Venture Capital Industry Index Portfolio Analysis

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Abstract

This paper using index analysis method, knowledge of venture capital as well as index funds investment ideas, successively set up the Markowitz model and the single index model of index investing. Markowitz model for the calculation of the risk of workload is too big and single-index model although accuracy is slightly lower, but can certainly be very well used in practice. Therefore, We use index invest to invest in Shanghai 10 index securities with the single-index model, and apply lingo software to figure out the venture capital portfolio which have different yields.

Keywords: portfolio, residual variance, Markowitz model, single-index model.

1. Introduction

On December 13, 2011, the Shanghai composite index closed at 2,249 points, almost falling to 2,245 points in 2001, ten-year stock market float is zero. In order to better avoid from risks and achieve the best combination of investment income, using index analysis method in this paper, by using the method of statistics and operational research to establish a mathematical model and empirical analysis of Shanghai series industry index model [1]-[3].

Based on business structure, business risk and macroeconomic variables that reflect on the similarities, listed companies can be divided into different trades or style; In the same style, business in every industry have similar reflection on changes to key macroeconomic variables, and reflected in higher relevance of price movement on the stock market; Every industries within the same style on the valuation method and the impact factors for expected earnings have high similarity. We assume that selecting n index to construct portfolios of investment, the weight for each industry index investing is X$_i$, earnings risk for index of every industry is $\sigma_i$ (variance), the relation between indices i and j is $\sigma_{ij}$ (covariance).The problem is converted to determine the weights of all industries index configuration [4]-[8].

The Introduction should provide a clear background, a clear statement of the problem, the relevant literature on the subject, the proposed approach or solution, and the new value of research which it is innovation. It should be understandable to colleagues from a broad range of scientific disciplines [9], [10].

By logging the Shanghai Stock Exchange SSE and using related stocks software we can acquire series 7 indexes of raw data. As these 7 major categories of 60 index calculation standard and the different initial value, thus collecting the absolute value of the original data is unreliable. Through analysis, 60 monthly price index method of sampling is more representative. Therefore, we will be on May 30, 2008 to December 31, 2009 of this period divided by month, class interval $\delta=1$ month. (Calculation of days in the month according to the actual month, ignoring the month differences between the number of days in each month) so we can get a 19-month indicators of price of the raw data. See Schedule I. (Note 1: data collection based on calendar month of k line chart records, as some of the index began to use later on May 30, 2008, which is part of the data cannot be collected. See Schedule II), (Note 2: the basic point of the article on statistics and calculation of price or yield is 1%, is recorded 8 as 8%, not tired after-mentioned).
2. The Model Assumptions
   Model 1. All investors are in the same period investment. Investor based on yields and risk selection portfolio. In the course of investment there is no transaction fee. Representatives of the index price index yields section headings are in boldface capital and lowercase letters. Second level headings are typed as part of the succeeding paragraph (like the subsection heading of this paragraph).
   Model 2. Industry index yields only under the influence of Shanghai stock index returns, and other effects in this document does not into consideration.
   (Note: the model 2 is based on the model 1 assumptions).

3. The Model
   The model is builded and explained the this section, and finally use the Markowitz model to analyze the risks of different yield portfolio.

