Delay-Dependent Observers for Uncertain Nonlinear Time-Delay Systems

Dongmei Yan*, Youwei Wang1, Jiuhong Wei2, Dongzhe Li1, Lei Chen1

1The Center of Network & Educational Technology, Jilin University, Changchun, China
2Computer Department, the Agricultural Division, JiLin University, Changchun, China
*Corresponding author, e-mail: ydm@jluhp.edu.cn

Abstract

This paper is concerned with the observer design problem for a class of discrete-time uncertain nonlinear systems with time-varying delay. The nonlinearities are assumed to satisfy global Lipschitz conditions which appear in both the state and measurement equations. The uncertainties are assumed to be time-varying but norm-bounded. Two Luenberger-like observers are proposed. One is delay observer and the other is delay-free observer. The delay observer which has an internal time delay is applicable when the time delay is known. The delay-free observer which does not use delayed information is especially applicable when the time delay is not known explicitly. Delay-dependent conditions for the existences of these two observers are derived based on Lyapunov functional approach. Based on these conditions, the observer gains are obtained using the cone complementarity linearization algorithm. Finally, a numerical example is given to illustrate the effectiveness of the proposed method.

Keywords: robust observer, Lipchitz nonlinear systems, uncertain systems, delay-dependent, linear matrix inequality

Copyright © 2013 Universitas Ahmad Dahlan. All rights reserved.

1. Introduction

The theory of state observers for linear systems has been receiving many researchers’ interests in the past a few decades. Rich literature have been published and different kind of observers have been proposed [1-4]. When uncertainties appear in the system model, a robust observer should be considered. Many results on this topic have been reported (see, e.g. [5-6], and references therein). On the other hand, time-delay is often encountered in many practical systems. The design problem of observers for time-delay systems has also been studied for many years. For example, Linear functional observers for discrete-time systems with state delays were designed and delay-dependent stability conditions were derived in [7]. Moreover, increasing attention has been paid to the robust observer design problem of linear uncertain time-delay systems. For example, Robust $H_\infty$ observers for linear time delay systems with parameter uncertainties were considered in term of the matrix Riccati-like equations in [8].

Recently, the design problem of observers for nonlinear systems, especially Lipschitz nonlinear systems, has received considerable attentions. A new observer design method based on a new Lyapunov-Krasovskii functional was proposed in [9]. An observer design method for discrete-time non-linear systems which have Lipschitz non-linearity and delayed output was proposed in [10]. However, most of the above literature assumed that the time delay is constant, which cannot be always the case in control systems [11-12]. To the best of the authors’ knowledge, the problem of designing delay-dependent observers for discrete-time uncertain Lipschitz nonlinear systems with time varying delay has not been fully investigated, which motivates this study.

In this paper, the design problem of observers for a class of discrete time delay systems with Lipschitz nonlinear perturbations and norm-bounded uncertainties is considered. Two Luenberger-like observers are proposed and new delay-dependent existence conditions for these two observers are derived. Numerical examples are provided to illustrate the validity and less conservativeness of the proposed methods.
2. Problem Formulation and Preliminaries

Consider a discrete-time uncertain nonlinear system with time-varying delay:

\[
x(k + 1) = (A + \Delta A(k))x(k) + (A_d + \Delta A_d(k))x(k - \tau(k)) + Gg(x(k))
\]

where \( \tau(k) \) is the state vector, \( y(k) \) is the measurement output, \( \phi(k) \) is the initial condition sequence, \( \tau_{\text{max}} \) is an integer representing the time-varying bounded delay and satisfies \( \tau_{\text{min}} \leq \tau(k) \leq \tau_{\text{max}} \), where \( \tau_{\text{min}} \geq 0 \) and \( \tau_{\text{max}} > 0 \) are constant scalars representing the minimum and maximum delays, respectively. \( A, A_d, C, G \) and \( H \) are known real constant matrices with appropriate dimensions, \( \Delta A(k), \Delta A_d(k) \) and \( \Delta C(k) \) denote parameter uncertainties and satisfy the following conditions:

\[
[\Delta A(k) \quad \Delta A_d(k)] = D_n F(k) \begin{bmatrix} E_1 & E_2 \end{bmatrix}, \quad \Delta C(k) = D_n F(k) E_1, \quad F^T(k) F(k) \leq I, \quad \forall k
\]

\[
g(\cdot): \mathbb{R}^n \rightarrow \mathbb{R}^n \quad \text{and} \quad h(\cdot): \mathbb{R}^n \rightarrow \mathbb{R}^n
\]

are known nonlinear functions. \( g(x(k)) \) and \( h(x(k)) \) meet the following global Lipschitz conditions:

\[
\|g(x_1(k)) - g(x_2(k))\| \leq \|R_g(x_1(k) - x_2(k))\|, \quad \|h(x_1(k)) - h(x_2(k))\| \leq \|R_h(x_1(k) - x_2(k))\|
\]

for all \( x_1, x_2 \in \mathbb{R}^n \), where \( R_g \) and \( R_h \) are known constant matrices with appropriate dimensions.

