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Abstract

This paper presents a critical survey of the reference current generation (RCG) methods for harmonic and reactive power compensation for the electrical distribution systems. A critical review of the p-q method, the d-q(id-iq) method, the p-q-r method, the UPF method and the FBD method are presented for the sake of comparison. In order to mitigate the deficiencies of these existing methods, the modified d-q method, the modified p-q-r method, and the modified p-q method are also presented. The MATLAB simulation of these algorithms is carried out for different grid voltage and load conditions scenarios. It is found that, all these algorithms show similar performance in case of ideal grid voltage scenario. In case of grid voltage unbalance or harmonics, the difference in steady state compensation accuracy among these algorithms is presented.

Keywords: reference current generation, instantaneous reactive power theory, harmonic compensation, grid distortion, power quality compensation

1. Introduction

With the emergence of sinusoidal voltage sources, the electric power network could be made more efficient if the load current were in phase with the source voltage. Therefore, the concept of reactive power was defined to represent the quantity of electric power due to the load current that is not in-phase with the source voltage. The average of this reactive power during one period of the line frequency is zero, which implies no energy transmission from the source to the load [1, 2]. One of the major concerns related to electric equipment was power factor correction, which could be done by capacitor banks or reactors. For all situations, the load acted as a linear circuit drawing a sinusoidal current from a sinusoidal voltage source. Hence, the conventional power theory based on active-, reactive- and apparent-power definitions was sufficient for design and analysis of power systems [3, 4].

However, with the widespread of the power electronics devices, the nonlinear loads that consume non-sinusoidal current have increased significantly, which generates nonlinear currents and corrupts the electric distribution network. The conventional power definition is no longer suitable to describe the nonlinear phenomenon in the electric network [2-4, 5]. As a result, the instantaneous reactive power theory (IRPT) was presented [6, 7], which defines a set of instantaneous powers in the time domain. The IRPT-theory is suitable to be applied to the three-phase systems with or without neutral conductors, as well as to generic voltage and current waveforms [4-6, 8].

Distribution loads like arc furnaces and thyristor rectifiers draw fluctuating and harmonic currents from the utility grid. These non sinusoidal currents cause a voltage drop across the finite internal grid impedance, and the voltage waveform in the vicinity becomes distorted. Hence, the normal operation of sensitive consumers is jeopardized. Active power filters (APFs) are a means to improve the power quality in distribution networks [6-8, 9]. In order to reduce the injection of non sinusoidal load currents shunt active filters are connected in parallel to disturbing loads. Figure 1 outlines the electrical circuit for the active filter investigated in this paper. Its main component is a voltage source inverter (VSI) with dc link capacitors. The VSI is
connected to the point of common coupling (PCC) via a decoupling inductor that is usually the leakage inductance of a transformer. The configuration is identical with an advanced static var compensator [10, 11]. These are various aspects regarding to the control strategies of the APFs, namely, the reference current generation (RCG) algorithms, the current tracking schemes, and the dc-link capacitor voltage regulation methods. Among these aspects, the RCG algorithm is the prerequisite for accurate compensation and stable operation of the APFs, hence it is systematically analyzed and discussed herein.

This paper presents a critical survey of the reference current generation (RCG) methods for harmonic and reactive power compensation for the electrical distribution systems. Section 2 presents a review of the p-q method, the d-q (id-iq) method, the p-q-r method, the UPF method and the FBD method [9-11] for the sake of comparison. In order to mitigate the deficiencies of these existing methods, the modified d-q method, the modified p-q method, and the modified p-q-r method are also presented. In Section 3, the MATLAB simulation of these algorithms is carried out for different grid voltage and load conditions scenarios. It is found that, all these algorithms show similar performance in case of ideal grid voltage scenario. However, in case of grid voltage unbalance or harmonics, these algorithms show different compensation accuracy.