3.1. Model Markowitz
   Markowitz’s portfolio theory not only reveals the portfolio risk decision factors, but also reveals the more important theory “Assets expectations income by its risk size to decide”, ie. assets (single assets and portfolio) is priced by its risk size, a single asset is priced by its variance, combination of asset price is priced by its covariance. Markowiz’s risk pricing thoughts in his creation of the “mean-variance” or “mean-standard deviation” two-dimensional space investment opportunity setting on the efficient frontier performance is most clearly.
   The theoretical is based on the following hypothesis:
   1. Investors consider every investment when each choice, it is based on a position time securities gains probability distribution.
   2. Investors is based on the securities expectations yield estimation portfolio risk.
   3. Investors’ decision is based on the securities of the risks and benefits.
   4. In a certain risk level, investors expect return most; The corresponding is in a certain income level, investors hope minimum risk.
   On the basis of the above hypothesis, this Markowitz model establishes portfolio expected return, risk calculation method and effective boundary theory, establishes the optimization allocation of assets of the mean-variance model:
   The objective function are:
   \[
   \min \sigma^2(rp) = \sum \sum x_i x_j Cov(r_i - r_j) \\
   \]
   \[
   rp = \sum x_i r_i \\
   \]
   Restrictions are:
   \[
   l = \sum X_i \text{ ( Short sales)} , \\
   \]
   \[
   or \ l = \sum X_i y_i \text{ ( Don't allow short sales)} . \\
   \]
   The rp is for combination earnings, \(r_i\) is for the first i stock income, \(x_i\), \(y_i\) is for securities investment proportion, \(\sigma^2(rp)\) is for portfolio variance (combined total risk), Cov\((r_i, r_j)\) is for two securities between covariance. The model for modern securities investment theory laid a foundation. The model is laid the fondation for the model modern securities investment theory. The type shows that under the condition of the limit to solve \(X_i\) securities return to make combination risk \(\sigma^2(rp)\) the minimum. It can be obtained by lagrangian lange objective function.
   We are considering four assumptions of the Markowitz model as follows.
Assumption 1. Investors expected rate of return (also known as the average yield) to measure the overall level of the actual yield to, and yield variance (or standard deviation) to measure the yield uncertainty (risk), and thus investors only care about the expected rate of return and the variance of the investment decision-making.

Assumption 2. Investors is insatiable and risk-averse, and investor always hoping that the expected rate of return as high as possible, while variance is the smaller the better.

Assumption 3. Investors to comply with the dominant principle: the same level of risk, higher yield securities; same income rate level, choose a low-risk securities.

Assumption 4. Investors know in advance that the probability distribution of the rate of return on investment.

We assume that select n index to construct portfolios of industry, the weight for each industry index investing is \( X_i \), earnings risk for index of every industry is \( \sigma_i^2 \) (variance), the relation between indices i and j is \( \sigma_{ij} \) (covariance). The problem is converted to determine the weights of all industries index configuration to achieve risk minimization for industry index portfolio income, that is to solve linear programming, shown as Eq. 5, Eq. 6, Eq. 7, Eq. 8:

\[
\begin{align*}
\min \sigma_p^2 &= \sum_{i=1}^{n} x_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{k=1}^{n} x_i x_k \sigma_{ik} \\
\text{s.t.} \sum_{i=1}^{n} x_i &= 1 \\
\sum_{i=1}^{n} x_i r_i &= E(r_p) \\
x_i &\geq 0 (i = 1, 2, \ldots, n)
\end{align*}
\]

Where \( \sigma_p^2 \) is Portfolio risk, \( r_i \) is i trade index yields, \( r_p \) is Portfolio yields.

Real investments, we are hard to find the probability distribution of industry index yields, but we can get the yield capacity of n samples \( (r_1, r_2, \ldots, r_N) \) by using historical data sample. Through two parameters of this sample—to estimate mean and variance. From the knowledge of probability theory, we can get Eq. 9, Eq. 10:

\[
\begin{align*}
\text{sample mean } \bar{r} &= \frac{1}{N} \sum_{i=1}^{N} r_i \\
\text{sample variance } \hat{\sigma}_r^2 &= \frac{1}{N-1} \sum_{i=1}^{N} (r_i - \bar{r})^2
\end{align*}
\]

Similarly, the covariance \( \sigma_{ij} \) is theoretical value, we still through sampling, using covariance of sample to estimate the relationship between \( r_A \) and \( r_B \). We assume respectively the sample of \( r_A \) and \( r_B \), then covariance of \( r_A \) and \( r_B \) for \( \sigma_{AB} \), \( r_B \) have good statistical properties for estimator[11], which shows as Eq. 11:

\[
\hat{\sigma}_{A,B} = \frac{1}{N-1} \sum_{i=1}^{N} (r_{Ai} - \bar{r}_A)(r_{Bi} - \bar{r}_B)
\]

Markowitz, respectively, with the variance of the expected rate of return and yield to measure the level of expected return and risk of the investment, mean - variance model, and thus the decision-making. Under the assumption that investors care only about the expected rate of return and variance, Markowitz theory and method is creating accurate and scientific, but
the biggest shortcoming of this theory and methods in the practical application of the process is
too large, especially in large-scale market, in the presence of thousands of securities and a
certain time requirements, the realization of the operation is almost impossible. Thus it severely
limits the Markowitz method application space.

