Backstepping Decentralized Fault Tolerant Control for Reconfigurable Modular Robots

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Abstract
For the actuators fault of reconfigurable modular robots, a backstepping decentralized fault tolerant control (DFTC) algorithm is proposed. The reconfigurable robot system is divied into a set of interconnected subsystems. The fault tolerant controller is designed based on backstepping method. It is hard to obtain the model parameters uncertainty term and interconnected term, so the adaptive fuzzy approximation is adopted to estimate the two terms. Simulation results show the controller can guarantee the tracking accuracy and stability of the system with the actuator faults.

Keywords: reconfigurable modular robot, backstepping, fault tolerant control, decentralized control

1. Introduction
Reconfigurable modular robots have been widely applied to many fields, such as industry, ocean, space, dangerous environments and so on. The control methods of reconfigurable modular robots have been studied in recent years. Decentralized control has the advantages of robustness and computational efficiency, which is convenient to design control law for large-scale system. The decentralized control has been used in reconfigurable modular robot. Furthermore, due to its sensors, actuators or other parts will inevitably go wrong, and the operator is unable to deal with the faults directly. If the robot fault can not be processed timely, the robot will work in an unpredictable way, which will shorten the service life of the robot, and even lead to catastrophic consequences. In order to improve the reliability and self repairing ability, the fault-tolerant control is one of important research directions of reconfigurable modular robots. Early literatures mainly focus on the hardware redundancy. In order to reduce costs and complexity of the system, software fault tolerant control methods have been widely appreciated. The robust control method of the motion of rigid robots is surveyed [1, 2], but the bound on the uncertainty used in the derivation of the robust-control law improperly depends on the controller gains. For linear time-invariant, SAKSENA et al.[3] and Jain et al. [4] addressed the problem of optimally controlling a large scale dynamic system modeled as an interconnection of subsystems, with each subsystem being controlled by a separate decision maker.

In [5], a new adaptive control scheme is employed to design decentralized adaptive regulators. The interconnected system to be regulated consists of N coupled subsystems having arbitrarily relative degrees. In all these output feedback schemes, the interconnections are assumed to be bounded by Lipschitz-type bounds. In [6] and [7], the overall system cannot be decomposed into subsystems whose states and control inputs are totally decoupled from one another because of the inherent coupling such as moment of inertia and Coriolis force. The system faults are becoming increasingly more important attention, Polycarpou [8] presents a general framework for constructing automated fault diagnosis and accommodation architectures using on-line approximation and adaptation/learning schemes. However, in the case of interconnected systems, a fault may occur not only in the subsystems but also in the interconnections. A decentralized adaptive approximation design for the fault tolerant control of a class interconnected feedback linearizable nonlinear systems is considered [9]. Recently, sliding mode control has attracted attention [10] [11]. In [12], a nonsingular fast terminal sliding mode controller that is based on consensus theory is designed for distributed cooperative

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attitude synchronization. However, there are still some questions, such as singularities in the system and the requirement of bounded uncertainty.

A decentralized adaptive fuzzy controller was designed [13], [14] for interconnected systems which implement the controller without any precise knowledge of the model. For large-scale system, the major advantages of decentralized control and backstepping design is to simplify the control design, but decentralized control method may be involve interconnection term on each subsystem. It is a challenge to deal with interconnection term.

In this paper, for the actuator fault of reconfigurable modular robot, a backstepping decentralized fault tolerant control (DFTC) algorithm is proposed. The reconfigurable robot system is represented by a set of interconnected subsystems. The fault tolerant controller is designed based on backstepping. For the model parameters uncertainty term and interconnected term, the adaptive fuzzy approximation is adopted. The proposed method doesn’t need fault information and the other joint module parameters, suitable for software modular design, and the method does not need to adjust controller parameters under different robot configuration. Simulation results show the controller can guarantee the tracking accuracy and stability of the system with the actuator fault.

2. Problem Statement

The reconfigurable modular robot is made up of joints and connecting rods. According to the Euler-Lagrange equation, the dynamics of a reconfigurable modular robot can be written as

\[ M(q)\ddot{q} + C(q, \dot{q})\dot{q} + F(q, \dot{q}) = u \]  \hspace{1cm} (1)

where \( q \in \mathbb{R}^n \) is the generalized coordinate of the system, \( M(q) \in \mathbb{R}^{n \times n} \) is the inertia matrix, \( C(q, \dot{q}) \in \mathbb{R}^{n} \) is the Coriolis and centrifugal matrix, \( F(q, \dot{q}) \) denotes the unmodeled dynamics term including friction, \( u \) is the generalized force.