The problem of interest is to design observers for the system (1)-(3) with norm-bounded uncertainties satisfying (4) and nonlinearities satisfying (5). The objectives are: (1) to design the following two different Luenberger-like observers to reconstruct the state \( x(k) \) based on measurement output \( y(k) \); (2) to provide an efficient procedure to compute the observer gains.

The two Luenberger-like observers have the following structures:

Delay observer: it has internal time delay, so it is applicable when the time delay is known.

\[
\dot{x}(k + 1) = A \dot{x}(k) + A_d \hat{x}(k - \tau(k)) + Gg(\hat{x}(k)) + L \left[ y(k) - C \hat{x}(k) - Hh(\hat{x}(k)) \right]
\]

Delay-free observer: it does not have internal time delay; hence, it is especially applicable when the time delay is not known explicitly.

\[
\dot{x}(k + 1) = A \dot{x}(k) + Gg(\hat{x}(k)) + L \left[ y(k) - C \hat{x}(k) - Hh(\hat{x}(k)) \right]
\]
Let the error vector be \( \varepsilon(k) = x(k) - \hat{x}(k) \), from (6), (8) and (10), it can be obtained that
\[
e(k+1) = (A-LC)e(k) + A_p e(k) - (\varepsilon + \tau(k)) + \left[G \quad -LH\right]\begin{bmatrix} g(x(k)) - g(\hat{x}(k)) \\ h(x(k)) - h(\hat{x}(k)) \end{bmatrix} (11)
\]

The following theorem gives a sufficient delay-dependent stability criterion for system (11).

**Theorem 1:** For given \( \tau_{\min} \) and \( \tau_{\max} \), if there exist scalars \( \varepsilon_i > 0 \), \( \lambda_i > 0 \) \( (i = 1, 2) \), matrices \( P > 0 \), \( Q > 0 \), \( S > 0 \), \( Z > 0 \) and any matrices \( M_i \), \( M_j \), \( N_i \) and \( N_j \) such that

\[
\Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12} \\ \ast & \Psi_{22} \end{bmatrix} < 0
\]

where

\[
\Psi_{11} = \begin{bmatrix} \Omega_1 & -M_i^T + N_i & 0 & 0 & 0 \\ * & \Omega_2 & -M_j^T + N_j & 0 & 0 \\ * & * & \Omega_3 & 0 & 0 \\ * & * & * & -E & 0 \\ * & * & * & * & -\tau_{\max} \Lambda \end{bmatrix}, \quad \Psi_{12} = \begin{bmatrix} A_i^TP & (A_i - I)^T & Z & \tau_{\max}M_i^T & 0 \\ A_j^TP & A_j^T & 0 & 0 & \tau_{\max}M_j^T \\ 0 & 0 & 0 & \tau_0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & G_i^TP & 0 & 0 & 0 \end{bmatrix}
\]

\[
\Psi_{22} = \text{diag}(-P, -Z/\tau_{\max}, -\tau_{\max}Z, -\tau_{\min}Z), \quad \Omega_1 = -P + (\tau_{\min} + 1)Q + (\varepsilon_1 + \tau_{\max} \lambda_1)R^T R_x + (\varepsilon_2 + \tau_{\max} \lambda_2)R^T R_y + M_1 + M_i^T + S, \quad \Omega_2 = -N_i^T - N_j - Q + M_j^T + M_j, \quad \Omega_3 = -N_i^T - N_j - S, \quad E = \begin{bmatrix} \varepsilon_1 I & 0 \\ 0 & \varepsilon_2 I \end{bmatrix}, \quad \Lambda = \begin{bmatrix} \lambda_1 I & 0 \\ 0 & \lambda_2 I \end{bmatrix}, \quad \tau_0 = \tau_{\max} - \tau_{\min}.
\]

then system (11) is asymptotically stable for any time-varying delay \( \tau(k) \) satisfying 

\[
\tau_{\min} \leq \tau(k) \leq \tau_{\max}.
\]

