Voltage and Reactive Power Control of Front-end Speed Controlled Wind Turbine via $H_\infty$ Strategy

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Abstract

According to the characteristics like rapidity, time variability and the uncertainty of the input mechanical torque of front-end speed control wind turbine (FSCWT) with directly grid-connected and electrically excited synchronous generator (EESG), a double-loop $H_\infty$ control approach was proposed. In which, a simplified structure of brushless excitation system was used. For the inner loop, first $H_\infty$ excitation controller was designed for realizing a fast excitation control; the second $H_\infty$ power system stabilizer (PSS) was designed by solving the Riccati equation for the purpose of eliminating the oscillation and desynchronization of generator may caused by fast excitation to improve the system transient stability, which ensured the generator with a stable operation in grid-connecting and effectively solved the contradiction between fast excitation and transient stability. Then the designed $H_\infty$ controllers were applied to the voltage and reactive power control system (VRCS), which realized the output voltage and reactive power control requirements by tuning the weighting functions. Simulation results show that the double-loop $H_\infty$ control approach was more effective than the single $H_\infty$ excitation control in voltage and reactive power control of FSCWT.

Keywords: voltage and reactive power control, $H_\infty$ control, transient stability, front-end speed controlled wind turbine (FSCWT), electrically excited generator (EESG).

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1. Introduction

The voltage and reactive power control of generators is of important significance to improve the stability of the power system, keep the voltage balance and minimize the transmission loss. As the proportion of grid-connected wind energy capacity in power system increased year by year, the grid-connecting of large-scale wind farms directly affect the power system stability. The wind generator output voltage and reactive power would be varied when a small signal disturbance or oscillations occured in power system. The voltage and reactive power control is necessary in order to keep a stable output voltage to maintain the power system voltage balance.

Scholars have made investigations on voltage and reactive power control of doubly fed induction wind generators and direct-drive permanent magnet synchronous wind generators [1-3]. However, these types of generators are all of poor performance in low voltage ride through (LVRT) and reactive power support capability compared with FSCWT proposed in this paper as presented in Figure 1, which used a hydro-dynamic controlled gearbox instead of the electrical inverter and directly connected to grid with a rated voltage of 10KV without using transformer [4].

![Figure 1. Structure of FSCWT](image)

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For FSCWT, the output voltage and reactive power can be regulated by excitation control. In [5], a fast excitation control structure was presented and a $H_{\infty}$ controller for output voltage real-time feedback control was designed accordingly, in which the influence of fast excitation control on the transient stability of synchronous generator was not considered; in [6-7], an exciter was proposed for electrically excited synchronous wind generator by using optimal module and symmetrical optimal design techniques, which didn’t take the effect of power system failure on voltage and reactive power into consideration. In this paper, a VRCS model was set up according to the fast excitation system and a $H_{\infty}$ excitation controller was designed accordingly; on the bases of which, an equivalent simple machine infinite bus (SMIB) system was refined and a $H_{\infty}$ PSS was designed in order to guarantee the transient stability.

2. Modelling of System

The unbalance of reactive power would cause a voltage deviation of power system from the set value, which results in an unstable operation of wind generators. It is the wind variation, the power system fault and the electrical equipment switching that lead to a reactive power change. For the purpose of meeting the requirements of voltage and reactive power control, an voltage and reactive power double loop control system was designed which structure can be seen in Figure 2. The speed deviation was used as input for $H_{\infty}$ PSS in the outer loop, the reference value was compared with the feedback voltage of generator and the output of PSS, then the error was used as input for inner $H_{\infty}$ excitation controller. So, the generator voltage can be regulated by controlling the output of PWM rectifier so that the generator voltage would maintain at a constant value.

\begin{align*}
\dot{v}_d &= -R_d i_d - d/dt \cdot \psi_d + \omega_0 \psi_q, \\
\dot{v}_q &= -R_q i_q + d/dt \cdot \psi_q + \omega_2 \psi_d, \\
\dot{v}_r &= d/dt \cdot \psi_r + R_e i_r, \\
0 &= d/dt \cdot \psi_D + R_d i_D, \\
0 &= d/dt \cdot \psi_Q + R_Q i_Q
\end{align*}

(1)

Where $i_d$ and $i_q$ are respectively the direct and transverse currents, $i_D$ and $i_Q$ are the direct and transverse damper’s currents and $i_r$ is the exciter current. $\psi_d$ and $\psi_q$ are the stator total...
flux, \( \psi_r \) is the main field total flux. \( \psi_d \) and \( \psi_q \) are the direct and transverse damper’s total flux. \( R_s \), \( R_f \) are respectively the stator resistance and the main field resistance, \( R_d \) and \( R_q \) are the damper’s resistances. \( \omega_e \) is the electrical speed corresponding to the time derivative of the stator electrical angle. The electrical torque equation of synchronous generator can be written as:

\[
P_e = [E'_q + (X_q - X'_q)i_d]i_q - R_s(i_d^2 + i_q^2)
\]

