Design of Interference Measurement System with Crystal

Shuyan Ren1, Yonggang Yang2, Hailong Duan1

1 Keylaboratory of Information Sensing & Intelligent Control, Tianjin University of Technology and Education, Tianjin, 300222, China, +862288181529
2 Aviation Engineering College, Civil Aviation University of China, Tianjin, 300333, China
*Corresponding author, e-mail: shuyan_ren@163.com

Abstract
A kind of interference measurement system with crystal is designed, which is not influenced much by other factors such as vibration for the property of common path. For the distance measurement, the phase of fringes is calculated on the oblique incident holography to avoid effects of fringe curvature, backup and light fluctuation. But the measurement result is affected by the uniformity of the crystal, and another method of adjusting position based on normal incident holography is also discussed. B-splines function is used to calculate zero position in normal incident holography, which is fitted over the patch image around gray extreme points of the stripes obtained by gray projection. Experiment results show that combining the normal and oblique incident holography reduce the repeatability error of the system and improve the measurement precision effectively.

Keywords: crystal, zero position, B-splines, fringe, phase

Copyright © 2013 Universitas Ahmad Dahlan. All rights reserved.

1. Introduction
In the interference measurement system with crystal, cone beam reflected from measured object is adopted to measure distance and 3D shape of the object, which is also named conoscopic holography [1, 2]. The method is one of common-path interference ways, so it has merits of common-path interference, such as restraining air turbulence and environment vibration [3]. It can measure inclined even spherical plane. Compared with laser trigonometry [4], it is easier to realize without compensation.

Hologram on CCD includes information of laser intensity and phase, and all of them can be utilized to measure. But laser intensity is influenced by the uneven beam amplitude and laser speckle, and phase of fringes is robust for measurement, which picks up numbers of fringes and phase of less one fringe. To calculate the phase of less one fringe, fringe projection with gradient center of mass is adopted to start points of less one fringe which needs to fit by cosine function.

From the measurement principle, errors will not be found in the results if the position of crystal has been moved. But the phenomenon of zero drift has been found in measurement process. The experiments show the repeatability turns down after using a period of time. So the method of zero calibration is proposed in this paper to reduce the repeatability error of the system.

2. Measurement Principle
Conoscopic holography is discovered by G. Sirat and D. Psaltis in 1985 [5, 6]. Optical path of it is shown in Figure 1.

According to Figure 1, laser from 9 goes through the collimation and expending system 10 to polarization spectroscope 4, and is divided into two parts with different polarization states. Reflection laser of the polarization spectroscope with one polarization state passes $\lambda/4$ wave plate 3 and object lens 2, then converges on the object 1. Reflection laser from focal point goes through object lens 2, $\lambda/4$ wave plate 3, polarization spectroscope 4, $\lambda/4$ wave plate 5 and the uniaxial crystal 6. Because of birefringence of the crystal, laser in 6 is divided into the ordinary o and the extraordinary e which have different velocity and polarization state. The velocity of e
laser is depending on the incident angles. Polarizing disc is taken as analyzer, after which both laser interfered. So holography can be received on CCD 8.

Figure 1. Optical structure of measurement system

Compared to the problems of light intensity on hologram, which is easy to be disturbed by light source fluctuation and noise, obtaining phase of fringe on hologram is much easier. Based on these analysis, measurement expression based on phase of Conoscopic Holography measurement system was proposed, which reduces the system demand on stability of light source. But the method depends on the character of crystal fully.

3. The Composition of the System
The measurement system is composed by the laser, optical path, the CCD camera system, software system and electrical control system which is used to compensate the temperature error and adjust the position of optical components. System block diagram is shown in Figure 2.

Figure 2. Block diagram of Conoscopic Holography system

After the collimated beam expander, circular beam with larger diameter is formed. Passing the shrink-beam system, the parallel light is emitted. And a diffuse disc will be found on the work piece. Diffuse reflection from the surface of the work piece goes through the optical path and imaged on the photosensitive surface of CCD. With an image acquisition card,
measured holography is sent to computer and reveal on the screen, and measured information can be calculated by the software.

Through controlling rotated angle of the crystal, the normal and oblique incident holography will be obtained. Normal incident holography is utilized to measure for improving the precision and oblique incident is used to adjust position of optical elements, such as the crystal. Laser and ambient temperature is controlled by the temperature control system adaptively, which avoids the affection to parameters of optical path and property of optical elements.

3.1. Rotation of the Crystal

According to the experiments shown as Figure 3 and Figure 4, if the light from the object is normal incident to the crystal, stripes on the image is much less than that on the oblique incidence conoscopic holography. So the measurement method of oblique incidence is adopted, and the crystal is fixed on the rotation platform, which will cause the movement of the crystal after rotation for a period of time.

