Non-certainty Equivalent Adaptive Exciting Control of Multi-machine Power Systems

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Abstract
Transient stability problem for multi-machine infinite bus system with the generator excitation was addressed via the non-certainty equivalent nonlinear re-parameterization method. The system need not to be linearized. The damping coefficient uncertainty was considered. A non-certainty equivalent excitation controller and a novel parameter updating law were obtained simultaneously via adaptive backstepping and Lyapunov methods to achieve stability of the error systems. Simulation results showed that the proposed controller had good transient performance.

Keywords: multi-machine power system, non-certainty equivalence, adaptive control, nonlinear re-parameterization.

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1. Introduction
The past decade has witnessed a rapid increase in the size and complexity of power systems. Maintaining power system stability is thus one of the main concerns [1-5], and see the references therein. The design of an advanced control system to enhance the power system stability margin so as to achieve higher transfer limits is one of the major problems in power systems, which has attracted a great deal of research attention in recent years [6-12]. Synchronous generator excitation control is one of the most important, effective and economic methods to enhance the stability of power systems [6]. Generator excitation control can not only enhance the power system static stability limit, but also attenuate low-frequency electromechanical oscillations inherent to power systems, during transient conditions.

With the development of power system research, there are a lot of references about single-machine power systems [8-13], and some researchers have increasing interesting in multi-machine system stability. Distributed exciting controllers based on adaptive backstepping are designed by using PSO optimization algorithm. A nonlinear adaptive \( L_2 \)-gain disturbance attenuation controller was given in [14] by using backstepping method. A state-feedback controller based on passivity for multi-machine power system has been proposed in [15]. The controller design method given in [12-15] follow the classical certainty equivalence principle, a new non-certainty equivalent adaptive controller design method was firstly proposed in [16], and it was further studied in [17-21]. It is always reasonable to expect availability of a priori information on the “structure” of uncertain plant parameters. For example, we may have a priori knowledge in terms of lower and/or upper bounds on the values of the uncertain parameters. One possible solution for these classes of systems is to ignore all such a priori available unknown parameter properties, thereby making feasible the application of certainty equivalence adaptive control methods with affine (linear) parameterization. The case against non-utilization of a priori information on uncertain parameter structure can be argued on the basis of the fact that the parameter search (estimation) process takes place outside the feasible region where the corresponding “true” parameters lie, ultimately leading to poor and slow tracking error convergence [22]. To avoid this problem, one could adopt the technique of projection [22] and thereby absorb the prior knowledge on bounds (lower and upper) of unknown parameters. However, an obvious disadvantage of any parameter projection method is the generation of non-smooth control laws, thereby potentially causing practical difficulties with respect to either requirement of actuators with large (infinite) bandwidths or possibility of exciting high-frequency unmodeled (flexible) dynamics. A new specific uncertain parameter structure, wherein the
parameters are known to lie within a priori specified intervals is given in [23]. We provide a new full state feedback adaptive control solution that is custom-built for this unknown parameter structure through a process very different from the parameter projection based conventional certainty equivalence approach. The new solutions derived here enforce a priori known bounds of the uncertain parameters on their corresponding estimates at all times, without compromising on control smoothness or global stability guarantees for the closed-loop dynamics.

In this paper, a non-certainty equivalent adaptive controller is designed for multi-machine power systems by using the method proposed in [23] when the damping coefficient is unknown, which can guarantee all the state trajectories of error systems are bounded and unknown damping coefficient is also bounded. The remainder of paper is as follows: In Section II, the general model for power system is given, we translate the presented model into necessary form in Section III, Section IV gives an important lemma, and a new non-certainty equivalent adaptive controller is designed, a numerical simulation is tested in Section V, Section VI gives some summarizing remarks and suggestions for future work.