The joint module is considered as a subsystem of the reconfigurable modular robot, and these subsystems are interrelated through interconnection term. The dynamics of a reconfigurable modular robot can be rewritten as:

\[
\sum_{j=1}^{n_i} [M_{ij}(q)\ddot{q}_j + C_{ij}(q, \dot{q})\dot{q}_j] + G_i(q) + F_i(q, \dot{q}_i) = u_i
\]

where \( M_{ij}(q), C_{ij}(q) \) denotes inertia matrix and coriolis, centrifugal matrix of the subsystem \( i \cdot M_i(q), C_i(q) \) represent matrix term of subsystem \( i \), respectively, \( M_{ij}(q), C_{ij}(q) \) are the \( ij \) th term of \( M(q), C(q) \).

To use decentralized control method, the dynamical equations of reconfigurable robot system are given by [6]

\[
u_i = M_i(q)\ddot{q}_i + C_i(q, \dot{q}_i)\dot{q}_i + F_i(q, \dot{q}_i) + Z_i(q, \dot{q}, \ddot{q}) \]

\[
Z_i(q, \dot{q}, \ddot{q}) = \left\{ \sum_{j \neq i} M_{ij}(q)\ddot{q}_j + \left[ M_{ii}(q) - M_i(q) \right] \dot{q}_i \right\} + \left\{ \sum_{j \neq i} C_{ij}(q, \dot{q})\dot{q}_j + \left[ C_{ii}(q) - C_i(q) \right] \dot{q}_i \right\} \]

where \( Z_i(q, \dot{q}, \ddot{q}) \) is the interconnection term of subsystem \( i \).

The parameter uncertainties of system \( i \) are given by

\[ M_i(q) = M_i(q) + \Delta M_i(q) \]

\[ C_i(q) = C_i(q) + \Delta C_i(q) \]  \hspace{1cm} (5)
where $M_i'(q_i)$, $C_i'(q_i)$ are the nominal value of subsystem $i$. $\Delta M_i(q_i)$, $\Delta C_i(q_i)$ represent the uncertainty value of subsystem $i$.

Substitute Eqs. (4)-(5) into Eq. (2), then Eq. (6) can be obtained.

$$\dot{u}_i = M_i'(q_i)\dot{q}_i + C_i'(q_i)q_i + \Delta M_i(q_i)\dot{q}_i + \Delta C_i(q_i)q_i + Z_i(q_i, \dot{q}_i, q_i)$$

(6)

Define $d_i(q_i) = \Delta M_i(q_i)\dot{q}_i + \Delta C_i(q_i)q_i$, then

$$\dot{q}_i = M_i''(q_i)[u_i - C_i'(q_i)q_i - d_i(q_i) - Z_i(q_i, \dot{q}_i, q_i)]$$

(7)

Suppose $x_i = [x_{i1}, x_{i2}]^T - [q_i, \dot{q}_i]^T$, The Eqs. (2)-(3) can be written as follows

$$\dot{x}_i = x_{i2}$$

$$x_{i2} = -M_i''(q_i)[C_i'(q_i)x_{i1} + d_i(q_i) + Z_i(q_i, \dot{q}_i, q_i) - u_i]$$

(8)

### 3. Backstepping Decentralized Control

**Step1:**

Assume $y_d$ is the desired trajectory, and define the tracking error as $z_{i1} = y_i - y_{i1}$.

Suppose $\alpha_{i1}$ is the estimation of $x_{i1}$, then the error can be given as $z_{i2} = x_{i2} - \alpha_{i2}$.

then

$$\dot{z}_i = x_i - \dot{y}_d = x_{i2} - \dot{y}_{i2} = z_{i2} + \alpha_{i2} - \dot{y}_d$$

define $\alpha_{i2}$ as

$$\alpha_{i2} = -\lambda_1 z_{i2} + \dot{y}_d$$

where $\lambda_1 > 0$

Chose Lyapunov function as

$$V_i = \frac{1}{2} z_{i1}^T z_{i1}$$

(9)

Differentiating Eq. (9) with respect to time, we have

$$\dot{V}_i = z_{i1}^T \dot{z}_{i1} = z_{i1}^T (\dot{y}_d - \dot{y}_d) = z_{i1}^T (\dot{x}_i - \dot{y}_d) = z_{i1}^T z_{i2} - \lambda_1 z_{i2}^T z_{i1}$$

(10)