![Figure 1. The schematic diagram of the active power filter](image)

2. Review of the Reference Current Generation Techniques

2.1. The p-q method

![Figure 2. The p-q method based on the instantaneous reactive power theory](image)
The p-q method based on the instantaneous reactive power theory is shown in Figure 2. In the three-phase three-wire system, transform the grid voltage \( (v_{sa}, v_{sb}, v_{sc}) \) and load current \( (i_{La}, i_{Lb}, i_{Lc}) \) from phase a-b-c coordinates to \( \alpha-\beta-0 \) coordinates by means of the Clack-transformation, which is represented by Eq. (1).

\[
\begin{bmatrix}
    v_{sa} \\
    v_{sb} \\
    v_{sc}
\end{bmatrix} = \frac{2}{\sqrt{3}} \begin{bmatrix}
    1 & -1/2 & -1/2 \\
    -1/2 & -1/2 & 1
\end{bmatrix}
\begin{bmatrix}
    v_{s\alpha} \\
    v_{s\beta} \\
    v_{s\gamma}
\end{bmatrix} = \frac{2}{\sqrt{3}} \begin{bmatrix}
    1 & -1/2 & -1/2 \\
    -1/2 & -1/2 & 1
\end{bmatrix}
\begin{bmatrix}
    i_{La} \\
    i_{Lb} \\
    i_{Lc}
\end{bmatrix}
\]

(1)

The instantaneous active power and reactive power at the load side can be defined by the following equation (2).

\[
\begin{bmatrix}
    p_L \\
    q_L
\end{bmatrix} = \begin{bmatrix}
    v_{s\alpha} & v_{s\beta} \\
    -v_{s\beta} & v_{s\alpha}
\end{bmatrix} \begin{bmatrix}
    i_{La} \\
    i_{Lb}
\end{bmatrix}
\]

(2)

For a three-phase system, the values of the instantaneous powers \( p_L \) and \( q_L \) contain dc and ac components depending on the existing active, reactive, and distorted powers in the system. The dc components of \( p_L \) and \( q_L \) represent the active and reactive powers in the load current and can be removed by the HPFs (high pass filter) with a cutting frequency between 5~25Hz. The ac components of \( p_L \) and \( q_L \) are afterwards calculated back to the abc coordinates to obtain the harmonic current distortion \( (i_{coref}, i_{qref}) \) as in Eq. (3), given as APF reference current for the inner current controller. The presence of the numerical filters influences the dynamic and the accuracy of the entire APF response [6, 8, 10-12].

\[
\begin{bmatrix}
    i_{coref} \\
    i_{qref}
\end{bmatrix} = -\frac{1}{v_{s\alpha}^2 + v_{s\beta}^2} \begin{bmatrix}
    v_{s\alpha} & -v_{s\beta} \\
    -v_{s\beta} & v_{s\alpha}
\end{bmatrix} \begin{bmatrix}
    p_L \\
    q_L
\end{bmatrix}
\]

(3)

The calculation in Eq. (2) is affected if the system has a zero sequence component due to an existing unbalance. In case of distorted voltages, the calculation of the instantaneous powers and the reference currents is influenced and the mitigation of the harmonics is not suitable, which will cause more serious distortion and call for filtering techniques for grid voltage.

2.2. The d-q method

The synchronous d-q reference frame method is shown in Figure 3. This method is based on the computation of the three-phase load current into a synchronous dq reference frame. In these coordinates, it can be seen that the fundamental and harmonic components are transformed into dc and alternating components, respectively, which is expressed in Eq. (4). The ac components \( i_d \) and \( i_q \) can be obtained by using HPF, thus the APF reference current in abc coordinates can be obtained by inverse Clack transformation.

\[ i_{coref} = i_d + i_d^{ref} \]

\[ i_{qref} = i_q + i_q^{ref} \]

Figure 3. The d-q method based on the instantaneous reactive power theory
Note that in order to obtain the load current in the synchronous domain, a synchronization signal is required, usually provided by a phase-locked loop (PLL). As shown in Eq. (4), \( \theta_{PLL} \) indicates the phase angle obtained from PLL, which is synchronized with the fundamental component of the grid voltage. Therefore, the errors on the APF reference current arise in case of distorted voltage due to PLLtracking errors.

\[
\begin{bmatrix}
i_d \\
i_q
\end{bmatrix} = \begin{bmatrix}
i_d + \bar{i}_d \\
i_q + \bar{i}_q
\end{bmatrix}
\]

\[
\frac{2}{3}
\begin{bmatrix}
\cos \theta_{PLL} & \cos(\theta_{PLL} - 2\pi/3) & \cos(\theta_{PLL} + 2\pi/3) \\
-\sin \theta_{PLL} & -\sin(\theta_{PLL} - 2\pi/3) & -\sin(\theta_{PLL} + 2\pi/3)
\end{bmatrix}
\begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix}
\]

\( (4) \)

2.3. The p-q-r method

![Diagram](a) ![Diagram](b)

Figure 4. the generation of the p-q-r reference frame

According to the definition of p-q-r theory proposed in [10], the two consecutive transformations are required to get the three coordinate variables rotating along with the grid voltage space-vector. The generation of the p-q-r reference frame is shown in Figure 4.

