Variable Weights in Assessment of Survival System

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Abstract
Weight is one of key parameters in survival assessment. Its veracity will directly affect the final evaluation result. After analyzed the relationships between survival incidents and induced factors, this paper draw into the natural weight, based on impact probability, to dynamically calculate the weights of assessment indexes. Based on membership of susceptible degree, appropriate distribution function is selected to describe the weight change of specific index. Combined with examples, the application process is present in detail. Experiments and analysis show that proposed method can truthfully and objectively reflect the impact of each index on survival system. It is valuable to apply and popularize in a variety of industries.

Keywords: dynamic weight, survivability, natural weight, survival incident, assessment index

1. Introduction
The technology of survivability is as a new generation of network security technology, which represents the new development direction of network security. It emphasizes the ability to provide continuous services, even in the events of attack, failure or accident. The comprehensive assessment of survivability is an important mean to improve the survivability of system, which provides scientific basis for development and operation of survival system by multi-levelly multi-anglely analyzing and understanding the relevant factors. The weight is one of important parameters in the process of assessment, which reflects the significance of each index and the relationships in survival system. How to calculate the weight directly impacts the objectivity and the creditability of assessment result.

At present, the calculation methods of weight mainly include three kinds: the method of subjective analysis, the method [1-3] of combining the subjective and objective and the method of neural network [4]. Most of them have some mortal shortcomings, such as subjectivity, arbitrary, constant weights, lack of dynamic and pertinence, and so on. It is difficult to adapt to evaluate complex or dynamic system. The idea of variable weights [5] was brought in to change the rigid calculated methods, reflected the variation of weights themselves. The evaluation method of risk probability has the perfect theoretical basis, and the means of logical analysis truly reflect the structured process of incidents. The best advantages of this method are the rigorous analysis process and the reliable results, but it needs to count and calculate mass data. After analyzing the probability of events in survival system, this paper proposes a method to dynamically calculate the weights, and gives the formal calculation method. After analysis, we can see that this method can scientifically and objectively reflect the status of evaluation indicators in survival system, and the calculation is simple.

This paper is organized as follows. In section 2 we will analyze the relationship between weights of indexes and survival incidents as well as the weight calculation formula. In section 3 the examples and analysis present the process of application. The conclusions are given in section 4.
2. The probability of survival incidents and the weights

The operations of information system are influenced by various factors. The state of information system at a moment is random \[6,7\]. The operating process of information system can be described as random process. The survival systems are the information system too.

Define 1: if \( \Omega \) is a set of all state in the operating process of survival system, \( S(t) \) is a subset of possible states at the moment \( t \), the operating process of survival system can be described as:

\[
\{ X(t,e) \mid e \in \Omega, t \in (-\infty, +\infty) \}
\]

Which is called a random process, based on \( \Omega \), abbreviated as \( X(t,e) \).

The survival assessment of system studies the possibility of incidents at different state, and the impact on the performance, the reliability and the security of survival system, to analyze the survivability of entire system. We define the influence degree to measure impact of incidents on survival system.

Define 2: in the survival system, if \( S \in \Omega \) is a set of possible states at the moment \( (t_0) \), \( e \in S \) is a possible state at the moment \( (t_0) \), \( S_e \) is the probability of occurrence of \( e \) state at the moment, \( E_e \) is the influence degree on system performance, the equation:

\[
F_e = S_e \times E_e
\]

is called impact degree of possible occurrence of \( e \) state at the moment \( (t_0) \).

However, in the assessment process, we mainly focus on the incidents, which have the serious impacts on system. The impacts include all aspects of survivability, such as availability, security, reliability, efficiency and so on.

if \( T \) is a subset of \( \Omega \), states in \( T \) have serious impacts on system. \( T_i \in T \) is serious state, namely survival incident. \( Q_i \) is the probability of occurrence of \( T_i \) at the moment \( (t_0) \); \( D_i \) is the serious degree of impact. Equation (2) can replace by

\[
R_{i} = Q_{i} \times D_{i}
\]

Define 3: At a fixed moment \( (t_0) \), the influence sum of possible incident is named total impacts of system. The total impacts of system at a fixed moment \( (t_0) \) can show as:

\[
R = \sum_{i \in T} R_i = \sum_{i \in T} Q_i \times D_i
\]

Where: \( T \) is a set of survival incidents that can happen at that moment, \( T_i \) is a survival incident of \( T \), \( T_i \in T \); \( Q_i \) is the probability of \( T_i \) at that moment, \( D_i \) is the degree of damage.