If $z_{i2} = 0$, then $V_i < 0$

**Step2:**

Taking the time derivative of $x_{i1}$

$$\dot{z}_{i1} = \dot{x}_{i1} - \dot{\alpha}_{i1} - \dot{M}_i''(q_i)[C_i'(q_i)x_{i1} + d_i(q_i) + Z_i(q_i, \dot{q}_i, q_i) - u_i] - \dot{a}_{i1}$$

(11)

In order to obtain uncertainty term and interconnected term, the fuzzy method is proposed. The adaptive fuzzy approximation can be expressed as

$$\hat{d}_i(q_i, \dot{q}_i) = \Phi_D(q_i, \dot{q}_i)\hat{\theta}_d, \hat{Z}_i(q_i, \dot{q}_i, q_i) = \Phi_Z(q_i, \dot{q}_i, q_i)\hat{\theta}_z$$

where $\Phi_D$, $\Phi_Z$ are fuzzy basis functions, and $\hat{\theta}_d$, $\hat{\theta}_z$ are adjustable parameter vector. $a_{i1}$, $a_{i2}$ represent the ideal values, then the error can be described as

$$\dot{\theta}_d = \theta_d - \hat{\theta}_d, \dot{\theta}_z = \theta_z - \hat{\theta}_z$$

**Assumption 1:** For positive Lipschitz constants $\beta_d$ and $\beta_z$, there exist

$$\|d_i(\theta_d) - \hat{d}_i(\hat{\theta}_d)\| \leq \beta_d \|\theta_d - \hat{\theta}_d\|, \|Z_i(\theta_z) - \hat{Z}_i(\hat{\theta}_z)\| \leq \beta_z \|\theta_z - \hat{\theta}_z\|$$
Define approximation error as
\[ w_{ei} = |d_i - \hat{d}_i| + |Z_i - \hat{Z}_i| \] (12)

The decentralized control law for reconfigurable robot is given as follows
\[ u_i = M_i^r(\ldots) - \lambda_{i1} - \lambda_{i2}z_{i1} + w_{ei} - \dot{\hat{w}} + \dot{\theta}_{i1}^T\hat{\theta}_{i1} + \dot{\theta}_{i2}^T\hat{\theta}_{i2} + y_{id} + C_i\dot{q}_i + \dot{\hat{Z}}_i + w_{ei} \] (13)

The Eq. (11) can be rewritten as
\[ \dot{z}_{i1} = -z_{i1} \text{sgn}(\cdot) - \lambda_{i2} z_{i2} + w_{ei} - \dot{\hat{w}} + \dot{\theta}_{i1}^T\hat{\theta}_{i1} + \dot{\theta}_{i2}^T\hat{\theta}_{i2} \] (14)

with the adaptive laws
\[ \dot{\hat{\theta}}_{i1} = \gamma_{i1} z_{i1} \hat{\theta}_{i1} \] (15)
\[ \dot{\hat{\theta}}_{i2} = \gamma_{i2} z_{i2} \hat{\theta}_{i2} \] (16)
\[ \dot{\hat{w}} = \gamma_{i3} z_{i2} \] (17)

where \( \gamma_{i1}, \gamma_{i2}, \gamma_{i3} \) are positive constants.

**Assumption 2:** the approximation error is bounded by
\[ |w_{ei}| \leq W, \quad \text{where } W > 0. \]

The error of \( W \) is defined as \( \dot{\hat{W}} = W - \hat{W} \).

**Theorem 1:** For reconfigurable system, as seen in Eq. (8), consider the decentralized robust controller, as seen in Eq. (13) with \( \dot{\hat{q}}, \dot{\hat{\theta}}, \dot{\hat{w}} \) updated by Eqs. (15)(16)(17), all signals in the close-loop are uniformly bounded, and asymptotical tracking is ensured.

**Proof:**
Consider the positive definite function
\[ V_i = \sum_{i=1}^{n} V_i = \sum_{i=1}^{n} \left( \frac{1}{2} z_{i1}^2 + \frac{1}{2} z_{i2}^2 + \frac{1}{\gamma_{id}} \dot{\hat{\theta}}_{id} \dot{\theta}_{i1} + \frac{1}{\gamma_{i2}} \dot{\hat{\theta}}_{i2} \dot{\theta}_{i2} + \frac{1}{\gamma_{iw}} \dot{\hat{w}} \dot{w} \right) \]

