicates the resources transported to demand points can't exceed the capacity of the distribution center.

(4) The resources from the suppliers can't be less than the resources provided to the demand points.

(5) Indicates the resources transporting to the distribution center can't exceed its capacity.

(6) Indicates the recourses transported from the distribution center to the demand points can't be less than the demanding volume.

(7) Indicates value \( x_i \) is 0–1, \( M \) is sufficiently large, \( c_i \) is sufficiently small.

3. Results and Analysis

3.1. Tabu Search Algorithm

Intensive covering is adopted in the location selection of distribution center, which is a NP hard, we therefore use Heuristic Algorithm to seek the solution. The commonly used Heuristic Algorithm includes Genetic Algorithm, Simulated Annealing, and Emergency Searching, etc. Here we use Tabu Search Algorithm. Tabu Search Algorithm is the expansion of Partial Algorithm. Glover [14] proposed the concept in 1986 and developed a set of completed algorithm. The Tabu Search Algorithm is characterized by the use of taboo technology, which means no repetition of the previous work, to avoid partial area searching being the partial optimal, for achieved partial optimal points in a taboo form record, Tabu Search Algorithm uses them in the next searching, where data in the record will not or selectively search for those points, in this way, partial optimal points is skipped, and larger searching area is reached.

In Tabu Search Algorithm, neighborhood must be defined. The formula of location selection is \( X = \left\{ \frac{z}{1 \cdots 1} \right\} \), in which any value is 0 or 1, 0 represents no distribution center at the given site, 1 represents the contrary, the solution represents a location proposal. The change of the value means the change from 0 to 1 or from 1 to 0. The neighborhood of each
solution X comprises solutions conforming to above rule with only one value changeable, the number of the solutions in the neighborhood is L. Tabu Search Algorithm uses taboo to avoid repeated work where the rule is defined as when one value changes, the change can't be repeated, for example, if the value i shifts from 0 to 1, then within the length of the taboo, the variable can't shift from 1 to 0. Length of the taboo means the iterative number the taboo target is not allowed to choose, here we select a commonly used rule, to make the length \( \sqrt{n} \), n is the number of solutions in the neighborhood; if it is not integer, upward rounding applies. When all the solutions in the neighborhood are banned, the ban on the change with minimum objective function value is lifted. Evaluation function selects objective function.

3.2. Steps of Algorithm

Step 1: Initial solution \( \mathcal{X}^0 \) presented for the given case

Step 2: Given the solution \( \mathcal{X}^0 \), the location selection and the corresponding covering are determined, then the solution model is used to get the distribution proposal, i.e. the optimal solution \( \mathcal{Y}^0 \), based on which the objective function \( G^0 \) is figured out.

Step 3: Based on the rule of neighborhood, look for the solution \( \mathcal{X}^i_k \) with minimum objective function value in the neighborhood \( \mathcal{X}^i \); if the objective \( G^i \) is less than the initial corresponding objective value \( G^0 \), the minimum solution for objective function value is \( \mathcal{X}^i = \mathcal{X}^0 \), and the searching for neighborhood is continued.

Step 4: Update the taboo record \( \mathcal{X} \), then indicate the solution without ban as \( \mathcal{X}^i_k \) in the neighborhood \( \mathcal{X}^i \), and calculate its corresponding objective function value \( G^{i+1} \).

Step 5: If the \( \mathcal{X}^i_k \) corresponding objective function value \( G^{i+1} \) is less than \( G^i \), then \( G^{i+1} \) is defined as the new starting point \( \mathcal{X}^{i+1} \) until the given number N is reached, then proceed to next step. Otherwise, return to the previous step.

Step 6: The corresponding solution \( \mathcal{X}^i \) and \( \mathcal{Y}^k \) of objective function value \( G^i \) are selected as the final solution.

4. Algorithm Example

Suppose 3 suppliers, 5 distribution center alternatives, and 6 demand points are available, we know that:

\[ r_1 = r_2 = r_3 = r_4 = r_5 = 1200; \]
\[ s_1 = 600, s_2 = 700, s_3 = 500; \]
\[ d_1 = 260, d_2 = 340, d_3 = 200, d_4 = 300, d_5 = 200, d_6 = 450; \]
\[ c_1 = 1000, c_2 = 1500, c_3 = 800, c_4 = 1400, c_5 = 1200; \]
\[ T^1_k = T^2_k = 6; \quad k = 1 \]

The matrix of transporting time from the suppliers to the distribution center alternatives is \( T^1 \), and that of transporting time from the distribution center to the demand points is \( T^2 \):

\[
T^1 = \begin{bmatrix}
4 & 5 & 8 & 6 & 9 \\
7 & 3 & 5 & 9 & 4 \\
6 & 8 & 6 & 5 & 5
\end{bmatrix}, \quad T^2 = \begin{bmatrix}
9 & 6 & 5 & 8 & 4 \\
6 & 4 & 7 & 8 & 5 \\
5 & 9 & 3 & 5 & 7 \\
9 & 3 & 4 & 7 & 6 \\
8 & 5 & 6 & 4 & 9 \\
4 & 7 & 8 & 6 & 8
\end{bmatrix}
\]
Matlab6.5 programming is used to develop $x_1 = x_2 = 1$, the overall cost is 17720,

$y^{11}_{11} = 200, y^{12}_{12} = 400, y^{22}_{22} = 700, y^{31}_{31} = 450$

$y^{12}_{12} = 260, y^{22}_{22} = 340, y^{31}_{31} = 200, y^{22}_{32} = 300, y^{32}_{32} = 200, y^{31}_{51} = 450$

The result suggests that the distribution center should be located at point 1 and point 2.
The suppliers for distribution center 1 should be 1 and 3, and the demand points should be 3 and 6;
The suppliers for distribution center 2 should be 1 and 2, and the demand points should be 1, 2, 4 and 5.

5. Conclusion

It is of great significance to conduct study on distribution center location as it plays important role in the logistics system. The optimal model is constructed and two coverings are adopted to minimize the overall cost. At the end of the essay, calculation example is provided to test and verify the validity of the model.

References