The Optimization Design of Six-bar Linkage Mechanism

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Abstract
At present, multi-bar linkage mechanism is one of the most important directions of mechanical presses’ development. Utilizing the typical multi-linkage drawing mechanism of the plunger slide and compounding its parameters scientifically are a fairly effective way to realize the drawing technology demand. This paper established kinematics mode of the six-bar drawing mechanism by bar-group method, and produced simulated system by Visual Basic, which simulated the actual motion of the mechanism. With the objective function-velocity fluctuation in drawing and drawing depth, using chaos genetic algorithm method, carried out optimization design of the mechanism, and acquired several groups’ data. The optimization results showed that their performance in kinematics was improved greatly and had exceeded the original mechanism.

Keywords: optimum design, chaos genetic algorithm, six-bar linkage mechanism

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1. Introduction
Multi-link mechanism instead of the general crank slider mechanism has become one of the important directions of structure development for mechanical press. To improve production efficiency, press must have the function of fast approaching parts and fast out of the parts. Multi-Linkage presses can meet the above requirements, but the productivity is relatively low. How to use the multi-Linkage to meet presses stretching operations as well as to achieve certain productivity have been the designers’ direction. Weck [1] designed a double-crank mechanism used in presses, Yossifon, Shivpuri [2], Hwang and other scholars have proposed a number of institutions and methods. In 1950, the “uniform stroke” press made by BLISS is described as “may provide a relatively slow stretching speed, and a faster upper stroke, thereby improved the productivity of the press”. The application of multi-Linkage has become the trend of the times, therefore, to study several typical multi-Linkage is a very meaningful work.

From the existing data, the optimal design of multi-Linkage is mostly from the principle on the kinematics, putting slider constant speed or velocity fluctuations as single optimization objective, using optimization method such as statistical testing method, fence round search method, constrained random, complex method, genetic algorithm, and so on [3].

Literature [4] used step search method to analysis and optimize double crank multi-bar mechanism press, and established the optimum design model according to the smallest speed fluctuations in slider working stroke; Literature [5] used statistical test method for the optimal design of multi-link double crank mechanism, slider speed uniform for the optimization objective; Literature [6] optimized six-bar linkage of deep drawing presses with complex method, it is also based on an uniform speed of sliding block as optimization objective; Literature [7] used penalty function method optimization method combined with the steepest descent, optimized double-action deep-drawing presses outer slider mechanism, it was based on the minimum fluctuation of sliding block displacement as optimized target; literature [8] used vector triangle method to analysis and calculate the displacement of multi-bar Linkage press, and optimized the mechanism with statistical test method; Literature [9] was genetic algorithm applied to mechanism optimization. With the rapid development of computer technology, more and more optimization design applied to press Linkage.
2. Motion Simulation

Through analysing six-bar inner sliding block mechanism of double-action drawing press with bar-group method, we know that it contains some of the basic group types, we can compile them into a series of calculation and graphical display subroutines to form a more complete set of common procedures [6], and then develop motion analysis system, which makes analysis work simple, reliable and convenient. Given the rod length and rack data of the six-bar linkage, $X_6 = 1330 \text{mm}$, $Y_6 = -200 \text{mm}$, $L_4 = 1020 \text{mm}$, $L_3 = 1250 \text{mm}$, $L_{20} = 1550 \text{mm}$, $L_4 = 1400 \text{mm}$, $e = -90 \text{mm}$, the mechanism as Figure 1 shows. Split the mechanism into one single rod group, two RRR rod groups and one RRP rod group, as Figure 2 shows. Calculated displacement, velocity and acceleration of the six-bar inner sliding block lignment with bar-group method, then compiled main program and develop the simulation according to the flow chart of program.

In the design of multi-bar linkage press, there are a large number of bars, so how to choose each bar size to ensure the best kinematical characteristic and to achieve longer drawing depth as soon as possible is a key to the design. Bar-group method was used in kinematic analysis. The solution of displacement, velocity, acceleration of six-bar inner sliding block mechanism is below.

Step 1. Establish the coordinate system, as Figure 1.