3.2. Single Index Model

Single index model (single-index model (SIM)) is a simple asset pricing models, it is
often used in the financial industry for a stock assessment of risk and return.

\[ r_{it} - r_f = \alpha_i + \beta_i (r_{mt} - r_f) + \epsilon_{it} \]  

\[ \epsilon_{it} \sim N(0, \sigma_i) \]

\( r_{it} \) is return to stock i in period t 
\( r_f \) is the risk free rate (i.e. the interest rate on treasury bills) 
\( r_{mt} \) is the return to the market portfolio in period t. 
\( \alpha_i \) is the stock's alpha, or abnormal return 
\( \beta_i \) is the stock's beta, or responsiveness to the market return 
\( \epsilon_{it} \) is the residual (random) return, which is assumed normally distributed with mean zero
and standard deviation \( \sigma_i \).

These equations show that the stock return is influenced by the market (beta), has a
firm specific expected value (alpha) and firm-specific unexpected component (residual). Each
stock's performance is in relation to the performance of a market index (such as the All
Ordinaries). Security analysts often use the SIM for such functions as computing stock betas,
evaluating stock selection skills, and conducting event studies [12].

In order to facilitate the analysis, a single index model assumes that there is only one
macro risk factors cause stock returns, which can be indicated by a rate of return on a market
index, such as the $ INX index 500 (S&P 500). Depending on the model assumptions, any stock
gains can be decomposed into individual shares of residual income expectations (here
represented by a company specific factor \( \alpha \)), that affecting the market benefits of
macroeconomic events and unpredictable consists of micro-events which only affect the
company.

\( \beta_i (r_m - r_f) \) said under the influence of stock market movement, \( \epsilon_i \) represents company
under the influence by factors of risk.

Macro events, such as changes in the interest rates, changes in labour costs can cause
systemic risk affecting the proceeds in the stock market as a whole. Company specific events
cause specific changes in corporate earnings of micro-events, for example, reducing the death
of important figures of the company's credit rating will affect the company's earnings, but the
impact on the economy as a whole is negligible. In a portfolio, the company refers to factors
arising from discretization of non-systematic risks can be reduced to 0.

The index is based on the following assumptions:

- Covariance that because most stocks are similar to their macro-incident response.
- however, some companies are sensitive to these factors is greater than any other
companies, which controlled by the coefficient \( \beta \) for the sensitivity.

- Covariance between the bonds is due to the different result of macro. So, in their beta
every poor stock multiply. \( \text{Cov}(R_i, R_k) = \beta_i \beta_k \sigma^2 \)

- The last equation covariance calculation reduces greatly, the amount or the portfolio
covariance bonds must be in terms of revenue in history, and must be calculated separately for
each bond. With this equation, only beta and the variance of the market can be. Single index
model reduces the amount of calculation greatly.

Single exponential model is a simple asset pricing model, which is usually used for
the assessment of the risks and rewards of the financial industry for a stock. We know that the stock
return is influenced by the market (beta), has a firm specific expected value (alpha) and firm-
specific unexpected component (residual). Each stock's performance is in relation to the
performance of a market index (such as the All Ordinaries). Security analysts often use the SIM
for such functions as computing stock betas, evaluating stock selection skills, and conducting event studies.

