Mathematical Analysis and Application on Mechanical Image of Hybrid Wavelet Transform Algorithm

Fuzeng Yang*1, Qiong Liu1,2, Mengyun Zhang1, Yuanjie Wang1, Yingjun Pu1
1College of Mechanical and Electronic Engineering, Northwest A&F University, Yangling, P.R. China; 2College of Information Engineering, Yangling Vocational & Technical College, Yangling, P.R. China.
*Corresponding author, e-mail: yfz0701@163.com

Abstract
To overcome the shortcomings such as significantly de-noising effect and easily losing the details of the image characteristics of the existing image de-noising methods, an image de-noising algorithm based on the hybrid wavelet transform was proposed. The algorithm integrated the advantages of wavelet de-noising retaining image details features and Wiener filter obtaining the optimal solution, and took the images processed by wavelet transform and Wiener filter as male and female of the initial population. The steps of the algorithm are as follows: mapping from image space to coding space, iterating to parents through selection, crossover and mutation operation until the offspring meeting the constraints was obtained, reducing the superior offspring to image space, gaining the approximate optimal solution. Theoretical analyses were made on the core of the algorithm, coding, crossover and mutation. The algorithm was applied to agricultural machinery parts image de-noising such as plough and disk harrow. The results showed that it had the advantages of high peak signal to noise ratio (PSNR), obvious edge characteristics, good vision effect, and so on. The result of the present work implied that the proposed algorithm is an effective and feasible exploration.

Keywords: hybrid wavelet transform, image de-noising, mathematical analysis, machinery image

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1. Introduction
With the popularization of digital scanners and cameras, images are becoming the most commonly used information carrier in human life and the main way to get information from the outside world. But in the processes of image acquisition, transmission, scanning, storage, etc, it is often disturbed by varieties of noises, leading to a decrease in image quality and affecting visual effects of subsequent image processing. It is necessary for image de-noising in order to obtain high quality digital image, reducing the noise disturbance in images and better reflecting the original information carried by the image. How to eliminate the noise reasonably has been one of the main subjects of the research field in image processing, so that the images not only maintain the original integrity of the information but also remove useless information, and adapt to the human observation.

Image de-noising has been a basic and important concern over image processing and detection of defects, also prerequisite conditions in the course of image analysis, feature extraction and pattern recognition [1, 2]. For a long time, people present and develop different de-noising algorithms according to the image and noise statistical characteristics and spectral distribution. The classic image de-noising algorithms are: Wiener filtering, median filtering algorithm, neighborhood average algorithm, and so on. Since the nineteen eighties, wavelet transforms have been successfully used in image de-noising, and its de-noising effects are better than the traditional method, and that causes a wide attention of scholars [3]. Many methods are to achieve the purpose of de-noising based on the optimal threshold of wavelet coefficients of image filtering. For example, in 2000 Walker and Chen proposed adaptive tree wavelet shrinkage method [4], they transformed the signal with noise for orthogonal wavelet, then got the de-noised signal by the coefficient threshold operation, the de-noising effects were quite good, in 2000 Jalobeanu, Thomas and Rodriguez et al improved the threshold method and presented the translation invariant wavelet de-noising method [5, 6], Chen and Bui proposed to multiple wavelet threshold de-noising of image noise by merging adjacent coefficients approach, the effects were better than single wavelet, superior to the traditional
methods [7], in 2002, Katkovnik, Kgiazarian and Astola described a novel approach based on the intersection of confidence intervals (ICI) rule to solve a problem of window size (bandwidth) selection for filtering an image signal given with a noise, the adaptive transforms with the adjusted threshold parameter perform better than the adaptive wavelet estimators [8], Vidya et al used a Wavelet Transform (WT) method with soft threshold for signal de-noising [9], Huang et al proposed Self-adaptive Decomposition Level De-noising Method Based on Wavelet Transform [10]. Some researchers had proposed de-noising methods that combine wavelet transform and other filtering methods [11, 12].