2. Problem Description

The multi-machine power systems with the generator excitation is considered, general model of multi-machine power system with excitation control consists of \( n \) generators can be built as follows [1].

\[
\begin{align*}
\dot{\delta}_i &= \omega_i - \omega_b, \\
\dot{\omega}_i &= \frac{D_i}{H_i} (\omega_i - \omega_b) + \frac{1}{H_i}(P_m - P_i), \\
E_{q_i} &= \frac{1}{T_{dii}} \left( X_{dq_i} - X_{dq_j} \right) E_{q_j} + \frac{1}{T_{dii}} \sum_{j=1,j\neq i}^{n} Y_{q_i} E_{q_j} \cos (\delta_j - \delta_i) + \frac{1}{T_{dii}} V_{qi},
\end{align*}
\]

where \( i = 1, 2, \ldots, n \) and the active power of generator \( i \) is

\[
P_i = E_{q_i}^2 G_i + E_{q_i} \sum_{j=1,j\neq i}^{n} Y_{q_i} E_{q_j} \sin (\delta_j - \delta_i).
\]

In system (1), where \( \delta_i \) is running angle of the generator rotor, \( \omega_i \) is the rotor speed of generator, which initial value is equal to \( \omega_b \); \( P_m \) and \( P_i \) are the mechanical power of generator \( i \) and the excitation power, respectively; \( H_i \) is the moment of inertia of the generator \( i \) rotor; \( D_i \) and \( E_{q_i} \) are the damping coefficient and then transient EMF in \( q \) axis of generator \( i \), respectively; \( T_{dii} \) and \( V_{qi} \) are the time constant of the exciting windings of the generator \( i \) and the voltage, respectively; \( B_{ij} \) is the mutual susceptance of generators \( i \); \( G_{ij} \) is the conductance of generator \( i \); \( Y_{ij} \) is the mutual admittance between the \( i \)th and \( j \)th generators; \( V_{qi} \) is the control electrical signal.

According to the generator theory, we have

\[
P_i = E_{q_i}^2 I_{q_i} + \left( X_{dq_i} - X_{dq_j} \right) I_{dq_j} I_{q_j},
\]

and

\[
E_{q_i} = \frac{1}{T_{dii}} E_{q_i} + \frac{1}{T_{dii}} V_{qi} = \frac{E_{q_i} + (X_{dq_i} - X_{dq_j}) I_{dq_j}}{T_{dii}} + \frac{1}{T_{dii}} V_{qi}.
\]

Then the system (1) becomes
\[
\begin{align*}
\dot{\delta}_i &= \omega_i - \omega_{i0}, \\
\dot{\alpha}_i &= \frac{D_i}{H_i}(\alpha_i - \alpha_{i0}) + \frac{\alpha_i}{H_i}(P_n - E'_q - (X'_q - X_{q0})I_dI_q), \\
E'_q &= -\frac{E''_q + (X'_q - X_{q0})I_q}{T_{sh}} + \frac{1}{T_{in}}V_i,
\end{align*}
\]

for \( i = 1, 2 \ldots n \).

3. Model Transformation

In this section, we will transform the system (2) into integrator chain form, which is necessary for designing controller.

Some definitions are given to simplify the system (2),

\[
\begin{align*}
x_{i1} &= \delta_{i1} - \delta_{i10}, \\
x_{i2} &= \omega_{i2} \quad \text{and} \\
x_{i3} &= E'_{q_i} - E'_{q_{i0}} \quad \text{in which} \quad \delta_{i1}, \omega_{i2} \text{and} \quad E'_{q_{i0}} \quad \text{are the initial value of corresponding variables.}
\end{align*}
\]