Define 4: if \( \xi \) is a set of all damage values, \( \mu_f(r) \) is a function, which maps any \( r \in \xi \) to \([0, 1]\),

\[
\mu_f : \xi \rightarrow [0,1]
\]

\[
r \rightarrow \mu_f(r)
\]

The \( \mu_f(r) \) is called a membership function base on \( \xi \); the \( f \) is a fuzzy set, which constitute by \( \mu_f(r) (r \in \xi) \); the value of \( \mu_f(r) \) is the degree of membership to \( f \).
Because of the duality between survivability and damage, there is membership between degree of survivability and degree of damage as follows:

Theorem 1: if $\mu_i(r)$ is the degree of survivability, and $\mu_f(r)$ indicates the degree of damage,

$$\mu_i(r) = 1 - \mu_f(r)$$

(6)

So we study the damages in survival system. From equation (3), we can see that the survivability of system includes the probability of occurrence of survival incidents and the damages. In the information system, there are many factors can trigger the survival incidents. But, in summary, all of them can divide into tow kinds: internal factors (leaks), external factors (dangerous behaviors). The external factors utilize internal factors lead to survival incidents. Therefore, the occurrence of survival incident can describe as the function between internal factors and external factors.

Theorem 2: $Q_{Ti}$ presents the probability of occurrence of survival incident ($T_i$). $A_i$ stands for the dangerous behaviors (external factors), $B_i$ indicates the state leaks (internal factors), who can lead to occur $T_i$ at the moment.

$$Q_{Ti} = P(A_i, B_i) = P(A_i \cap B_i)$$

(7)

According to the probability principle [9], equation (4) can change as:

$$Q_{Ti} = P(A_i \cap B_i) = P(A_i | B_i)P(B_i) = P(B_i, A_i)P(A_i)$$

(8)

In the survival system, internal factors indicate defects and deficiencies in the process of design, implementation, operation and control; external factors present factors or incidents which may damage the survivability of system. In generally, external factors always employ internal factors to occur survival incidents to damage system. Internal factors are objective existence, which only are applied by external factors to cause survival incidents; if internal factors are null, the dangerous behaviors can’t trigger survival incidents. Namely, both are independent of each other. According to the probability principle,

$$P(B_i | A_i) \times P(A_i) = P(B_i)$$

$$P(A_i | B_i) \times P(B_i) = P(A_i)$$

(9)

We can get equation (10) from equation (9)

$$Q_{Ti} = P(A_i \cap B_i) = P(A_i) \times P(B_i)$$

(10)

From equation (3), we can get:

$$R_{Ti} = (D_{Ti} \times P(A_i)) \times P(B_i)$$

(11)

The equation (4) can rewrite by:

$$R = \sum_{i \in \tau} R_{Ti} = \sum_{i \in \tau} P(A_i) \times P(B_i) \times D_{Ti} = \sum_{i \in \tau} (P(A_i)D_{Ti}) \times P(B_i)$$

(12)

So, a conclusion can be obtained that the probability of occurrence of a survival incident has the proportional relationship with the probability of leak in dangerous state. The ratio is the
probability of leak in dangerous state at the moment \((t_0)\), which can trigger survival incidents. After normalization,

Define 5: if \(P(B_{i_j})\) indicates the probability of occurrence of a survival incident \((T_i)\),

\[
k_i = \frac{P(B_{i_j})}{\sum_{i \in T} P(B_{i_k})} \tag{13}
\]

\(k_i\) is the natural weight of the survival incident \((T_i)\), based on damaged probability.

So, there are proportional relationships between natural weights of factors and probability of factors in dangerous states. They reflect the probability of themselves in dangerous states, and indicate the importance of those factors in survival system at the same time. They are the appearance of the natural vulnerability of factors. After above analysis, we can see that weights of factors are changing with the system operating status, and the natural weights can represents this situation.