Now taking the derivative of \( V_i \), we obtain
\[ \dot{V}_i = \sum_{i=1}^{n} \dot{V}_i = \sum_{i=1}^{n} \left( z_{i1}\dot{z}_{i1} + z_{i2}\dot{z}_{i2} - \frac{1}{\gamma_{id}} \dot{\theta}_{id} \dot{\theta}_{i1} - \frac{1}{\gamma_{i2}} \dot{\theta}_{i2} \dot{\theta}_{i2} - \frac{1}{\gamma_{iw}} \dot{w} \dot{w} \right) = \sum_{i=1}^{n} \left( z_{i1}\dot{z}_{i1} - z_{i2}\dot{z}_{i1} - \lambda_{i1} z_{i1}^2 - z_{i1}\dot{\hat{w}} \right) \]

Apply Eq. (14) to \( \dot{V}_i \), then
\[ \dot{V}_i = \sum_{i=1}^{n} \left( z_{i1}\dot{z}_{i1} + z_{i2}\dot{z}_{i2} - \frac{1}{\gamma_{id}} \dot{\theta}_{id} \dot{\theta}_{i1} - \frac{1}{\gamma_{i2}} \dot{\theta}_{i2} \dot{\theta}_{i2} - \frac{1}{\gamma_{iw}} \dot{w} \dot{w} \right) = \sum_{i=1}^{n} \left( z_{i1}\dot{z}_{i1} - z_{i2}\dot{z}_{i1} - \lambda_{i1} z_{i1}^2 - z_{i1}\dot{\hat{w}} \right) \]

Substituting the adaptive laws, as seen in Eqs. (15) (16), into above equation,
\[ \dot{V}_i \leq \sum_{i=1}^{n} \left( -\lambda_{i1} z_{i1}^2 - \lambda_{i2} z_{i2}^2 - z_{i1}\dot{\hat{w}} - \frac{1}{\gamma_{iw}} \dot{w} \dot{w} + z_{i2} (\beta_{i1} \| \beta \dot{\theta} - \dot{\theta} \|) + \beta_{i2} \| \beta \| \right) \]
\[ \leq \sum_{i=1}^{n} \left( -\lambda_{i1} z_{i1}^2 - \lambda_{i2} z_{i2}^2 + z_{i2} \dot{w} - z_{i2}\dot{\hat{w}} - \frac{1}{\gamma_{iw}} \dot{w} \dot{w} \right) = \sum_{i=1}^{n} \left( -\lambda_{i1} z_{i1}^2 - \lambda_{i2} z_{i2}^2 + \dot{\hat{w}} (z_{i2} - \frac{1}{\gamma_{iw}} \dot{w}) \right) \]

using Eq. (17), yields
\[ \dot{V}_i \leq \sum_{i=1}^{n} \left( -\lambda_{i1} z_{i1}^2 - \lambda_{i2} z_{i2}^2 \right) \leq 0 \] (18)

Integrating inequality (18) from \( t = 0 \) to \( t = \infty \), (19) can be obtained.
\[ \sum_{i=1}^{n} \int_{0}^{\infty} (\lambda_i z_i^2 + \lambda_2 z_2^2) dt = V(0) - V(\infty) \]  

(19)

Applying Barbalat’s Lemma, for \( t \to \infty \), then \( z_i = 0, z_2 = 0 \)

4. Fault-tolerant Control

A key objective in engineering is the design of reliable systems that can operate even in the presence of faults. Considering the actuator fault-tolerant control of reconfigurable robot system, the system (2) can be rewritten as follows.

\[ \rho_i \mu_i = M_i(q_i) \ddot{q}_i + C_i(q_i, \dot{q}_i) \dot{q}_i + F_i(q_i, \dot{q}_i) + Z_i(q_i, \dot{q}_i) \]  

(20)

where \( \rho_i \) is the fault factor of the \( i \)-th subsystem, and \( 0 < \rho_i \leq 1 \). \( \mu_i \) represents the fault-tolerant control input.

Following as method in Part 3, the fault-tolerant control is given by

\[ \mu_i = \frac{1}{\rho_i} \mu_i \]

(21)

where \( \mu_i \) is the same as Eq.(13)

Theorem 2: For reconfigurable system, as seen in Eq. (20), consider the decentralized fault-tolerant controller, as seen in Eqs. (13) (21) with \( \dot{q}_i, \ddot{q}_i, \dot{\hat{W}} \) updated by Eqs. (15) (16) (17), all signals in the close-loop are uniformly bounded, and asymptotical tracking is ensured.