Let \( k = \frac{\alpha_i}{H_i}, b_0 = X_q - X'_{q_i}, b_0 = X_q - X'_{q_i}, \) are known constants, and \( b_{i0} = -\frac{D_i}{H_i} \) is an unknown constant, then the system (2) can be transformed into (3),

\[
\begin{align*}
x_{i1} &= x_{i2} \\
x_{i2} &= b_{i1}x_{i1} + k_{i}(P_{n} - (X_{i3} + E'_{q_{i0}})I_{q_i} - b_{i2}I_dI_{q_i}) \\
x_{i3} &= \frac{-(X_{i3} + E'_{q_{i0}}) + b_{i2}I_{q_i}}{T_{sh}} + \frac{1}{T_{in}}V_i, \\
\end{align*}
\]

where, \( z_i = [q_{i1}, q_{i2}, q_{i3}]^T \) is the regulation output, \( q_1 \) and \( q_2 \) are the non-negative weighting coefficients satisfied \( q_1^2 + q_2^2 \leq 1 \). Furthermore, let

\[
\begin{align*}
X_{i1} &= x_{i1}, \\
X_{i2} &= x_{i2}, \\
X_{i3} &= b_{i1}x_{i1} + k(P_{n} - (X_{i3} + E'_{q_{i0}})I_{q_i} - b_{i2}I_dI_{q_i});
\end{align*}
\]

We have \( (X_{i3} + E'_{q_{i0}})= [b_{i1}x_{i1} + k(P_{n} - b_{i2}I_dI_{q_i}) - X_{i3}]/(kI_{q_{i0}}) \). The system (3) can be transformed into (4),

\[
\begin{align*}
\dot{X}_{i1} &= X_{i2}, \\
\dot{X}_{i2} &= X_{i3}, \\
\dot{X}_{i3} &= \theta_q(X_{i2} + X_{i3}) + u_i, \\
z_i &= [q_{i1}X_{i1}, q_{i2}X_{i2}]^T
\end{align*}
\]

where \( \theta_q \) is unknown constant, \( u_i = \frac{1}{kI_{q_{i0}}}X_{i3} + u_i \). Note that system (4) is a multi-machine power system with unknown parameters.

4. Main Theorem

In the next, we will give an important lemma. We consider an nth-order nonlinear single input system linearly parameterized through constant uncertain scalar parameters with the following structure.
\[
\dot{x}_1 = x_2,
\dot{x}_2 = x_3,
\vdots
\dot{x}_n = \sum_{k=1}^{n} \theta_k f_k(x) + u,
\]

where the state \( u \in \mathbb{R} \), with the state \( x = [x_1 \ldots x_n]^T \in \mathbb{R}^n \), and the input \( u \in \mathbb{R} \). The parameters \( \theta = [\theta_1 \ldots \theta_n] \) are assumed uncertain. However, a priori bounds on each \( \theta_i \) are assumed known, i.e., the parameters lie within \( (\theta_{\text{min}}, \theta_{\text{max}}) \).

The reference systems are chosen

\[
\begin{align*}
\dot{x}_{m,1} &= x_{m,2}, \\
\dot{x}_{m,2} &= x_{m,3}, \\
&\vdots \\
\dot{x}_{m,n-1} &= x_{m,n} \\
\end{align*}
\]

Define

\[
\varepsilon(t) = x(t) - x_w(t)
\]

Error systems are constructed

\[
\begin{align*}
\dot{\varepsilon}_1 &= e_2, \\
\dot{\varepsilon}_2 &= e_3, \\
&\vdots \\
\dot{\varepsilon}_n &= \sum_{k=1}^{n} \theta_k f_k(x) + u - \dot{x}_{m,n}.
\end{align*}
\]

The method of using nonlinear re-parameterization for adaptive control of error systems has been proposed in [23]. The following result from [23] will be instrumental for our developments.

**Theorem 1 [23]:** Consider the trajectory tracking problem associated with the nth-order single input nonlinear system of Eq. (5) where the constant uncertain parameters \( \theta \), re-parameterized in terms of \( \phi \), in accordance with Eq. (10), satisfy a priori specified lower and upper bounds of the form \( \theta \in (\theta_{\text{min}}, \theta_{\text{max}}) \). Then the following smooth adaptive controller ensures global stability and asymptotic convergence \( \lim_{t \to \infty} [\varepsilon_1(t), \varepsilon_2(t), \ldots, \varepsilon_n(t)] = 0 \) for all initial conditions:

\[
u = \dot{x}_{m,n} - \hat{\theta}^T(t) f(x) - \sum_{j=1}^{n} \alpha_j e_j
\]