![Figure 1. The six-bar linkage of press](image1)

![Figure 2. Dividing into bar group forms](image2)
Step 2. Split the mechanism into several bar-groups as shown in Figure 2.
Step 3. Determine the position of each double-bar group.
Factor M can be seen from Figure 2, for the RRR double-rod group, composed by bar 2 and 3, \( M = +1 \); rod BCE RRR double-rod group, \( M = +1 \); member 4 and the slider dual-rod group composed of the RRP, because EFA is less than 90°, \( M = +1 \).
Program flowchart of six-bar mechanism is shown in Figure 3.
Step 5. Calculate the crank AB single rod member to obtain the position of the point B, as in formula (1) below, the velocity as the formula (2) shown.

\[
\begin{align*}
\dot{x}_B &= L_1 \cos \varphi_i \\
\dot{y}_B &= L_1 \sin \varphi_i \\
\dot{v}_{Bx} &= -\alpha \dot{\theta} (y_B - y_A) \\
\dot{v}_{By} &= \alpha \dot{\theta} (x_B - x_A)
\end{align*}
\]  

Velocity is the time derivative of the position equations, and acceleration is the time derivative of the velocity equations, as the formula (3) shown below.

\[
\begin{align*}
\ddot{x}_B &= -\alpha \dot{\theta}^2 (x_B - x_A) \\
\ddot{y}_B &= -\alpha \dot{\theta}^2 (y_B - y_A)
\end{align*}
\]
Step 6. Acquire the point C motion parameters.
Rod BC and rod CD constitute RRR bar-group. Point C equation as in formula (4), velocity equation as in formula (5) and acceleration equations, as the formula (6) shown below.

\[
\begin{align*}
\frac{\pi}{2} & = \frac{\pi}{2} \\
2 & = 2 \\
\sin \phi & = \sin \phi \\
L_{yy} & = L_{yy} \\
L_{xx} & = L_{xx} \\
& = \phi \\
\end{align*}
\]

Velocity equation as in formula (5):

\[
\begin{align*}
\frac{\pi}{2} & = \frac{\pi}{2} \\
2 & = 2 \\
\sin \phi & = \sin \phi \\
L_{yy} & = L_{yy} \\
L_{xx} & = L_{xx} \\
& = \phi \\
\end{align*}
\]

Acceleration equations, as the formula (6) shown below:

\[
\begin{align*}
\frac{\pi}{2} & = \frac{\pi}{2} \\
2 & = 2 \\
\sin \phi & = \sin \phi \\
L_{yy} & = L_{yy} \\
L_{xx} & = L_{xx} \\
& = \phi \\
\end{align*}
\]

Step 7. Finding the motion parameters of the point E.
Rod BCE can be regarded as a RRR bar-group composed of three rods - BC, BE, and CE. The motion parameters of the point B and point C has been determined, then the point E motion parameters which can be calculated.

Step 8. Acquire the motion parameters of the F-point.
Rod EF and sliders constitute the RRP double rod group. For the abscissa of the slider is known, that is offset, the rod EF can be considered a single rod member, position vertical coordinate of the point F can be obtained by the single rod member positions equation, and then seek the speed and acceleration of the point F.

Figure 4. Main interface of six-bar linkage mechanism

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The press six-bar kinematic analysis and optimization software were developed by VB, the main interface of the mechanism as Figure 4 shown. Using the developed software to track movement characteristic curve of the drawing press, can not only test the reasonableness of the design program, but also test whether the mechanism parameter design is reasonable and provide a powerful tool for multi-link mechanism design of drawing press. It will produce a certain significance for studying the optimization of multi-link design and machine performance parameters.

This software can realize the data input, modification of rod length and geometrical parameters, playing movement animation, calculating and displaying the six-bar linkage slider displacement, velocity and acceleration curves according to the input data, outputing displacement, velocity and acceleration data of crank position corresponding into a Word document, outputing link optimized parameters data to Excel spreadsheets, and conducting optimal design of the six-bar inner block mechanism. According to the given design performance, optimized several times, and output optimized slider speed graph curves, select the appropriate optimization, output the mechanism parameters and kinematics graph.