Firstly, the grid voltages and load current are transformed to \( \alpha-\beta-0 \) coordinates, which is expressed in Eq. (1), and then the voltage space-vectors in p-q-r coordinate is defined as:

\[
\begin{bmatrix}
v_p \\
v_q \\
v_r
\end{bmatrix} = \frac{1}{v_{\alpha0}}
\begin{bmatrix}
v_u & v_p & v_0 \\
v_{\alpha0} & v_{\alpha0} & v_{\alpha0} \\
v_{\beta0} & v_{\beta0} & v_{\beta0}
\end{bmatrix}
\begin{bmatrix}
v_a \\
v_b \\
v_c
\end{bmatrix}
= \begin{bmatrix}
v_{\alpha0} \\
v_0 \\
v_0
\end{bmatrix}
\]

\( (5) \)

The p-q-r coordinates are rotating along with the grid voltage space-vector, thus the load currents in p-q-r coordinates are defined as:

\[
\begin{bmatrix}
i_p \\
i_q \\
i_r
\end{bmatrix} = \frac{1}{v_{\alpha0}}
\begin{bmatrix}
v_u & v_p & v_0 \\
v_{\alpha0} & v_{\alpha0} & v_{\alpha0} \\
v_{\beta0} & v_{\beta0} & v_{\beta0}
\end{bmatrix}
\begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix}
\]

\( (6) \)
where $v_{a0b} = \sqrt{v_a^2 + v_b^2 + v_\beta^2}$ and $v_{a\alpha\beta} = \sqrt{v_\alpha^2 + v_\beta^2}$, from Eqs. (5) and (6), the instantaneous real power $p$ is defined as a scalar product of the voltage and current vectors as:

$$p = \vec{v}_{pp} \times \vec{i}_{pp} = v_p v_p$$

(7)

And the instantaneous imaginary power $q$ is defined as a vector product of the voltage and current vectors as:

$$q = \vec{v}_{pp} \times \vec{i}_{pp} = \begin{bmatrix} 0 \\ -v_p i_q \\ v_p i_q \end{bmatrix}$$

(8)

Therefore, the load currents in p-q-r coordinates can be derived as:

$$\begin{bmatrix} i_p \\ i_q \\ i_r \end{bmatrix} = \frac{1}{v_p} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} p_u \\ q_r \\ q_q \end{bmatrix}$$

(9)

According to the definition of p-q-r method, the APF reference current in $\alpha\beta$-0 frame can be expressed as:

$$\begin{bmatrix} i_{q0} \\ i_{a0} \\ i_{c0} \end{bmatrix} = \frac{1}{v_{a\alpha\beta}} \begin{bmatrix} v_0 \\ -v_\alpha v_\beta \\ v_\beta \end{bmatrix} \begin{bmatrix} p_{Lu}/v_p \\ q_{Lu}/v_p \\ -q_{Ld}/v_p + v_0 p_{Lu} \end{bmatrix}$$

$$= -k \begin{bmatrix} 0 \\ v_\alpha \\ v_\beta \end{bmatrix} \begin{bmatrix} 0 \\ v_\alpha v_\beta \\ v_\beta v_\alpha \end{bmatrix}$$

(10)

2.4 The unity power factor method (UPF)

The objective of unity power factor method is to make the source current in phase with the PCC voltage whether the PCC voltage is ideal or unbalance, distorted etc., and the reference current $i_{ref}$ and power $P_s$ in the grid side will be expressed as in Eqs. (11) and (12).

$$i_s = k \cdot \vec{u}$$

(11)

$$P_s = \vec{u} \cdot \vec{i}_s = \vec{u} \cdot k \cdot \vec{u} = k(u_0^2 + u_\alpha^2 + u_\beta^2)$$

(12)

where $k$ is a constant which is related to the PCC voltage and load current.

Due to the power delivered by the source equals the dc componet of the instantaneous active power of load, then the reference source current can be got:

$$\begin{bmatrix} i_{s0ref} \\ i_{s\alpha\beta ref} \\ i_{s\gamma ref} \end{bmatrix} = k \begin{bmatrix} u_0 \\ u_\alpha \\ u_\beta \end{bmatrix} = \frac{P_{Ref} + P_{L0}}{(u_0^2 + u_\alpha^2 + u_\beta^2)_{dc}} \begin{bmatrix} u_0 \\ u_\alpha \\ u_\beta \end{bmatrix}$$

(13)

$$k = \frac{P_{Ref} + P_{L0}}{(u_0^2 + u_\alpha^2 + u_\beta^2)_{dc}}$$

(14)
It can be seen that in three-phase four-wire system, if the zero sequence voltage \( u_0 \neq 0 \), then the zero sequence active power exist at the grid side.