But these methods are mostly to an image using a method of image processing [13-16], there are few combining multiple methods and making full use of their respective advantages of de-noising. In last few years, our work is been supported by National Natural Science Foundation of China with reference 30971690. We attempt to make a train of thought, with reference to the thought of biological hybridization breeding and the genetic algorithm ideas. In this paper, we cross wavelet transform and Wiener filtering respectively as male and female hybrid with different advantages of de-noising, extract their de-noising dominant genes, achieve better image de-noising effect by using proposed hybrid wavelet transform algorithm[13]. But we use no mathematical theory analysis at all to prove this algorithm, only apply to agriculture image de-noising. So it is very necessary for mathematical analysis of coding completeness and algorithm convergence, and the applications range are expanded from agricultural product image to agricultural machinery image in order to obtain a better image. We describe a novel approach to the problem of mechanical image de-noising, and it is proved to be a beneficial exploration.

2. Research Method

Hybrid wavelet transform algorithm begins with two parents (also called the populations) of respective advantage genes, every population is made up of a certain number of individuals obtained by gene code. Therefore, the coding task from phenotype to genotype mapping needs to be accomplished at the outset. After the generation of the initial population, hybridization can be carried out. In the process of hybridization, considering the two parental advantages, according to the principle of survival of the fittest, better and better new individuals will be produced from generation to generation. In every generation, individuals are selected according to individual fitness value in the problem domain, then crossover and mutation are made among populations, new population is generated at last. The last population is considered as optimal and the approximate optimal solution after decoding operation.

On the view of macroscopic, algorithm requires the following steps: mapping from image space to coding space and reduction from coding space to image space, this is so-called coding and decoding. In image space, two parental images are mainly obtained. While the specific selection, crossover and mutation operation are completed in coding space, the operation is iterated to parents through fitness values using the selection, crossover and mutation operator according to respective advantages of the two parents, until the offspring meeting the constraints is obtained. In the end, reducing the superior offspring to image space, filial generation images having the dominant gene of two parents are gained.

The implementation of hybrid wavelet transform algorithm is: firstly encoding the two images needing hybridization operation into two populations recorded as P1(0) and P2(0), the superscript 0 is the initial population, and so on. New population P1(0*) is obtained after hybridization, then mutation is operated on P1(0*) in order to get more compatible population P1(0**); at last, P1(0**) is considered as initial population of next generation P1(1) until the optimal population P1(n) is obtained. See details in literature [13]. The core of the algorithm are coding, crossover and mutation, the key are the completeness of coding and the convergence of crossover and mutation.

2.1. Coding

The algorithm selects float encoding method, calculates according to gray scale image pixel value. Considering a gray image of 256 pixels x 256 pixels, the corresponding numerical matrix are denoted as \( M(256, 256) \). The elements in \( M(256, 256) \) are displaced by row, divides into sections containing a fixed number (such as 20), each segment will be a new row.
vector \( r_i(1, 20) \), here \( i = [256 \times 256 \div 20 + 1] = 3277 \), then these vectors \( r_i(1, 20) \) will form a new matrix \( P(3277, 20) \) in columns.

\[
P(3277, 20) = \begin{pmatrix} r_1(1, 20) \\ \vdots \\ r_{3277}(1, 20) \end{pmatrix}
\] (1)

That is to say, a matrix of pixels \( M(256, 256) \) is converted into a new matrix \( P(3277, 20) \) formed by row vectors \( r_i(1, 20) \) in columns, \( i = 1, 2, ..., 3277 \). At this time, the original image is encoded into a corresponding matrix containing 3277 individuals. It is described in genetic algorithms that the initial population can be thought of a collection \( P \) containing 3277 individuals, the population size is 3277, and the individual (or chromosome) length in the population is 20.

2.2. Selection

Selection is used to determine the individuals needing crossover or mutation based on ranking ranking-selection[17]. Firstly, sort the individuals according to the fitness; secondly, determine a threshold value by selection rate. The specific method is as follows:

(1) Calculate all the individual fitness value \( V(i) \)

\[
V(i) = \sum_{k=1}^{n} | r_{(i)}(m) - \bar{r}_{(i)} |
\] (2)

(2) Sort the individual fitness value from small to big (or from big to small), the results are recorded as \( V' \).

(3) Determine threshold value \( t_s \) according to the selection rate \( ra \), the fitness value greater than \( t_s \) will be required to the following operation. Threshold calculation methods are as follows:

\[
t_s = V'(k), (k = j \ast (1 - ra))
\] (3)

Here: \( k \) is threshold value corresponding to the chromosome location;

\( V'(k) \) is the staining fitness value corresponding to the location \( k \).