**Proof:** Define \( d(k) = e(k+1) - e(k) \) and choose a Lyapunov functional candidate as follows:

\[
V(k) = V_1(k) + V_2(k) + V_3(k) + V_4(k)
\]

where

\[
V_1(k) = e^T(i) Pe(k), \quad V_2(k) = \sum_{i = k}^{k + \tau_{\max} + 1} e^T(i) Se(i), \quad V_3(k) = \sum_{i = k}^{k + \tau_{\max} + 1} e^T(i) Q e(i),
\]

\[
V_4(k) = \sum_{i = k - \tau_{\max} - 1}^{k - 1} \sum_{j = i + \tau_{\max} + 1}^{k - 1} e^T(i) Q e(i), \quad V_5(k) = \sum_{i = k}^{k + \tau_{\max} + 1} (i - k + \tau_{\max} + 1) d^T(i) Z d(i),
\]

then, one can obtain that

\[
\Delta V_1(k) = e^T(i)(A_i^T PA_i - P)e(k) + 2e^T(k)A_i^T PA_i e(K) + e^T(K)A_i^T PA_i e(K) + 2\left[A_i e(k) + A_i e(K)\right]^T P G_i B - B^T T B + B^T E B
\]

where

\[
T = E - G_i^T P G_j, \quad E = \text{diag}(\varepsilon_1 I, \varepsilon_2 I), \quad \xi = \xi(x(k), \hat{x}(k)).
\]

From (5), (13), it is easy to obtain

\[
\Delta V_1(k) \leq e^T(i) \Xi e(k) + 2e^T(k) \Pi e(K) + e^T(K) \Omega e(K)
\]

where

\[
\Xi = A_i^T PA_i - P + A_i^T P G_i^T G_j^T P A_i + \varepsilon_1 R_x^T R_x + \varepsilon_2 R_y^T R_y, \quad K = k - \tau(k), \quad \Pi = A_i^T PA_i + A_j^T P G_i^T G_j^T P A_j, \quad \Omega = A_j^T PA_j + A_j^T P G_i^T G_j^T P A_j,
\]

and
\[ \Delta V_z(k) = e^T(k)S_e(k) - e^T(k - \tau_{\max})S_e(k - \tau_{\max}) \quad (15) \]
\[ \Delta V_z(k) \leq e^T(k)Qe(k) - e^T(k - \tau(k))Qe(k - \tau(k)) + \sum_{i=1}^{k-\tau} e^T(i)Qe(i) \quad (16) \]
\[ \Delta V_z(k) = (\tau_{\min} - \tau_{\max})e^T(k)Qe(k) - \sum_{i=k+1}^{k+\tau_{\max}} e^T(i)Qe(i) \quad (17) \]
\[ \Delta V_z(k) = \tau_{\max}d^T(k)Zd(k) - \sum_{i=k+1}^{k+\tau_{\max}} d^T(i)Zd(i) - \sum_{i=k+1}^{k+\tau_{\max}} d^T(i)Zd(i) \quad (18) \]

It can obtained that
\[ \Delta V_z(k) \leq \tau_{\max}d^T(k)Zd(k) + E_0^T\Delta_0E_0 + \tau_{\min}E_0^T\Omega_0E_0 + \tau_{\max}\Delta_2E_0 + (\tau_{\max} - \tau_{\min})E_0^T\Omega_2E_0 \quad (19) \]

where
\[ E_0 = \begin{bmatrix} e(k) \\ e(k - \tau(k)) \end{bmatrix}, \quad e_0 = \begin{bmatrix} e(k - \tau_{\max}) \\ e(k - \tau_{\max}) \end{bmatrix}, \quad \Delta_0 = \begin{bmatrix} M_1^T + M_1 & -M_1^T + N_1 \\ * & -N_1^T - N_1 \end{bmatrix}, \quad \Omega_0 = \begin{bmatrix} M_1^T \\ N_1 \end{bmatrix} \quad (i = 1, 2) . \]

From (14)-(19), using Schur complements, (12) can be obtained. (12) implies \( \Delta V(k) < 0 \), which guarantees the error state system (14) is asymptotically stable. □