\[
\hat{\theta} = \Delta_k \left(1 - \tanh(\hat{\phi}_k + \beta_k(x))\right) + \theta_{\text{min}}^k
\]

\[
\beta_k(x) = -\gamma \int_{0}^{t} f_k(x_1, x_2, \ldots, x_{n-1}, \tau) d\tau
\]
\[
\dot{\phi}(t) = \left[ \frac{\partial \phi}{\partial x} \right] \left( \begin{array}{c} x_2 \\ x_3 \\ \vdots \\ \hat{x}_{m,n} - \sum_{j=1}^{n} \alpha_j e_j \end{array} \right)
\]

where the index \( k \) ranges from \( k = 1, \ldots, N \); \( \gamma \) is any positive scalar, \( \Delta_k = \frac{1}{2}(\theta_{\text{max}} - \theta_{\text{min}}) \) and the positive scalars \( \alpha_1, \alpha_2, \ldots, \alpha_n \) are the coefficients of any \( n \)-th order monic and Hurwitz polynomial.

In the following, we will design non-certainty equivalent adaptive excitation controller for multi-machine power systems by using the preceding lemma.

Defining the variable as \( \hat{X}_{m,1} = X_{m,1} \), \( \hat{X}_{m,2} = X_{m,2} \), \( e_i(t) = X_i(t) - X_{i,m}(t) \), we have

\[
\begin{align*}
\dot{e}_1(t) &= e_1(t), \\
\dot{e}_2(t) &= e_2(t), \\
\dot{e}_3(t) &= \theta_i (X_{i,1} + X_{i,2}) + u_i - X_{i,m,3}.
\end{align*}
\]

We begin by re-parameterization of each \( \theta_i \) in Eq. (12) in terms of an associated new uncertain variable \( \phi_i \) as follows:

\[
\theta_i = \frac{1}{2}(\theta_{\text{max}} - \theta_{\text{min}})(1 - \tanh \phi_i) + \theta_{\text{min}}.
\]

It can be seen from re-parameterization that the obvious advantage for such a benefit is that for all values of \( \phi_i \), the uncertain parameter \( \theta_i \) is restricted to lie in \( (\theta_{\text{min}}, \theta_{\text{max}}) \).

However, the system governing equations in Eq. (12), which are linear (affine) in terms of \( \theta_i \), immediately become nonlinear in terms of \( \phi_i \). In what follows, we develop a new class of smooth adaptive controllers that handle the nonlinear parameterization of Eq. (13) and at the same time, ensure satisfaction of the tracking control objective.

\[
\begin{align*}
\dot{e}_1(t) &= e_1(t), \\
\dot{e}_2(t) &= e_2(t), \\
\dot{e}_3(t) &= \Delta(1 - \tanh \phi_i) + \theta_{\text{max}}(X_{i,1} + X_{i,2}) + u_i - X_{i,m,3}.
\end{align*}
\]

where \( \Delta = \frac{1}{2}(\theta_{\text{max}} - \theta_{\text{min}}) \) and \( \phi_i(t) = \hat{\phi}(t) + \beta_i(x) \), \( \hat{\phi}_i = \frac{1}{2}(\theta_{\text{max}} - \theta_{\text{min}})(1 - \tanh(\hat{\phi}_i + \beta_i(x))) + \theta_{\text{min}} \).

We define an auxiliary variable \( z_{i,1}(t) \) such that \( z_{i,1}(t) = \hat{\phi}_i(t) - \phi_i(t) + \beta_i(x) \), so

\[
\theta_i = \frac{1}{2}(\theta_{\text{max}} - \theta_{\text{min}})(1 - \tanh(\hat{\phi}_i + \beta_i(x) - z_{i,1})) + \theta_{\text{min}}.
\]