### 2.5. The FBD Method

Based on the IPRT-based RCG scheme, the FBD method was presented in [6], which is also called vector method. The essence of the FBD method is to represent the load as the ideal equivalent conductance at each phase, and the power consumption is totally absorbed by this equivalent conductance. This method avoids the complicated coordinates transformation, and the instantaneous reactive power is denoted as:

\[
\tilde{q}(t) = \tilde{v}^* \tilde{i}_q = \frac{1}{\sqrt{3}} \begin{bmatrix} v_b - v_c \\ v_r - v_c \\ v_b - v_r \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \tilde{q}_q(t)
\]

where \( \tilde{v} \) denotes the voltage vector after the zero-sequence component is eliminated, and the relations between the currents and the instantaneous power are as follows:

\[
\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \frac{p}{v^*} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} + \frac{1}{\sqrt{3}} \begin{bmatrix} p_a \\ p_b \\ p_c \end{bmatrix} + \frac{1}{\sqrt{3}} \begin{bmatrix} q_a \\ q_b \\ q_c \end{bmatrix} \left( \begin{array}{c} v_b - v_c \\ v_r - v_c \\ v_b - v_r \end{array} \right)
\]

From the above equation, it can be noticed that the current vector is composed of three parts. The first part is related to the voltage vector and the instantaneous active power. The second part is related to the zero-sequence voltage components and the instantaneous power. The third part is related to the instantaneous reactive power and line voltage. The target of the FBD method is to make the grid currents in-phase with the grid voltages without zero-sequence components, i.e., without the zero-sequence currents. In the previous literature, The average load conductance \( G_e \) was proposed by Fryze et al., which denotes the relations between the reference grid currents with the grid voltages, as:

\[
G_e = \frac{P_{L\mu}}{V^2}
\]

where \( V \) denotes the root-mean-square (RMS) value of the grid voltage without zero-sequence components, and \( P_{L\mu} \) denotes the active power at the load side. The conductance \( G_e \) can be calculated by the grid voltages and the load side currents:

\[
G_e = \frac{\int v(t) i(t) dt}{(V_{i^+}^*)^2} = \frac{\sum_{k=1}^{\infty} V_k I_k \cos(\phi_k - \theta_k)}{(V_{i^+}^*)^2}
\]

where \( \phi_k, \theta_k \) denotes the phase angle of the \( k \)th order harmonic voltage and harmonic current, respectively.

Since the target of harmonic compensation by active filters is to achieve sinusoidal waveform of the grid currents in-phase with the grid voltages, hence the compensating currents needs to be modified in case of grid voltage unbalance or asymmetry, as denoted by:

\[
\begin{bmatrix} i_{srefa} \\ i_{srefb} \\ i_{srefc} \end{bmatrix} = G_e \begin{bmatrix} u_{srefa} \\ u_{srefb} \\ u_{srefc} \end{bmatrix} = G_e \tilde{u}^+_i = \frac{P_{L\mu}}{(V_{i^+}^*)^2} \tilde{u}^+_i
\]

where \( U_{i^+}^* \) denotes the RMS value of the fundamental grid voltage, and \( \tilde{u}^+_i \) denotes the positive sequence fundamental component of the grid voltages.
2.6. The modified p-q method

In order to compensate harmonic currents as well as the fundamental reactive power component, the existing p-q method can be further enhanced. Meanwhile, the grid voltage unbalance can be avoided, and the grid currents are in-phase with the fundamental components of the grid voltages, hence the reference signal of the grid side current can be denoted as:

\[ i_{\text{ref}} = k \cdot u_i^+ \]  \hspace{1cm} (20)

Therefore, the active power of the grid side can be derived as:

\[ p_x = u \cdot i_{\text{ref}} = u \cdot k \cdot u_i^+ = k(u_a^+ u_{a1}^+ + u_b^+ u_{b1}^+) \]  \hspace{1cm} (21)

Assuming the grid side power equals to the dc component of the load side instantaneous power, the reference signals of the grid side currents are:

\[
\begin{bmatrix}
  \frac{i_{\text{ref}0}}{u_{a1}^+ + u_{b1}^+} \\
  \frac{i_{\text{ref}a}}{u_{a1}^+ + u_{b1}^+} \\
  \frac{i_{\text{ref}b}}{u_{a1}^+ + u_{b1}^+} \\
\end{bmatrix}
= \begin{bmatrix}
  0 \\
  u_{a1}^+ \\
  u_{b1}^+ \\
\end{bmatrix}
\]  \hspace{1cm} (22)

where \( u_{a1}^+ \) and \( u_{b1}^+ \) denotes the fundamental positive-sequence components in the \( \alpha \beta \) coordinates.