By ignoring the damper winding response, the transient response of stator winding and the effect on rotor speed variation, a simplified synchronous generator in Park’s framework can be defined as:

\[
\begin{align*}
T_{eq} &= E'_q - (X_d - X'_d)i_d \\
\nu_d &= -R_s i_d + X_d i_q \\
\nu_q &= -R_s i_q - X'_d i_d + E'
\end{align*}
\]

(2)

Where \( P_e \) the electromagnetic power of synchronous generator is, \( E'_q \) is the transverse electromotive force, \( X_d \) and \( X_q \) are respectively the direct and transverse resistances, \( X'_d \) is the direct transient resistance. The model of PWM rectifier in d-q axis can be described as [11]:

\[
\begin{align*}
\nu_{sd} &= R_s i_{sd} + \psi_{sd} \cdot d/dt - \omega_r \psi_{sq} \\
\nu_{sq} &= R_s i_{sq} + \psi_{sq} \cdot d/dt + \omega_r \psi_{sd}
\end{align*}
\]

(3)

Where \( \psi_{sd} \) and \( \psi_{sq} \) are the fluxes and \( \omega_r \) is the angular speed of the generator shaft.

In order to decouple the d-axis and q-axis components of PWM rectifier, a feedforward decoupling control method was used [12]. Where \( G(s) \) is the transfer function of current regulator, \( i_{sd}^* \) and \( i_{sq}^* \) is the set values of \( i_{sd} \) and \( i_{sq} \). So, the control equation of \( u_{sd} \) and \( u_{sq} \) is:

\[
\begin{align*}
u_{sd} &= -G(s)(i_{sd}^* - i_{sd}) - \omega_r \psi_{sq} + e_d \\
\nu_{sq} &= -G(s)(i_{sq}^* - i_{sq}) - \omega_r \psi_{sd} + e_q
\end{align*}
\]

(4)

(5)

the (5) can be rewritten as:

\[
\begin{align*} e_1 = [L \cdot d/dt + R_s \quad \omega_r \cdot L]_i [i_{sd}]_i + [u_{sd}]_i \\
0 = -[G(s) + R_s \quad L] \cdot [i_{sq}]_i + [u_{sq}]_i
\end{align*}
\]

(6)

substitute formula (4) into (6) resulted in:

\[
\begin{align*} d/dt [i_{sd}]_i = [G(s) + R_s \quad L]_i [i_{sd}]_i + [u_{sd}]_i \\
0 = -[G(s) + R_s \quad L] \cdot [i_{sq}]_i + [u_{sq}]_i
\end{align*}
\]

(7)

Formula (7) shows that the feedforward decoupling control method realized the decoupling of \( i_{sd} \) and \( i_{sq} \). The measurement unit in PWM rectifier can be equivalent to a small inertial unit \( G_m(s) = K_m/(1 + \tau_m s) \), combined with PWM rectifier unit, we can get:

\[
\frac{dv_r}{dt} = -\frac{v_r}{\tau_m + 3T_s} + \frac{K_{PWM}}{1 + (\tau_m + 3T_s)s}
\]

(8)
Consider \( v_i = \frac{R_l}{X_{sd}} E_{sd} \) and Substituted into (8) we can get:

\[
E_{sd} = \frac{E_{sd}}{\tau_m + 3T_s} + \frac{K_{pwu} X_{sd}}{(\tau_m + 3T_s)R_y} v_i
\]

(9)

Combined with (3), we can get a 3rd order excitation control model of electrically excited synchronous generator:

\[
\begin{align*}
\frac{dE'}{dt} &= \frac{1}{T_{so}} E_u - \frac{1}{T_{so}} E' - \frac{(X_u - X'_s)}{T_{so}} I_u \\
\frac{dE''}{dt} &= c \frac{1}{T_{st}} E_u + \frac{1}{T_{st}} E' - \frac{(X_u - X'_s)}{T_{so}} I_u + \frac{cX'_u - cX'_s}{T_{so}} I_u \\
\frac{dE_q}{dt} &= -\frac{1}{T_{so}} E'_q + (X'_q - X'_s) I_q \\
\frac{E_q}{\tau_m + 3T_s} + \frac{K_{pwu} X_{sd}}{(\tau_m + 3T_s)R_y} U_c
\end{align*}
\]

(10)

Where \( U_c \) is the output voltage of excitation controller, \( c = (X'_q - X'_s)/(X'_d - X_d) \).