Define the error vector as \( \hat{e}(k) = x(k) - \hat{x}(k) \). From (1), (3) and (6), it is easy to obtain
\[ e(k+1) = (A - LC)e(k) + A_0e(k - \tau(k)) + (\Delta A(k) - L\Delta C(k))e(k) + \Delta A_0(k)e(k - \tau(k)) + G_0\hat{e}(x(k), \hat{x}(k)) \quad (20) \]

where
\[ G_0 = [G_{D}, -LH], \quad \hat{e}(x(k), \hat{x}(k)) = \begin{bmatrix} g(x(k)) - g(\hat{x}(k)) \\ h(x(k)) - h(\hat{x}(k)) \end{bmatrix} . \]

Let the augmented state vector be \( z(k) = \begin{bmatrix} x(k) \\ e(k) \end{bmatrix} \), then
\[ z(k+1) = (A_1 + \Delta A_1(k))z(k) + (A_2, + \Delta A_2(k))z(k - \tau(k)) + G_0\hat{e}(x(k), \hat{x}(k)) \quad (21) \]

where
\[ A_1 = \begin{bmatrix} A & 0 \\ 0 & A-LC \end{bmatrix}, \quad G_1 = \begin{bmatrix} G & 0 \\ 0 & G \end{bmatrix}, \quad \hat{e}_1(x(k), \hat{x}(k)) = \begin{bmatrix} g(x(k)) \\ g(x(k)) - g(\hat{x}(k)) \\ h(x(k)) - h(\hat{x}(k)) \end{bmatrix} , \quad A_{2,} = \begin{bmatrix} A_0 & 0 \\ 0 & A_2 \end{bmatrix} . \]

\[ \Delta A_1(k) = \Delta A_1(k) \quad \Delta A_2(k) = \Delta A_2(k) \quad \Delta A_3(k) = \Delta A_3(k) \quad \Delta A_4(k) = \Delta A_4(k) \quad \Delta A_5(k) = \Delta A_5(k) \quad \Delta A_6(k) = \Delta A_6(k) \quad \Delta A_7(k) = \Delta A_7(k) \quad \Delta A_8(k) = \Delta A_8(k) . \]

The following theorem gives a sufficient delay-dependent stability condition for augmented system (21).

**Theorem 2:** For given \( \tau_{\max} \) and \( \tau_{\min} \), if there exist scalars \( \varepsilon_1 > 0 \), \( \lambda > 0 \) \((i = 1, 2, 3)\), \( \delta_1 > 0 \) and \( \delta_2 > 0 \), matrices \( P > 0 \), \( Q > 0 \), \( S > 0 \), \( Z > 0 \) and any matrices \( M_i, M_j, N_i \) and \( N_j \) such that
\[ \Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ * & \Sigma_{22} \end{bmatrix} < 0 \quad (22) \]

where
\[ \Sigma_{11} = \begin{bmatrix} \Omega_1 & -M_1^T + N_1 & 0 & 0 \\ * & \Omega_2 & -M_1^T + N_2 & 0 \\ * & * & \Omega_3 & 0 \\ * & * & * & \Omega_4 \end{bmatrix}, \quad \Sigma_{12} = \begin{bmatrix} A_1^T P & (A_1 - I)^T Z & \tau_{\max} M_1^T & 0 & 0 & 0 \\ A_2^T P & A_2^T Z & \tau_{\min} N_1^T & \tau_{\max} M_2^T & 0 & 0 \\ 0 & 0 & 0 & 0 \\ G_0^T P & 0 & 0 & 0 & 0 \\ 0 & G_0^T Z & 0 & 0 & 0 \end{bmatrix}, \quad \Sigma_{22} = \begin{bmatrix} \Omega_1 & -M_1^T + N_1 & 0 & 0 \\ * & \Omega_2 & -M_1^T + N_2 & 0 \\ * & * & \Omega_3 & 0 \\ * & * & * & \Omega_4 \end{bmatrix} . \]
\[ \Omega_1 = -P + Y_L + Y_k + S + (r_0 + 1)Q + M_i^T + M_i + \delta \hat{E}_1, \]
\[ \Omega_2 = -N_1 - Q + M_i^T + M_i + \delta \hat{E}_2, \]
\[ \Omega_3 = \text{diag}(-E_1, -r_{max}, -r_{max}, -r_{max}, -r_{max}, -r_{max}, -r_{max}), \]
\[ E_z = \text{diag}(e_1, e_1, e_1), \]
\[ \Sigma_{22} = \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix}, \quad X_{12} = \begin{bmatrix} 0 & P \hat{D}_1 & P \hat{D}_2 \\ 0 & Z \hat{D}_1 & Z \hat{D}_2 \\ 0 & 0 & 0 \end{bmatrix}, \]
\[ X_{21} = \begin{bmatrix} Z \hat{D}_2 & Z \hat{D}_1 & 0 \end{bmatrix}, \]
\[ X_{11} = \text{diag}(-P, -Z / r_{max}, -r_{max}, -r_{max}), \quad X_{22} = \text{diag}(-r_0 Z, -\delta I, -\delta I), \]
\[ r_0 = r_{max} - r_{min}, \]
\[ Y_g = \begin{bmatrix} \bar{e}_i R_g^T R_g & 0 \\ 0 & (\bar{e}_i + r_{max}) R_g^T R_g \end{bmatrix}, \]
\[ Y_e = \begin{bmatrix} r_{max} \bar{e}_i R_g^T R_g & 0 \\ 0 & (\bar{e}_i + r_{max}) R_g^T R_g \end{bmatrix}. \]