![Figure 3. Normal incident conoscopic holography](image)

![Figure 4. Oblique incident conoscopic holography](image)

3.2. Volume of the Crystal

To design the measurement system, volume of the crystal is an important. Along the crystal optical axis, birefringence phenomenon will not be produced. and when the crystal axis is perpendicular to incident surface of the crystal, interference fringes are concentric rings. Optical path of the system is shown as Figure 5.

![Figure 5. Light translation in the optical path](image)

According to the principle of geometrical optics, the size of the crystals incident surface can be determined by the light spot size on the incident surface, which can be denoted:

\[
\tan \theta^' = \left( 1 + \frac{f - Z_s/2}{f} \right) \cdot \frac{d^1}{\frac{f}{2} - \frac{Z_s}{2}}
\]

(1)
\[ D = 2P \tan \theta' + p' \]  

\( p' (1.145 \text{mm}) \) is the entrance pupil of the focusing objective, and \( Z_s (10 \text{mm}) \) is the measurement scope. \( f \) is the focal length of the focusing objective. Considering the sizes of other elements and demands of installment with eq.1 and eq.2, the volume of the crystal is \( 10 \text{mm} \times 10 \text{mm} \times 20 \text{mm} \). It is too difficult to keep the uniformity of the crystal. When the position of the crystal is changed, errors will be brought into the results. So zero position (the center of the concentric rings) need to be calibrated after the system is used for a period of time. To improve the positioning accuracy, B-splines is adopted to realize sub-pixel positioning.

4. Measurement and Calculation of Zero Position

Numbers of fringes on the normal incident holography are much less than those on the oblique incident holography, shown as Figure 3 and Figure 4, and the later is utilized to measure. For different measurement scope and precision, the rotated angle of the crystal will be changed. So it is difficult to realize auto-measurement with undefined stripe width. Using scanning to determine the extreme points, noise point is easy to be regarded as the extreme point. But the frequency of noise is higher than that of fringes, so the spacing between noisy extreme points is quite different to real fringe width. So the real fringe width is needed to be estimated to exclude noise points.

On analysis of optical transmission model, the lowest theoretical width of the fringe is computed. Automatic stripe scanning within a local window can be realized with the lowest theoretical width. According to the theory of crystal optics, fringe spacing is relatively uniform under a bigger rotated angle of the crystal. So the lowest fringe width is defined taken as the standard spacing in scanning process. The lowest fringe width with different angle is shown in Table 1.

<table>
<thead>
<tr>
<th>Rotated angle(°)</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fringe width(pixel)</td>
<td>64</td>
<td>51</td>
<td>39</td>
<td>31</td>
<td>25</td>
<td>23</td>
</tr>
</tbody>
</table>

4.1. Measurement Based on Phase of Fringes

Measurement method combines the rough position and exact position. Gray projection directed by the lowest fringe width is used to get the rough gray apex, around which, a set of points are selected for polynomial fitting to find the exact gray apex on a sub-pixel level. For the measurement with oblique incident holography, second-order polynomial can satisfy the precision demand, and run fast compared to the splines.

It is known to all, interference intensity complies with cosine distribution. Cosine function can work well at fitting the phase of fringe less than a period, denoted as eq.3, which is better than polynomial and splines and avoids uncertain multi-extremes.

\[ y = A + B \cdot \cos(Tx + \varphi) \]

\[ \text{Object} = \sum_{i=1}^{n} (y_i - y)^2 \]  

The phase of fringe less than a period distribute on both sides of the image shown in Figure 6. Considering the differences of holography, the pixels in next integral period are selected to fit together if points in the fringe less than a period is much less. So the tripe gray of enlarged part in Figure 6 is fitted as Figure 7. The measurement phase will be obtained by adding the phase of integral fringes and the phase of fringe less than a period in both sides.

4.2. Calculate the Zero Position Based on B-Splines.

For a measurement image, shown as Figure 3, it is hard to find the zero position. So we adopted the B-spline method to fit the other stripes with better quality near the zero position.
B-splines functions [7] were investigated by a number of researchers in the 1940s. But they did not gain popularity in industry until de Boor published his work. His recurrence formula to derive B-splines is still the most useful tool for computer implementation. Given \( n+1 \) control vertices (or de Boor points) \( B, (i = 1, 2, \ldots, n + 1) \), a B-splines curve of order \( k \) is defined as:

\[
P(u) = \sum_{i=1}^{n+1} B_i N_{i,k}(u), u \in [u_{\text{min}}, u_{\text{max}}], 2 \leq k \leq n + 1
\]

(4)

Where, \( N_i, k(u) \) are polynomials of degree \( k-1 \) known as B-splines of order \( k \). In most practical applications, the B-splines basis functions are derived from the following knot vector (or knot sequence):

\[
x_1 = x_2 = \ldots = x_k, x_{k+1} \leq \ldots \leq x_{n+1} = x_{n+2} = \ldots = x_{n+k+1}
\]

(5)