In the study of the first question we found when index yields of Shanghai stock significantly increase or decline, nearly all industry index yields are increased or decreased accordingly, although some index movements are out of sync with Shanghai index, but on the whole they are the same tendency. This means that the Shanghai index changes can fully reflected the common trend of all industry index. Therefore, the covariance of the calculation between indexes can be instead of each industry index and Shanghai index of covariance. Using single-index model, we assume that the industry index yields only under the influence of Shanghai stock index returns, in order to determine the portfolio's weighting.

Assuming industries index yields with simple linear structure, which yields $r$ and Shanghai stock index portfolio yields $r_M$ has Eq. 13:

$$r = A + Br_M + \varepsilon.$$ (13)

Where: $a$, $b$ are assessment parameters; $\varepsilon$ is Residual variance.

Suppose there are $n$ industry index, according to the above structure, $i$ yield on the index met:

$$r_i = A_i + \beta_i r_M + \varepsilon_i, i = 1, 2, \ldots, n.$$ (14)

Under the single index model, Markowitz combination investment model are:

$$\min \sigma_p^2 = \left( \sum_{i=1}^{n} x_i \beta_i \right)^2 \sigma^2(r_M) + \sum_{i=1}^{n} x_i^2 \sigma^2(\varepsilon_i).$$ (15)

$$s.t. \sum_{i=1}^{n} x_i = 1.$$ (16)

$$\sum_{i=1}^{n} x_i A_i + \left( \sum_{i=1}^{n} x_i \beta_i \right) E(r_M) = E(r_p).$$ (17)

In practice during the calculation we regard $B_i$ as an estimate value.

4. Model Solution.

Use the second model portfolios to solve, will bring a 10 industries index data has been evaluated $A_i, B_i$, and $\sigma S2$ into the index, use Lingo software for solving linear programs. (Note: as the industry index starts at January 09, so this example of the portfolio is investment for January 09). The result shown as table 1 and table 2.

<table>
<thead>
<tr>
<th>Industry code</th>
<th>Industry name</th>
<th>Proportion of investment</th>
<th>Proportion of investment</th>
<th>Proportion of investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>SSE energy</td>
<td>4.0E-02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>SSE materials</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>SSE industry</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>SSE optional</td>
<td>0.07</td>
<td>0.36</td>
<td>0.55</td>
</tr>
<tr>
<td>5</td>
<td>SSE consumer</td>
<td>0.22</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>SSE Medicine</td>
<td>0.71</td>
<td>0.59</td>
<td>0.41</td>
</tr>
<tr>
<td>7</td>
<td>SSE financial</td>
<td></td>
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<td></td>
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<tr>
<td>8</td>
<td>SSE Information</td>
<td></td>
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<tr>
<td>9</td>
<td>SSE Telecom</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>SSE public</td>
<td>31.88</td>
<td>41.12</td>
<td>59.09</td>
</tr>
</tbody>
</table>

Table 1. Yield is less than 9%
Table 2. Yield is more than 10 %

<table>
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<tr>
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<td>10</td>
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<td></td>
</tr>
<tr>
<td>Portfolio risk</td>
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</tr>
</tbody>
</table>

5. Model Evaluation

In the solution process of Model Markowitz, you must select covariance among industry index to estimate. If the index number is too large, it would require massive covariance estimation, which takes a lot of time. But there is no doubt that such estimates are more accurate. Markowitz risk pricing groundbreaking ideas and models, laid the theoretical foundation of modern finance, investment, and even financial management science. However, this theory has shortcomings, is the more complex mathematical model, the practice is not easy.

In the solution process of Single index model, the estimated value greatly reduced, their workload is smaller than model with an order of magnitude, but the degree of precision is not as good as the model 1, and it relies on every industry index yields of the rational single index structure hypothesis. However, Single index model greatly reduces the portfolio return and variance Markowitz portfolio selection process to estimate the number of parameters.

6. Conclusion

According to the requirements of different yields, then come to the portfolio and the result is shown as Table 1 and Table 2. By analyzing the data obtained in the table we it can be seen that with the required yield enhancement, risk of the portfolio increased dramatically. At the same time when the yield is lower, the risk of the portfolio can reduce.

References