The above delay observer contains delayed states, so it cannot be applicable when the delay is not available. The delay-free observer (10) does not need the exact value of the delay, so it is applicable especially when the delay is not known explicitly.

Define the error vector as \( \hat{e}(k) = x(k) - \hat{x}(k) \). From (1), (3) and (7), the error dynamics is described by:
\[ \hat{e}(k + 1) = (A - LC)e(k) + (A_0 + \Delta A_0(k))e(k - \tau(k)) + (\Delta M(k) - L \Delta C(k))x(k) + G \zeta(\hat{x}(k), \check{\hat{x}}(k)) \]

Let the augmented vector be \( \hat{\theta}(k) = \begin{bmatrix} x(k) \\ e(k) \end{bmatrix} \). Combining (1), (3) with (24) yields:

\[ \hat{\Sigma} = \begin{bmatrix} \hat{\Sigma}_{11} & \hat{\Sigma}_{12} \\ \hat{\Sigma}_{21} & \hat{\Sigma}_{22} \end{bmatrix} < 0 \]
\[
\theta(k+1) = (A_d + \Delta A_d(k))\theta(k) + (A_{\phi} + \Delta A_{\phi}(k))\theta(k - \tau(k)) + G_{\phi}\varphi(x(k), \tilde{x}(k)) +
\]
\[
\xi_{\phi}(x(k), \tilde{x}(k)) = \begin{bmatrix} g(x(k)) \\ g(x(k)) - g(\tilde{x}(k)) \\ h(x(k)) - h(\tilde{x}(k)) \end{bmatrix},
\]
(25) where \( A_d = \begin{bmatrix} A & 0 \\ 0 & A - LC \end{bmatrix}, \ G_d = \begin{bmatrix} G & 0 \\ 0 & G_{\phi} \end{bmatrix}, \ A_{\phi} = \begin{bmatrix} A_d & 0 \\ A_d & 0 \end{bmatrix}, \]
\( \xi_{\phi}(x(k), \tilde{x}(k)) =
\]

The following theorem gives a sufficient stability condition for augmented system (25).

**Theorem 3:** For given \( \tau_{\text{min}} \) and \( \tau_{\text{max}} \), if there exist scalars \( \varepsilon > 0 \), \( \lambda > 0 \) (i = 1, 2, 3), \( \delta_i > 0 \) and \( \delta_\Delta > 0 \), matrices \( P > 0 \), \( Q > 0 \), \( S > 0 \), \( Z > 0 \) and any matrices \( M, M_\Delta, N_i \) and \( N_\Delta \) such that
\[
\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} < 0
\]
(26)
where
\[
\Phi_{11} = \begin{bmatrix} \Omega_{\lambda} & -M_{\lambda} + N_i & 0 & 0 \\ * & \Omega_{\lambda} & -M_{\lambda} + N_i & 0 \\ * & * & \Omega_{\lambda} & -E_0 \\ * & * & * & -\tau_{\text{max}} \Lambda_{\lambda} \end{bmatrix},
\]
\[
\Phi_{12} = \begin{bmatrix} A_{\phi}P & (A_d - I)^T Z \tau_{\text{max}} N_\Delta & 0 & 0 \\ A_{\phi}P & A_{\phi}^T Z & \tau_{\text{max}} N_\Delta & 0 \\ G_{\phi}^T P & 0 & 0 & 0 \\ G_{\phi}^T Z & 0 & 0 & 0 \end{bmatrix}, \Phi_{22} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix},
\]
\[
Z_{11} = \text{diag}(-P, -Z, \tau_{\text{max}} - \tau_{\text{max}}, Z) \quad Z_{12} = \text{diag}(-P, -Z, \tau_{\text{max}} - \tau_{\text{max}}, -\tau_{\text{max}}, Z) \quad Z_{21} = \begin{bmatrix} 0 & 0 \\ 0 & Z_{\Delta} \end{bmatrix} \quad Z_{22} = \begin{bmatrix} 0 & 0 \\ 0 & Z_{\Delta} \end{bmatrix},
\]
\[
Y_{\phi} = \begin{bmatrix} \varphi R_{\phi}^T R_{\phi} \\ 0 \\ \varphi R_{\phi}^T R_{\phi} \end{bmatrix}, \quad Y_d = \begin{bmatrix} \tau_{\text{max}} \lambda_\Delta R_{\phi}^T R_{\phi} \\ 0 \\ \tau_{\text{max}} \lambda_\Delta R_{\phi}^T R_{\phi} \end{bmatrix},
\]
then augmented system (25) is asymptotically stable for any time-varying delay \( \tau(k) \) satisfying
\[
\tau_{\text{min}} \leq \tau(k) \leq \tau_{\text{max}}.
\]
Proof: The proof is similar to that of Theorem 2; hence, it is omitted here. \( \square \)