Probability of damages is an important indicator to assess the survival system, but it is difficulty to calculate. Because the damage and survivability are fuzzy and relative concepts, we redefine natural weights to simplify calculation, according to equation (5).

Define 6: At the moment \((t_0)\), if \(K_{T_i}\) indicates susceptible degree of survival incident \(T_i\) \((T_i \in T)\) in survival system, then

\[
K_{T_i} = \frac{\mu_i(P(B_{i_j}))}{\sum_{i \in T} \mu_i(P(B_{i_k}))} \tag{14}
\]

Is the natural weight of survival incident \(T_i\) based on susceptible degree, namely, natural weight. \(T_i\) presents a specific survival incident at the moment \((t_0)\); \(T\) indicates a set of survival incidents at a fixed period, \(T_i \in T\).

So, the weight can be calculated by susceptible degree of factors, and reflect the status proportional relationship of each factors in survival system. And, the calculated method is changed form probability calculation to calculating membership of susceptible degree, which greatly reduces the difficulty of computing. There are many methods to calculate membership, such as fuzzy statistical method, comparison sorting method, expert evaluation method, and so on. In information system, probability of occurrence of survival incident is always positive real number. If the membership function takes real number as discussion domain, it is called fuzzy distribution. And, there are several fuzzy distribution functions to fit for Define 6, such as half-normal distribution, half kosit distribution, and so on.

The function of half-normal distribution:

\[
\mu_i(\mu) = \begin{cases} 0 & \mu \leq a \\ 1 - e^{-k(\mu-a)^2} & \mu > a, k > 0 \end{cases} \tag{15}
\]

The function of half kosit distribution:

\[
\mu_i(\mu) = \begin{cases} 0 & \mu \leq a \\ \frac{a(\mu-a)^\theta}{1 + (a(\mu-a))^{\theta}} & \mu > a, k > 0 \end{cases} \tag{16}
\]

From equation (15) and equation (16), we can see that only three parameters \((k, a, n)\) need to determine in order to get the weight function. In practical applications, it is necessary to select appropriate function to determine the weight distribution, according to practical statistical property.
3. Example and Analysis

Weights are the ratio of states and importance of indicators in the whole survival system, whose specific meanings are determined with the indicators or the projects. At present, there is no unified standard of survivability assessment and index system. The researchers in accordance with their own ideas and research object design different index system, which reflect performance of survival system from different angles and at different levels. As show in figure 1, reference [8] described the index system in three-dimensional space, which reflects the inner relationship between different index systems.

![Figure 1. Model of Evaluating Index System for System Survivability](image)

At present, researchers are more agreeable that there are four characteristics in survival system, namely: resistance, recognition, recovery, adaptation. Combing with classification of survival incidents, we take these characteristics as indexes to present the weight calculation process. According to the survey and analysis a survival system of third party payment, we select the function of half-normal distribution to describe change of weights in this survival system. In practice, the values of statistical probability are positive real number, and change in the range between 0 and 1. Some increase with the increase of values, while others reduce with the increase of values. So, we select positive half-litter function or positive half-off function:

\[
\mu_f(\mu) = \begin{cases} 
0 & \mu \leq a \\
1 - e^{-k(\mu-a)} & \mu > a, k > 0 
\end{cases}
\]  
(17)

\[
\mu_f(\mu) = \begin{cases} 
0 & \mu \leq a \\
e^{-k(\mu-a)} & \mu > a, k > 0 
\end{cases}
\]  
(18)

3.1. Weight Function of Each Index

1) The weight function of resistance

Because various information systems employ different defensive tools, and deployment environment vary widely, the resistances are different. The capability of resistance reflects its

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weight in whole system. With strengthen of resistance, the viability of survival system increase. There is positive change relationship between resistance and survivability. Therefore we select equation (17) as distribution function of its weight. As show in equation (17), three parameters \((k, a, n)\) need to determine.