If \( V'(k) > t_s \), it will need crossover and mutation operation for the selected individuals.

2.3. Crossover

The purpose of crossover operation is to generate new individuals by gene recombination. We firstly identify locus position of inferior gene in every individual, and then hybridize the gene directly in order to quickly obtain excellent new individuals. The gene values are different from so-called bad genes to other genes in the same chromosome.

(1) Calculate absolute deviation by each locus gene values relatively to the mean values of gene on chromosome, that is calculated loci \( m \) genetic fitness \( u_{(i)}(m) \). The method is as follows:

\[
u_{(i)}(m) = | r_{(i)}(m) - \bar{r}_{(i)} |
\] (4)

(2) Sort \( u_{(i)}(m) \) from small to big, the result is recorded as \( u_{(i)}' \).

(3) Determine locus position of needing to crossover operation, it is the position of inferior gene. Fix a threshold value \( s \) according to the hybrid rate \( hra \). The fitness value is greater than the value of the gene was thought to be inferior gene. The method to determine the threshold is as follows:
Here, $s$ is fixed threshold value; $k$ is threshold value corresponding to the location of gene; $u_{(i)}'(k)$ is the position $k$ corresponding to the chromosome fitness value; $n$ is gene number containing in chromosome; $hra$ is hybrid rate.

(4) Hybrid the gene whose fitness value is greater than the threshold value $s$. The method is as follows:

$$r_{(i)}(m) = R_{(i)}(m)$$

(6)

Here, $r_{(i)}(m)$ is gene $m$ on chromosome $i$ from Parent One $P1$; $R_{(i)}(m)$ is gene $m$ on chromosome $i$ from Parent Two $P2$.

2.4. Mutation

The offspring is mutated after hybrid operation; this is genes on a chromosome change with a very small probability. It is a local random search which makes the algorithm itself also has the capability of local random search. At the same time, new individuals are generated through variation. It is the basis of guaranteeing the diversity of population, and it can reduce premature convergence of algorithm.

In order to reduce the variability of random, we use the following methods:

(1) Calculate absolute value $b$ by gene values on chromosome chain relatively to mean deviation, that is calculation of gene fitness value,

$$b_i(m) = |r_{(i)}(m) - \bar{r_{(i)}}|$$

(7)

Here, $b_i(m)$ is gene fitness value of calculated gene locus.

(2) Sort the calculated absolute value of the deviation from small to big, the result is marked as $b'$. Determine the threshold by the variation rate, metamorphosis the gene which is greater than this threshold. The way is the same as determining the location of dissociation gene.

(3) Mutation operation, the method is as follows:

$$r_{(i)}(m) = \begin{cases} r_{(i)}(m+1) & m = 1 \\ r_{(i)}(m-1) & m = n \\ (r_{(i)}(m-1) + r_{(i)}(m+1))/2 & 1 < m < n \end{cases}$$

(8)

2.5. Termination Rule

If the population generated by the algorithm satisfied the constraints, the algorithm stops. The population is optimal, it is considered as the approximate optimal solution after decoding operation. Decoding is the inverse operation for coding.

2.6. Algorithm Process

Step 1: initial population Take the images processed by wavelet transform and Wiener filter as male and female of the initial population of a hybrid wavelet transform. They are recorded as “parent one” and “parent two”, the superscript 0 represents the initial population, and so on.

Step 2: Coding Code the two images to be crossed according to the methods given in section 2.1, and form two populations.

Step 3: Selection Make the selection operation in accordance with section 2.2, and then determine the individuals needing crossover or mutation.

Step 4: Crossover Make the genes recombination to generate new individuals according to section 2.3.

Step 5: Mutation Make the mutation operation according to section 2.4.
Step 6 Judgment Judge the hybrid operation is terminated or not according to the termination rule given in section 2.5. If it is terminated, the approximate optimal solution is output and the hybrid operation stops; otherwise, transferring to step 4.

3. Results and Analysis

3.1. Theoretical analysis

3.1.1. Coding Completeness

**Theorem 1 (Bolzano’s theorem)** If \( \{P_m\} \) is a bounded sequence in \( R^n \), there are convergent subsequences in \( \{P_m\} \). \( \{P_m\} \) is called Cauchy sequence. If \( \forall \varepsilon > 0, \exists N, n > N, m > N \), there \( \|P_n - P_m\| < \varepsilon \).