The \( n-k \) knots \( x_{k+1}, x_{k+2}, \ldots, x_{n+k} \) are named the interior knots. If the interior knots are all distinct, then the B-splines curve has \( n-k+1 \) non-vanishing spans. Non-uniform knot vectors lead to Non Uniform B-splines (NUBS). They may have either unequally spaced and/or multiple internal knot values. The present study on curves is of this category of B-splines. Fitness is to construct a smooth parametric curve \( P(u) \) passing through a sequence of data points \( P_i, i = 1, 2, \ldots, s \). If a data point lies on a B-splines curve, then it must satisfy eq.3. Rewriting eq.3 for each of the \( s \) data points yields a set of equations in matrix form as:

\[
P = CB
\]

(6)

With \( 2 \leq k \leq n+1 \leq s \), \( P, C, B \) are the point data, basis and defining polygon matrices respectively. In case of curve approximation \( C \) is not square, The problem is over-specified and can be solved in some mean sense. Recalling that a matrix times its transpose is always square, the defining polygon vertices for a B-spline curve that smoothes the data is given by:

\[
B = C^T C^{-1} C^T P
\]

(7)

5. Results and Analysis

Design and install the system, and set the rotated angle of the crystal is 8°. According to Tab1, the lower fringe width is 25 pixels. For the same measured distance, pick the holography every 10 minutes and 15 days respectively, and the results are shown in Table 2.

From the results, measurement error is apparent. With repeated the experiment and analysis, zero position shifting of the crystal should take the responsibility for the error. So it is a key step to adjust zero position of the crystal after some time.

On basis of gray projection, the rough location of the center of the sub-image (the linear scanning window near the center of the concentric rings [8]). The second gray apex from left of
the image and the second apex from the right of the image are chosen. In order to calculate the zero position, two patches around the apexes are selected on image as Figure 8, and B-splines is used to fit the intensity of the stripe in the patch. After getting the two apexes, calculate the zero position denoted by two-dimensional coordinates, which is $O(379.13, 268.07)$. Repeat the above process, and zero position deviation calculated is shown in Table 3, according to which, the position of crystal needs to be adjusted. So the most measurement error can be removed.

### Table 2. Phase of fringes under different interval

<table>
<thead>
<tr>
<th>Index</th>
<th>Phase of fringes (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10 minutes</td>
</tr>
<tr>
<td>1</td>
<td>57.53</td>
</tr>
<tr>
<td>2</td>
<td>57.51</td>
</tr>
<tr>
<td>3</td>
<td>57.55</td>
</tr>
<tr>
<td>4</td>
<td>57.59</td>
</tr>
<tr>
<td>5</td>
<td>57.56</td>
</tr>
</tbody>
</table>

6. Conclusion

The conoscopic holography is designed. Normal incident holography is applied to adjust zero position of measurement, and oblique incident holography is used to measure distance. With the big volume, the crystal has poor uniformity. Because of the rotation structure, loose installation is inevitable in measurement, which make the crystal shift and affect results. From above analysis, shifting of zero position which influence the precision and repeatability of conoscopic holography system can be adjusted based on image. To improve the precision of adjustment, B-splines is adopted to find the gray apex of the stripe on image and zero position will be calculated, which is the basis to adjust position of the crystal. For measurement, second-order polynomial can satisfy the precision demand, and run fast compared to the splines. Combine the normal and oblique incident holography work well to reduce the repeatability error of the system and improve the measurement precision effectively.

### Table 3. Results of experiments

<table>
<thead>
<tr>
<th>Index</th>
<th>Zero position deviation [pixel,pixel]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[25.7,17.5]</td>
</tr>
<tr>
<td>2</td>
<td>[61.4,35]</td>
</tr>
<tr>
<td>3</td>
<td>[29.3,7.8]</td>
</tr>
<tr>
<td>4</td>
<td>[23.2,11.5]</td>
</tr>
<tr>
<td>5</td>
<td>[13.7,10.6]</td>
</tr>
</tbody>
</table>

### Figure 8. Image for experiment

6. Conclusion

The conoscopic holography is designed. Normal incident holography is applied to adjust zero position of measurement, and oblique incident holography is used to measure distance. With the big volume, the crystal has poor uniformity. Because of the rotation structure, loose installation is inevitable in measurement, which make the crystal shift and affect results. From above analysis, shifting of zero position which influence the precision and repeatability of conoscopic holography system can be adjusted based on image. To improve the precision of adjustment, B-splines is adopted to find the gray apex of the stripe on image and zero position will be calculated, which is the basis to adjust position of the crystal. For measurement, second-order polynomial can satisfy the precision demand, and run fast compared to the splines. Combine the normal and oblique incident holography work well to reduce the repeatability error of the system and improve the measurement precision effectively.

### Acknowledgements

This work was financially supported by the Science and Technology Development Program of Tianjin Higher Education (20100711), the National Natural Science Foundation of China (61178081), the Research and Development Fund of Tianjin University of Technology and education (kjy11-09).

### References