The FBD method and the modified p-q method are effective for harmonic current and reactive compensation in case of grid voltage unbalance or asymmetry, and sinusoidal voltage can be achieved at the grid side. The difference between these two methods exists, i.e., the modified p-q method acquires the reference signals by using the load side fundamental active power and the fundamental grid voltage. And the FBD method acquires the reference signals by using the load side fundamental active power, harmonic power and the grid side fundamental voltage. As a result, the reference signals may show some small difference in case of grid distortion.

2.7. The modified d-q and p-q-r method

Followed by the idea of the previous subsection, the conventional d-q and p-q-r methods can also be enhanced by using similar approach. In other words, the fundamental positive sequence components of the grid voltages can be utilized to furnish the reference signals of these two RCG methods in case of grid distortion or harmonics.

3. The comparison of the various RCG methods

In order to evaluate the similarities and differences among the various RCG methods discussed in the previous subsections, namely, the p-q method, the d-q(i-r-l0) method, the p-q-r method, the UPF method and the modified d-q method, the modified p-q-r method, the FBD and the modified p-q method, the MATLAB simulation is carried out for different grid voltage and load conditions scenarios. The following four cases are presented:

1. The grid voltages are contaminated by 10% fundamental frequency negative-sequence and 20% 5th order harmonic component, and the harmonic loads are assumed to be balanced.

2. The grid voltages are contaminated by 10% fundamental frequency negative-sequence and 20% 5th order harmonic component, and the harmonic loads are assumed to be unbalanced.

3. The grid voltages are contaminated by 20% fundamental frequency zero-sequence and 20% 5th order harmonic component, and the harmonic loads are assumed to be balanced.

4. The grid voltages are contaminated by 20% fundamental frequency zero-sequence and 20% 5th order harmonic component, and the harmonic loads are assumed to be unbalanced.

The simulation waveforms of the grid voltages \( v_a, v_b, v_c, v_0 \), load currents \( i_{La}, i_{Lb}, i_{Lc}, i_{L0} \) and the grid side reference currents \( i_{sa}, i_{sb}, i_{sc}, i_{s0} \) are shown in Figures 5-8.
Figure 5. Performance comparison among the algorithms in case 1: the grid voltage, load current and grid side reference current

Figure 6. Performance comparison among the algorithms in case 2
From Figures 5-8, it can be observed that the conventional $p$-$q$, $d$-$q$($i_d$-$i_q$) and $p$-$q$-$r$ methods suffer from insufficient performance in case of grid voltage unbalance or harmonics.
The well-known p-q-r method shows poor accuracy in case of grid voltage zero-sequence and harmonic distortion, and the grid side currents are still asymmetric after compensation. Furthermore, the other methods achieve grid side balanced currents compensation, but the current waveforms are highly distorted. The UPF method results in exact same waveform of grid currents with the grid voltages.

The modified methods, on the other hand, achieve satisfactory compensation performance and the neutral-wire current is also mitigated. The FBD method shows some deficiency compared to the modified p-q, modified p-q-r and PHC methods in case of grid voltage distortion, owing to the fact that the instantaneous harmonic power is taken into consideration when calculating the reference signals. Notably, all the aforementioned methods show similar performance in case of ideal grid voltages scenarios.

4. Conclusion

This paper presents a critical survey of the reference current generation (RCG) methods for harmonic and reactive power compensation for the electrical distribution systems. A review of the p-q method, the d-q(id iq) method, the p-q-r method, the UPF method and the FBD method are outlined for the sake of comparison. In order to mitigate the deficiencies of these existing methods, the modified d-q method, the modified p-q-r method, and the modified p-q method are presented, and the MATLAB simulation of these algorithms is carried out for different grid voltage and load conditions scenarios. It is found that, all these algorithms show similar performance in case of ideal grid voltage scenario. However, in case of grid voltage unbalance or harmonics, these algorithms show different compensation accuracy.

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