When defensive system prevents 95% survival incidents, it almost avoids all incidents. When 60% of survival incidents are let off, it hardly does something to the survivability. So, According to experience of actual operating and expert analysis, the weight is 0.9 at 95%, 0.5 at 70%, and 0.1 at 40%. If these data is put into equation (18), parameters are

\[
a = -0.365856, \quad k = 0.481914, \quad n = 5.697996
\]

The weight function of resistance is obtained

\[
\mu_{\text{res}}(r) = \begin{cases} 
0 & \mu \leq -0.365856 \\
1 - e^{-0.481914(r+0.365856)^{5.697996}} & \mu > -0.365856, k > 0
\end{cases}
\]

(19)

2) The weight function of recognition

Because of the same reason, equation (17) is select to represent the weight function of recognition. When recognition rate is up to 85% and appropriate measures are used to handle survival incidents, the performance is well, the weight is 0.9; when recognition rate is at 60%, the weight is 0.5; when recognition rate fall 30%, weight is to 0.1. So, the weight function of recognition is

\[
\mu_{\text{rec}}(r) = \begin{cases} 
0 & \mu \leq -0.465856 \\
1 - e^{-0.465856(r+0.465856)^{5.697996}} & \mu > -0.465856, k > 0
\end{cases}
\]

(20)

3) The weight function of recovery

The half-liter function is suitable for weight function of recovery. Because recovery is an auxiliary function in this system, so when recovery rate is up to 95% the weight is 0.6; when recovery rate is at 60%, the weight is 0.3; when recovery rate drop to 20, the weight is 0.1. After these values substituted into equation (17), the weight function of recovery show as:

\[
\mu_{\text{rec}}(r) = \begin{cases} 
0 & \mu \leq -0.116871 \\
1 - e^{-0.785833(r+0.116871)^{2.37292}} & \mu > -0.116871, k > 0
\end{cases}
\]

(21)

4) The weight function of adaptation

Adaptation is less mature in this system. Therefore, when adaptation achieves 95%, the weight is 0.4; when adaptation is at 50%, the weight is 0.2; when adaptation is at 8%, the weight is 0.05. After Calculation, The weight function of adaptation is as:

\[
\mu_{\text{ad}}(r) = \begin{cases} 
0 & \mu \leq -0.234714 \\
1 - e^{-0.380734(r+0.234714)^{1.733781}} & \mu > -0.234714, k > 0
\end{cases}
\]

(22)

3.2. Calculation and analysis

By analyzing the log of a period, we obtain the values of resistance, recognition, recovery, adaptation (94%, 80%, 70%, and 20%). After substituted into equation (19)-(22), the weight vector is as:

\[
w'=(0.8897, 0.8422, 0.3851, 0.0859)
\]

Normalized:

\[
w=(0.4039, 0.3823, 0.1748, 0.0390)
\]
In the process of assessment, each index has some evaluation parameters; the same method can get their weight function. For the space, here do not present in detail. After above analysis, we can see that each index has different impact on the survivability of system and the degree of impact change with its value. That is, its value reflects its influence on information system. Proposed method is more scientific and objective than traditional methods in calculating weights. For example, weights \((0.98, 0.5, 0.2, \text{and} 0.3)\) means that the resistance is excellent which almost defense all the attack. Therefore, this system is a nice survival system. But in traditional methods, this system is not a well survival system.

In order to extend the application of proposed method to adapt the complex system or system with different purpose, it is necessary combine variable weight and fixed weight.

\[
W = \mathbf{W} \cdot \mathbf{W}_f = \{w_{1i}, w_{2i}, \ldots, w_{ni}\} \cdot \{w_{1f}, w_{2f}, \ldots, w_{nf}\}
\]

Where \(n\) is the number of index.

4. Conclusion
Solution of weight is a key step in survival assessment. Its veracity will directly affect the final evaluation result. Weight is changing with the operating of survival system. It is improper to employ traditional method with fixed value. In order to objectively reflect the status of each index in survival system, this paper proposed a novel calculation of weight. Proposed method brings in natural weight, which is the essential reflection of survival incident, to calculate the weight. Case studies shown that the weight, calculated by proposed method, can scientifically and effectively reflect the importance of assessment index. It is a better solution with small amount of calculation to application and popularization.

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