### 4. Numerical examples
Consider the following uncertain discrete-time Lipschitz nonlinear system with
\[
A = \begin{bmatrix} -0.08 & 0.3 & 0.1 \\ 0.2 & -0.1 & 0.2 \\ -0.2 & 0.2 & -0.1 \end{bmatrix}, \quad A_d = \begin{bmatrix} 0.3 & 0.1 & 0 \\ -0.1 & 0 & -0.05 \\ 0 & 0.3 & 0.05 \\ 0.2 & 0.1 & -0.2 \\ 0 & 0.2 & 0.1 \\ 0.1 & 0.2 & 0.1 \end{bmatrix}, \quad G = \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.1 & 0.2 & 0.2 \\ 0 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 \\ 0 & 0.1 & 0.1 \\ 0 & 0.1 & 0.1 \end{bmatrix}, \quad D = \begin{bmatrix} 0.05 & 0 \\ 0.2 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}, \quad H = \begin{bmatrix} 0.2 & 0.1 & 0.2 \\ 0.1 & 0.1 & 0.2 \\ 0.1 & 0.2 & 0.1 \end{bmatrix}, \quad F(k) = \begin{bmatrix} \sin(k) \\ 0 \end{bmatrix}, \quad \]
\[
C = \begin{bmatrix} 0.1 & 0.2 & 0.1 \\ 0.1 & 0.2 & 0.1 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.2 \\ 0 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 \\ 0 & 0.1 & 0.1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.1 & 0.2 & 0.2 \\ 0.2 & 0.1 & 0.2 \\ 0.2 & 0.1 & 0.2 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 \end{bmatrix}, \quad \]
\[
H = \begin{bmatrix} 0.2 & 0.1 & 0.2 \\ 0.1 & 0.1 & 0.2 \\ 0.2 & 0.1 & 0.2 \end{bmatrix}.\]
Assume $0 \leq \tau(k) \leq 4$. Using results in this paper, the delay observer and the delay-free observer are respectively obtained as

$$L_1 = \begin{bmatrix} 0.0420 & 0.3674 \\ -0.1429 & 1.4533 \\ -1.6470 & 1.2910 \end{bmatrix}, \quad L_2 = \begin{bmatrix} -0.2497 & 0.0966 \\ 1.0011 & -0.4147 \\ -0.9737 & 0.5439 \end{bmatrix}.$$  

Assume the initial value $x(0) = [1 \ 2 \ -1]^T$ and denote the error state $e_i = x_i - \hat{x}_i$, $(i = 1, 2, 3)$. For the above two observer gains, the initial responses of error dynamics are shown in Figure 1 and 2, respectively. The simulation results show that the methods proposed in this paper are valid and effective.

![Figure 1. Estimation error of delay observer](image1)

![Figure 2. Estimation error of delay-free observer](image2)
5. Conclusion
In this paper, the design problem of observers for a class of nonlinear uncertain time-delay systems has been studied. The nonlinearities are assumed to satisfy the global Lipschitz condition and the uncertainties are assumed to be time-varying but norm-bounded. Delay and delay-free observers have been designed for this class of nonlinear systems. Delay-dependent existence conditions for these two observers have been derived. Since the obtained conditions are non-convex, a cone complementarity linearization algorithm has been presented to calculate the observer gains. A numerical example has illustrated the effectiveness of the proposed methods.

References