3. Chaos Genetic Algorithm

Chaos is a non-linear phenomenon widespread in nature, it has randomness, ergodicity, sensitivity of initial conditions, and has been widely used in stochastic optimization. Moreover, it has superior performance in areas of local optimization. The chaotic mapping Logistic iterative equation[10,11] in the study is as following:

\[ \beta_j^{k+1} = \mu \beta_j^k (1 - \beta_j^k), k = 1, 2, ..., \beta_j \in (0,1), \]
\[ \beta_j \neq 0.25, 0.5, 0.75. \]

\( \beta_j \) is the j-th chaos variable of particle \( X_j \). When \( \mu = 4 \), Logistic equation is completely in a chaotic state. Chaotic iteration practiced for the best individual in the population, \( P_{ibest} = [x_1, x_2, ..., x_j, ..., x_n] \). The steps are as follows:

Step1. Random assumption \( \beta_j^k, k = 1 \).

Step2. Acquire \( \beta_j^{k+1} \) through Logistic equation.

Step3. Determine variation scale as follows:

\[ P_c = x_{j_{min}} + \beta_j^{k+1} (x_{j_{max}} - x_{j_{min}}) \]

Step4. The new position of \( x_j \) can be acquired by the equation:

\[ x_j^{k+1} = (1 - \lambda_g) x_j^k + \lambda_g P_c \]

Step5. Recalculate the fitness value of \( P_{ibest} \), if the value is less than the original adaptation or chaotic iteration reaches a certain number of steps, then stop chaotic search; Otherwise, go to step 2. \( \lambda_g \) is the shrinkage factor, which determines the variation \( x_j \) of the variable space, obtained by the following formula \( \lambda_g = 1 - ((g - 1)/g)^m \), g is the evolution of the genetic algorithm algebra, m is used to control the convergence rate.

We can see from the formula, the search space of the best individual variable \( P_{ibest} \) is reduced with increasing iteration algebra. Thus, the variation scale in the initial stage of evolution is relatively large, which is conducive to search the global optimum solution in a vast space; in the later stage of evolution is small-scale, and fine search tightly around the local pole, helps to improve the accuracy of the solution in a small space.

4. Optimization Design

Because \( L_{20} \) and \( L_4 \) are obtained by the iteration of known stroke h, the optimization variables are \( X = [X_6, Y_6, L_1, L_2, L_3, h, e]^T \).
Corresponding to the six speed curve, the maximum fluctuation of drawing can be expressed as formula (7), \( V'(x) \) is the first extreme point in drawing range, and \( V''(x) \) is the second extreme point in drawing range. Drawing depth can be expressed as the displacement difference from starting drawing (speed of 0.3m/s, corresponding to the crank angle \( \theta_1 \)) to the the end of the drawing (corresponding crank angle \( \theta_2 \)); drawing angle can be expressed as the angle difference from \( \theta_1 \) to \( \theta_2 \).

For the six-bar press execution mechanism design, the maximum speed fluctuation amount in drawing stage should be as small as possible and drawing depth as large as possible, so the speed fluctuation in the forging punch stroke and drawing depth are chosen as optimization goal, as following:

\[
V_b(x) = \frac{V''(x) - V'(x)}{\left(V' + V(x)\right)/2} \tag{7}
\]

\[
H_L(x) = S_\theta (x) - S_\theta (x) \tag{8}
\]

The expression of the objective function : \( F(x) = k_1V_b(x) + k_2H_L(x) \)

\( k_1 \) and \( k_2 \) are weighting coefficients. They can be selected based on experience and calculation analysis.

Constraints

The six-bar slider mechanism must satisfy the presence of crank and mechanical transmission conditions, as following:

\[
\begin{align*}
L_1 &< L_2 \\
L_1 &< L_3 \\
L_1 &< d \\
L_1 + Mx &\leq L_3 + L_4 \\
amax &\leq \varphi_a
\end{align*}
\]

\( Mx \) is the maximum 0f \( L_2, L_3 \) and \( d; L_3 \) and \( L_4 \) are another two bars except \( L_1 \) and \( d \); amax is the maximum pressure angle of the slider; \( \varphi_a \) is the limit of maximum pressure angle.