**Theorem 2 (Cauchy criterion)** Necessary and sufficient conditions for convergence of subsequences \( \{P_m\} \) is that \( \{P_m\} \) is Cauchy sequence.

Numerical matrix \( M(256, 256) \) corresponding to a gray image is converted to a new matrix \( P(3277, 20) \) which is formed by row vectors \( r_i(1, 20) \) in columns, here \( i = 1, 2, \ldots, 3277 \). So we can treat the matrix \( P(3277, 20) \) as a space collection constituted by \( r_1, r_2, \ldots, r_{3277} \).

Firstly, prove \( V_n \) can be considered as a metric space. Define the distance between \( r_1 \) and \( r_2 \) as:

\[
d(r_1, r_2) = \|r_1 - r_2\| = \sqrt{(r_{11} - r_{21})^2 + (r_{12} - r_{22})^2 + \ldots + (r_{1n} - r_{2n})^2}
\]

then \( d(r_1, r_2) = \|r_1 - r_2\| \) satisfies the following conditions is called Euclidean metric in \( V_n \),

- Symmetry: \( d(r_1, r_2) = d(r_2, r_1) \);
- Non negative: \( d(r_1, r_2) \geq 0, d(r_1, r_2) = 0 \) is equivalent to \( r_1 = r_2 \);
- Triangle inequality: \( \forall r_1, r_2, r_3 \in V_n \), it is the constant that \( d(r_1, r_2) \leq d(r_1, r_3) + d(r_3, r_2) \), and the necessary and sufficient conditions of establishment of the equation is \( r_1, r_2, r_3 \) are on the same straight line.

Secondly, prove the point sequences in \( V_n \) are Cauchy sequences.

\( V_n = \{(x_1, x_2, \ldots, x_{20}) | x_i \in R, i = 1, 2, \ldots, 20\} \), because \( x_i \) is from the gray image pixel matrix, \( x_i \) are real numbers between 0 and 255, for any sequence \( r_i = (x_1, \ldots, x_{20}) \in V_n \) is a bounded sequence, that is to say \( \{r_i\} \) is a bounded sequence in \( V_n \). Learning from Theorem 1 (Bolzano's theorem), there is a convergent subsequence in \( \{r_i\} \). \( \{r_i\} \) is called Cauchy sequence, fulfill the following condition: if \( \forall \varepsilon > 0, \exists N, n > N, m > N \), then \( \|P_n - P_m\| < \varepsilon \). This shows that, when the parameters in \( r_i \) are fixed, \( r_n \) is a Cauchy sequence. Learning from Theorem 2 (Cauchy criterion), there is a \( r_0, r_m \to r_0 (m \to \infty) \).

Last, prove \( V_n \) is Complete metric space.

In fact, in inequality \( \|r_n - r_m\| < \varepsilon \), if let \( n \to \infty \), \( \|r_m - r_0\| < \varepsilon \). That explains that \( r_m \) uniformly converges to \( r_0 \). By mathematical analysis, \( r_0 \) is a continuous function over an interval. For any \( r_0 \in V_n \), and when \( m > N \), \( r_m \to r_0 (m \to \infty) \). That explains that \( V_n \) is a complete metric space. Therefore, the coding operation is completed. QED.
3.1.2. Algorithm Convergence

There are the related definition and theorem.  
**Definition 1:** If \( P\{MC(x) = x'\} > 0 \), \( MC(x) \) represents point sequence generated by the crossover and mutation operators, \( P\{\cdot\} \) represents the probability of a random event \( \{\cdot\} \), the individual \( x' \) is called reachable from \( x \) through crossover and mutation. 

**Theorem 3:** Let a Genetic Algorithm fulfill the following conditions[18]: 

1. The population sequence \( P(0), P(1), \ldots \) is monotone, i.e. \( \forall t : \)  
   \[
   \min \left\{ \Phi \left( \tilde{a} \left( t + 1 \right) \right) \left| \tilde{a} \left( t + 1 \right) \in P \left( t + 1 \right) \right\} \leq \min \left\{ \Phi \left( \tilde{a} \left( t \right) \right) \left| \tilde{a} \left( t \right) \in P \left( t \right) \right\} ,
   \]
   \( \text{(10)} \)

2. \( \forall \tilde{a}, \tilde{a}' \in I, \tilde{a}' \) is reachable from \( \tilde{a} \) by means of mutation and recombination. Then:  
   \[
   P\{\lim_{t \to \infty} \tilde{a} \in P(t)\} = 1
   \]
   \( \text{(11)} \)

The proof of the convergence of the algorithm. 