5. Simulation Result

The reconfigurable robots consist of joint module and connecting rod module with a set of standard interface, and these modules can be assembled quickly to adapt to different task environment. Figure 1 shows one of the reconfigurable robots with two joints. In order to verify the effectiveness of proposed fault tolerant algorithm in this paper, the experiments were simulated for two-joint reconfigurable modular robot, see Figure 1. The reconfigurable robot is divided into subsystem. Every subsystem has their own control law. These subsystem is coupled by interconnection.

![Figure 1. Reconfigurable modular robot](image)

The dynamical model of two-joint reconfigurable modular robot can be defined [15] by

\[
M_1(q) = \begin{bmatrix}
\frac{4}{3}m_1r_1^2 + m_2l_1^2 + \frac{4}{3}m_2r_2^2 + 2m_2l_1l_2 \cos \theta_2 \\
\frac{4}{3}m_2r_1^2 + m_2r_2l_2 \cos \theta_2 \\
\end{bmatrix}^T, \\
M_2(q) = \begin{bmatrix}
\frac{4}{3}m_2r_2^2 + m_2l_1l_2 \cos \theta_2 \\
\frac{4}{3}m_2r_2^2 \\
\end{bmatrix}^T
\]
where \( m_1, m_2 \) represent the qualities of the first modular and the second modular, \( l_1, l_2 \) represent the length of the first modular and the second modular, \( \alpha_1, \alpha_2 \) represent the angles of the first modular and the second modular, \( r_1 \) represents the distance between the joint 1 and gravity of the first modular, \( r_2 \) represents the other distance. Select the reference inputs as follows. 

\[
y_{d1} = \sin(2\pi t) + \sin(0.5\pi t), \quad y_{d2} = \sin(2\pi t) + \sin(\pi t)
\]

The parameters are \( \lambda_1 = [20 30], \lambda_2 = [30 50], \gamma_\alpha = 2, \gamma_\omega = 5, \gamma_\omega = 10, m_1 = 1, m_2 = 1, \lambda_2 = 0.4, \lambda_1 = 0.4, r_2 = 0.2, r_1 = 0.2 \), the initial speeds were \( q_1(0) = q_1(0) = 0 \), the fault factors of joint 1 and joint 2 were 1 and 0.8, respectively. The fuzzy member functions were chosen as follows. 

\[
\mu_\xi^1 = \exp[-0.5((x_i - 1.25)/0.6)^2], \quad \mu_\xi^2 = \exp[-0.5(x_i/0.6)^2], \quad \mu_\xi^3 = \exp[-0.5((x_i - 1.25)/0.6)^2]
\]

Figure 2(a). Position tracking performance of the joint 1

Figure 2(b). Position tracking performance of the joint 2

Figure 2. Position tracking performance of two-joint reconfigurable modular robot

Figure 2, Figure 3 and Figure 4 are the simulation results. Figure 3(a) and Figure 3(b) show position tracking performance of joint1 and joint2, respectively. Figure 3 shows the control inputs of two-joint reconfigurable modular manipulator. Figure 4(a) and Figure 4(b) show position tracking error of joint1 and joint2, respectively. Figure 5(a) and Figure 5(b) show speed tracking error of joint1 and joint2, respectively.
From simulation results, the tracking error is large at the start, which reason is computing time delay and initial state. The system is steady in less 0.2 second. To verify the tolerant control performance, we assume that fault develops in joint 2 and joint 1 is normal. The position tracking error is small enough to satisfy requirement. It is observed that the decentralized control method presented in the paper has good tracking performance and robustness, and it can be applicable to different reconfigurable modular manipulator.
We obtain the reconfigurable robot model from common robot model, then divided the robot system into multiple subsystem and design controllers. To simplify the design about DFTC, the backstepping method is adopted. The decentralized control method presented in the paper is for reconfigurable robot, which also can be applied to common robot control.

Note that, in practical applications, the joint angle output of the reconfigurable robot should be turned to position trajectory by inverse kinematics. In the theory, we don’t discriminate totally between the joint angle and position trajectory, which needs to pay attention.

6. Conclusions

For the actuator fault of a reconfigurable modular robot, the decentralized fault tolerant control (DFTC) method based on the adaptive fuzzy system is put forward. The actuator fault factors are integrated into the system dynamics model. The controller is consists of backstepping fuzzy logic approximation improving the adaptive capacity of parameter uncertainties term and interconnections term. The adaptive update parameter is constructed based on Lyapunov stability theory, which can guarantee the system stability and tracking performance. The results of numerical simulation show that the proposed algorithm is effective.

The future work is to apply the proposed algorithm to actual experiment and validate its effectiveness.

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