Regard an existing press as a study object, the mechanism parameters of original scheme are showed in Table 1.

| Table 1. Comparison between superior schemes and the original scheme |
|-------------------|-----------------|----------------|----------------|-----------------|-----------------|------------------|
| Variable          | \( X_6 \) (mm)  | \( Y_6 \) (mm) | \( L_1 \) (mm) | \( L_2 \) (mm)  | \( L_3 \) (mm)  | \( h \) (mm)     | \( e \) (mm)     |
| Original scheme   | 1320            | 240            | 240            | 1000           | 1260           | 960              | -87             |
| Scheme 1          | 1340            | 180            | 280            | 1020           | 1250           | 950              | -92             |
| Scheme 2          | 1320            | 220            | 280            | 1040           | 1250           | 950              | -82             |
| Scheme 3          | 1340            | 260            | 280            | 1020           | 1270           | 960              | -92             |

<table>
<thead>
<tr>
<th>Variable</th>
<th>( L_{20} ) (mm)</th>
<th>( L_4 ) (mm)</th>
<th>( H_L ) (mm)</th>
<th>( V_b )</th>
<th>Drawing range(°)</th>
<th>Drawing angle(°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original scheme</td>
<td>1793.1</td>
<td>1255.5</td>
<td>254.48</td>
<td>0.167</td>
<td>112~202</td>
<td>90</td>
</tr>
<tr>
<td>Scheme 1</td>
<td>1495.4</td>
<td>1445.7</td>
<td>280.24</td>
<td>0.167</td>
<td>104~189</td>
<td>85</td>
</tr>
<tr>
<td>Scheme 2</td>
<td>1479.1</td>
<td>1461.7</td>
<td>257.34</td>
<td>0.074</td>
<td>108~188</td>
<td>80</td>
</tr>
<tr>
<td>Scheme 3</td>
<td>1484.5</td>
<td>1456.6</td>
<td>254.96</td>
<td>0.084</td>
<td>108~188</td>
<td>80</td>
</tr>
</tbody>
</table>

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The speed in drawing range required less than 0.4m / s. Write VB program based on above mathematical model, then optimize the simulation, and optimization results are output to an Excel table, after selection, three preferred schemes can be acquired.

Compared with the original scheme, optimization schemes 1 to 3 have the following advantages:

In the case of the same speed of the crank, the same chassis parameters, and the same speed fluctuation amount in drawing range, drawing depth of optimization scheme significantly increased. The drawing depth of the original mechanism is 254.48mm, but for scheme 1, the drawing depth is 280.24mm, increased 25.76mm, thereby improved the performance level of the mechanism.

In the case of the same speed of the crank, the same chassis parameters, the slider speed of the drawing stroke stage can be more uniform, and the drawing depth is greater than the original scheme, such as the scheme 2, and scheme 3, their draw depth are slightly larger than the original scheme, and speed fluctuation amount of the drawing area are smaller, only 0.074 and 0.084.

In the case of the same speed of the crank, the maximum pressure angle reduces much compared with the original scheme, the original scheme is 44.071°; scheme 1 is 40.337°; scheme 2 is 40.232°; scheme 3 is 38.367°; the maxim um pressure angle of optimization schemes are all less than the original scheme. The smaller of the pressure angle, the better of the force transfer performance of the mechanism, the higher of the efficiency.

In the given stroke h and allowable speed $V_X$ conditions, by adjusting the parameters of the lever member, found the structural parameters of the drawing smaller angle. For example, with the same stroke, compared the optimization scheme 1 with 2, drawing angle were respectively 90° and 80°, so scheme 2 is very useful for double-action press.

4. Conclusion

Adopted velocity fluctuation quantity and drawing depth as pros and cons indicators to evaluate six-bar drawing mechanism performance. For single action press, we can use bigger drawing angle to acquire longer drawing depth; for double action press, given drawing depth, drawing angle must be as small as possible.

Optimization schemes are better than the original scheme in terms of velocity fluctuation quantity, drawing range, drawing speed, drawing depth, and drawing angle respects, and the maximum pressure angle reduces more than the original scheme.

The optimum design model and mechanism design optimization examples in this article are available to be referred to for similar mechanism designs.

References