Then the error systems (13) becomes (14)

\[
\begin{align*}
\dot{e}_1(t) &= e_1(t), \\
\dot{e}_2(t) &= e_2(t), \\
\dot{e}_3(t) &= \Delta(1 - \tanh(\hat{\phi}_i(t) + \beta_i(x) - z_{i,1})) + \theta_{\text{min}}(X_{i,1} + X_{i,2}) + u_i - X_{i,m,3}.
\end{align*}
\]

For the system (7), Hurwitz matrix is constructed and systems (7) can be rewritten as
\[ \dot{v} = A e + \begin{bmatrix} 0 \\ 0 \\ m \end{bmatrix}, \]  

(15)

where \( m = u_i - \dot{X}_{i\alpha} + \sum_{j=1}^{n} \alpha_j e_j + (\Delta(1 - \tanh(\phi_i(t) + \beta_i(X_i) - Z_i) + \theta_{\text{min}})) f_i(X_i) \).

\[ f_i(X_i) = X_i \cdot z_i + X_i \cdot \alpha \cdot \left( \begin{array}{ccc} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\alpha_4 & -\alpha_2 & -\alpha_3 \end{array} \right) \]

**Theorem 2:** For the multi-machine power systems with uncertain parameter which can be represented in the form of (4), the designed controller can guarantee the closed systems (1) and (16) are globally stable and \( \lim_{t \to \infty} [e_i(t), e_j(t), e_k(t)] = 0 \)

\[ u_i = \dot{X}_{i\alpha} - \sum_{j=1}^{n} \alpha_j e_j - (\Delta(1 - \tanh(\phi_i(t) + Z_i) + \theta_{\text{min}})) f_i(X_i) \]  

(16)

\[ \dot{\theta} = \frac{1}{2}(\theta_{\text{max}} - \theta_{\text{min}})(1 - \tanh(\theta_i + \beta_i(x))) + \theta_{\text{min}} \]  

(17)

\[ \beta = \gamma \int_{0}^{x_i} f_i(X_{i\alpha}, X_{i\beta}, \chi)d\chi \]  

(18)

**Proof:** Choosing Lyapunov function

\[ V = e^T Pe + \frac{1}{\gamma}(\rho + \frac{2p_i^2}{q_m}) \Delta \cdot \left( \ln(\cosh(Z_i + \phi_i)) - Z_i \cdot \tanh(\phi_i) \right) \]  

(19)

The derivative of Lyapunov function is computed

\[ \dot{V} = e^T Pe + e^T \dot{P} e + \frac{1}{\gamma}(\rho + \frac{2p_i^2}{q_m}) \Delta \cdot \left( \tanh(Z_i + \phi_i) - \tanh(\phi_i) \right) \cdot Z_i \]  

(20)

Choosing

\[ u_i = \dot{X}_{i\alpha} - \sum_{j=1}^{n} \alpha_j e_j - (\Delta(1 - \tanh(\phi_i(t) + Z_i) + \theta_{\text{min}})) f_i(X_i) \]

\[ m = \eta_i \cdot f_i \cdot \eta \cdot \Delta \cdot (\tanh(Z_i + \psi_i) - \tanh(\psi_i)) \]

We have \( m = \eta_i \cdot f_i \cdot \eta \cdot \Delta \cdot (\tanh(Z_i + \psi_i) - \tanh(\psi_i)) \).

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\[\dot{Z}_i(t) = \dot{\phi}_i(t) + \dot{\beta}_i(X_i)\]
\[= \dot{\phi}_i(t) + \frac{\partial \beta_i}{\partial X_{i1}} \cdot \dot{X}_{i1} + \frac{\partial \beta_i}{\partial X_{i2}} \cdot \dot{X}_{i2} + \frac{\partial \beta_i}{\partial X_{i3}} (\theta_i f_i + u_i)\]
\[= \dot{\phi}_i(t) + \frac{\partial \beta_i}{\partial X_{i1}} \cdot \dot{X}_{i1} + \frac{\partial \beta_i}{\partial X_{i2}} \cdot \dot{X}_{i2} + \frac{\partial \beta_i}{\partial X_{i3}} \left( \Delta (1 - \tanh(\dot{\phi}_i(t) + \beta_i(X_i) - Z_i) + \theta_{i_{\text{min}}}) f_i + u_i \right)\]