Firstly, prove that two random individuals \( x' \) and \( x \) in individual space, \( x' \) is reachable from \( x \) through crossover and mutation. 

In order to prove this, that is to prove \( P\{MC(x) = x'\} > 0 \), here \( MC(x) \) represents point sequence generated by the crossover and mutation operators, \( P\{\cdot\} \) represents the probability of a random event \( \{\cdot\} \). Supposing that event A(X) represents the individuals generated by crossover operator, event B(X) represents the individuals generated by mutation operator, \( m \) expresses the number of inferior gene locus determined by crossover operator, \( k \) expresses the number of variant gene locus determined by mutation operator, here \( m, k = 1, 2, \ldots, 20 \). Therefore, 

\[
P\{MC(x) = x'\} = P(A(x) = x') + P(B(x) = x') - P((A(x) = x')(B(x) = x')) = P(A(x) = x') + P(B(x) = x') - P(A(x) = x')P((B(x) = x') \mid A(x) = x'))
\]

\[
= \frac{m}{20} + \frac{k}{20} - \frac{m}{20} \times \frac{k}{20}
\]

\[
= \frac{m}{20}
\]

\[
\therefore m = 1, 2, \ldots, 20 \text{ so, } m > 0
\]

\[
\therefore \frac{m}{20} > 0 \text{ that is } P\{MC(x) = x'\} > 0
\]

Consequently, two random individuals \( x' \) and \( x \) in individual space, \( x' \) is reachable from \( x \) through crossover and mutation. Then prove population sequence \( p(1), p(2), \ldots, p(t) \) is monotonic, that is for \( \forall t \), any solution in \( p(t + 1) \) is non inferior or at least not worse than any solution in \( p(t) \). Learning from optimum maintaining strategy of mutation and selection operator, offspring population sequence produced by this algorithm is monotonic. That is for \( \forall t \) any solution in \( p(t + 1) \) is non inferior or at least not worse than any solution in \( p(t) \). 

Last, learning from Theorem 3, the algorithm is convergence. 

3.2 The image de-noising application for Agricultural machinery parts
High-definition images of plough and disk harrow were selected in the experiment. The image resolutions of plough and disk harrow were respectively 2133×1979 and 2272×1864. After adding Gauss white noise with mean 0, variance 0.05 to images, we used a variety of de-noising methods to verify the accuracy and validity of the algorithm. Then images of disk harrow were reduced to 1000×820 and 400×328. And adding Gauss white noise with mean 0, variance 0.05 to images, we viewed effects of this algorithm on different resolution images.

Based on hybrid wavelet transform algorithm, coding frequency image which is after wavelet transforming was recorded as P1, wiener filtering image coding was recorded as P2; parameters were listed in table 1. As shown in table 1, the optional length of a chromosome was respectively 4, 8, and 16; the selection rate and hybridization rate were 0.3 and step length was 0.3, and the rates were set to 0.3, 0.6, and 0.9; the mutation rate was 0.1 and step length was 0.2, the rate was set to 0.1, 0.3, and 0.5 (the mutation rate exceeding 0.5 will be reduced to a random search algorithm); and the hybrid generation were 2, 5, and 10.

Four parameters were fixed and only one parameter was altered in every experiment. Programming by setting the initial value and step length, 27 images after hybridization can be obtained through altering one parameter every time. Select image with the best visual effect and the highest peak signal-to-noise ratio (PSNR) as the optimal solution.