And
\[\dot{\phi}_i(t) = -\frac{\partial \beta_i}{\partial X_{i1}} \cdot \dot{X}_{i1} - \frac{\partial \beta_i}{\partial X_{i2}} \cdot \dot{X}_{i2} - \frac{\partial \beta_i}{\partial X_{i3}} \left( \Delta (1 - \tanh(\dot{\phi}_i(t) + \beta_i(X_i))) + \theta_{i_{\text{min}}}) f_i + u_i \right)\]

We can obtain
\[Z_i(t) = -\frac{\partial \beta_i}{\partial X_{i1}} \cdot \eta_i \cdot f_i\]  \hfill (21)

Choosing
\[\frac{\partial \beta_i}{\partial X_{i1}} = \gamma f_i\]  \hfill (22)

Substituting (21) and (22) into (20),
\[\dot{V} = e^T P e + e^T P e + \frac{1}{\gamma} \left( 2 p_u^2 / q_m \right) \Delta \cdot (\tanh(Z_i + \phi) - \tanh \phi) \cdot \dot{Z}_i\]
\[= e^T (A^T P + AP) e + 2 e^T P [0, 0, f_\eta \| \eta \|^2 + \frac{1}{\gamma} (\rho + \frac{2 p_u^2}{q_m}) \eta \dot{Z}_i\]
\[\leq -d_m \| e \|^2 + 2 n_u \| f_\eta \|^2 - (\rho + \frac{2 p_u^2}{q_m}) \| f_\eta \|^2\]
\[\leq -q_m \| e \|^2 - \rho \| f_\eta \|^2 - (\frac{q_m}{2}) \| \eta \|^2 - 2 n_u \| f_\eta \|^2 + \frac{2 p_u^2}{q_m} \| f_\eta \|^2\]
\[\leq -q_m \| e \|^2 - \rho \| f_\eta \|^2 - (\frac{q_m}{2}) \| \eta \|^2 - \sqrt{\frac{2 p_u^2}{q_m}} \| f_\eta \|^2\]
\[\leq -q_m \| e \|^2 - \rho \| f_\eta \|^2\]

From lemma 1, we can obtain that the closed systems (1) and (16) are globally stable and \[\lim_{t \to \infty} [e_1(t), e_2(t), e_1(t)] = 0\]  .

5. Simulation Analysis
This section gives the comparative simulation analysis for the two-machine power systems (2) between proposed controller and the method given in [11]. The partial parameters for simulation are selected as in Table 1.

<table>
<thead>
<tr>
<th>Table 1 Physical parameters</th>
</tr>
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<tbody>
<tr>
<td>Generator 1</td>
</tr>
<tr>
<td>( \omega_1 )</td>
</tr>
<tr>
<td>( \omega_2 )</td>
</tr>
<tr>
<td>( \omega_3 )</td>
</tr>
<tr>
<td>( x_1 )</td>
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<tr>
<td>( x_2 )</td>
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</table>

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Suppose that the boundary of uncertain parameters are known, generator 1 and generator 2 are running at the stable equilibrium point. Dynamic responses of error systems for the generator 1 and generator 2 using different controller are shown in Figure 1 and Figure 2, respectively.

![Figure 1. Dynamic responses of error systems for the generator 1](image1)

![Figure 2. Dynamic responses of error systems for the generator 2](image2)

From Figure 1 and Figure 2, it can be seen that the error systems for the generator 1 and generator 2 have faster convergence speed using the proposed controller when the damping coefficients are unknown.

6. Conclusions
For the generator excitation system with the damping coefficient uncertainty, an adaptive controller has been designed via a kind of nonlinear re-parameterization method to guarantee asymptotic tracking stability of the system. At the stage of controller design, we can design adaptive law and controller separately by applying nonlinear re-parameterization method. Simulations results verify the effectiveness of the proposed controller.

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