Table 1. Parameters included in algorithm (plough)

<table>
<thead>
<tr>
<th>Original Image Size</th>
<th>Chromo-some Length</th>
<th>Chromo-some Number</th>
<th>Selection Rate</th>
<th>Crossover Rate</th>
<th>Mutation Rate</th>
<th>Generation of Hybrid</th>
</tr>
</thead>
<tbody>
<tr>
<td>2133×1979</td>
<td>4</td>
<td>16384</td>
<td>0.3/0.6/0.9</td>
<td>0.3/0.6/0.9</td>
<td>0.1/0.3/0.5</td>
<td>2/5/10</td>
</tr>
<tr>
<td>2133×1979</td>
<td>8</td>
<td>8192</td>
<td>0.3/0.6/0.9</td>
<td>0.3/0.6/0.9</td>
<td>0.1/0.3/0.5</td>
<td>2/5/10</td>
</tr>
<tr>
<td>2133×1979</td>
<td>16</td>
<td>4096</td>
<td>0.3/0.6/0.9</td>
<td>0.3/0.6/0.9</td>
<td>0.1/0.3/0.5</td>
<td>2/5/10</td>
</tr>
</tbody>
</table>

Programming by setting the initial value and step length, 27 images after hybridization can be obtained through altering one parameter every time. Select image with the best visual effect and the highest peak signal-to-noise ratio (PSNR) as the optimal solution.

Figure 1. De-noising Results of the Plough Image
3.2.1. Comparison hybrid wavelet transform algorithm with conventional de-noising methods

In order to compare the algorithm with the conventional de-noising methods, we de-noise the images of plough and disk harrow using Gaussian filter, the Wiener filter, the median filter, and the average filter at the same time, de-noising effects are shown in Figure 1c to Figure 1f and Figure 2c to Figure 2f.

![Image](image_url)

Figure 2. De-noising Results of the Disk Harrow Image

Median filter, neighborhood averaging method and other conventional de-noising method weak noise to some extent, but the whole image is fuzzy, the edge feature is not apparent, and de-noising effect of the background is not good; while using the proposed method, the noise is significantly reduced, and it keeps many details of image, the edge feature is clear. To analysis the advantages and disadvantages of various de-noising methods, the numerical values are given in table 2. We can see that the peak signal to noise ratio (PSNR) values were 164.64 and 162.03dB for plough and disk harrow using hybrid wavelet transform algorithm, it is significantly higher than that of conventional de-noising methods, relative to wavelet transform, PSNR is also raised.

3.2.2. The Application Effect of Different Resolution Image

In order to view the application effects of different resolution images, this algorithm was applied to the narrowing disk harrow image, the image resolution were 1000×820and 400×328, and added Gauss white noise with mean 0, variance 0.05 to images. At the same time we compared with the resolution of 2272×1864 disk harrow image as shown in Figure 3, a to c were for noisy images, d to e were for de-noising images.

From table 3 we can see that with the reducing of resolution, the peak signal to noise ratio (PSNR) of the processed images are also decreased. Combined with figure 3, de-noising effect is obviously decreased.
4. Conclusion

(1) By the inspiration of hybridization breeding, this article proposes image de-noising algorithm based on hybrid wavelet transform. The analysis from mathematical theory shows that coding operation is complete, and the algorithm is convergent.

(2) The method is applied to mechanical image de-noising such as plough and disk harrow, the peak signal to noise ratio (PSNR) of the de-noised image were 164.64 (plough) and 162.03 (disk harrow), it is the best in some traditional de-noising methods such as neighborhood averaging method (157.69 and 159.25), median filter(159.69 and 158.54).

(3) The experimental results show that image de-noising algorithm based on hybrid wavelet transform applied to mechanical field has a high PSNR value, apparent edge characteristics, and good visual effect and so on. Experimental data verifies the validity and feasibility of this algorithm.

(4) The experiment also showed that with the reducing of resolution, the algorithm processing effect is also decreased, so it remains to be improved. But for the high resolution image, the algorithm can be used effectively.
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Notation

\( r_I(1, 20) \): A row vector constituted of twenty elements, in this algorithm it represents an individual (or chromosome);

\( j \): The number of chromosomes. \( j = [256 \times 256 \div 20 + 1] = 3277 \);

\( i \): The location of the current chromosome, \( i = 1, 2, ..., j \) and \( i \) is an integer;

\( n \): The gene number of chromosomes, \( n = 20 \);

\( m \): The locus location of the current chromosome, \( m = 1, 2, ..., n \) and \( m \) is an integer;

\( r_{i,j}(m) \): The locus location \( m \) of the current chromosome \( r_{i,j} \);

\( r_{i,j} \): The mean values of all genes the current chromosome \( r_{i,j} \).

References