For the purpose of taking the effect of excitation control on the system stability into account, using the swing equation of synchronous generator in [13]:

\[
\begin{align*}
\Delta\omega &= 1/(2Hs + K_0) \\
\Delta\delta &= 2\pi f_0 \Delta\omega / s
\end{align*}
\]

(13)

The torque equation is:

\[
\Delta T_q = K_1 \Delta\delta + K_2 \Delta E'_q
\]

(14)

And the excitation voltage equation is:

\[(1 + s T_{do} K_3) \Delta E'_q = -K_3 (\Delta E_i + K_4 \Delta\delta) \]

(15)

Also the stator voltage equation is:

\[
\Delta u_i = K_5 \Delta\delta + K_6 \Delta E'_q
\]

(16)

The Low-frequency oscillation matrix equation of synchronous generator for transient stability analysis can be obtained from formula (3), (13), (14), (15) and (16) as is shown in below:

\[
\begin{bmatrix}
\Delta\omega \\
\Delta\delta \\
\Delta E'_q \\
\Delta E_i
\end{bmatrix} =
\begin{bmatrix}
-K_0 / 2H & -K_1 / 2H & -K_2 / 2H & 0 \\
2\pi f_0 & 0 & 0 & 0 \\
0 & -K_3 / T_{do} & -K_3 / T_{do} & -K_3 / T_{do} \\
0 & -K_4 K_s / T_{so} & -K_4 K_s / T_{so} & -K_4 K_s / T_{so}
\end{bmatrix}
\begin{bmatrix}
\Delta\omega \\
\Delta\delta \\
\Delta E'_q \\
\Delta E_i
\end{bmatrix}
\]

(17)

where \( K_0 \sim K_6 \) represent internal influence factors within the system, which were all described detailly in [13].
3. Design of $H_\infty$ controller

3.1. Design of Excitation Controller

Figure 4 shows the standard $H_\infty$ feedback structure, where $r$ is the system input while $y$ is the output, $e$ is the input error, $u$ is the output of controller, $K_{c1}$ is the controller and $P$ is the general controlled plant. $W_1$, $W_2$, and $W_3$ are respectively the weighting functions of sensitivity function $S$, input sensitivity function $R$ and penalty sensitivity function $T$.

![Figure 4. Structure of Standard $H_\infty$ feedback](image)

In the $H_\infty$ control theory, the general controlled plant $P$ is:

$$P = \begin{bmatrix} W_1 & -W_1 G \\ 0 & W_2 \\ 0 & W_3 G \\ I & -G \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix}$$

(18)

And the state space is described as:

$$\begin{align*}
\dot{x} &= Ax + B_1 \omega + B_2 u \\
z &= C_1 x + D_{11} \omega + D_{12} u \\
y &= C_2 x + D_{21} \omega + D_{22} u
\end{align*}$$

(19)

Where $x$ is an $n$-dimensional state variable, $\omega$ is an $r$-dimensional signal vector and $u$ is a $p$-dimensional control vector. $z$ and $y$ are respectively the expected and measured output. The closed-loop transfer function from $w$ to $z$ can be written as:

$$T_{zw}(s) = LFT(P(s), K(s)) = P_{11} + P_{12} K(I - P_{22} K)^{-1} P_{21}$$

The $H_\infty$ standard problem is to find out whether there is a controller $K$ with the given $H_\infty$ performance $\gamma$ to the generalized plant $P$ or not to ensure that the closed-loop system have an internal stability, and $\left\| T_{zw}(s) \right\| < 1$ in addition. For the generalized plant shown as formula (18), the $H_\infty$ standard problem based on solving linear matrix inequalities is design a feedback $H_\infty$ controller $u(s) = K(s)y(s)$, which will make the output from $w$ to $z$ is bounded [14].

The inner loop control structure of VRCS uses the stator voltage and field voltage as feedback signals to compare with the set value, and then the $H_\infty$ controller will take the compared error as input signal for field current regulation so as to control the generator output voltage.

In the formula (10), let $y = E_\phi^s$, the equation of the electrically excited synchronous containing the excitation system can be written as:

$$\begin{align*}
\dot{X} &= A_\phi E + B_\phi I + B_{\phi\phi} U_s \\
y &= C_\phi E
\end{align*}$$

(20)
Where:

\[
X = \begin{bmatrix} E_d^* \\ E_q^* \\ E_d^* \\ E_q^* \end{bmatrix}, \quad I = \begin{bmatrix} i_d \\ i_q \end{bmatrix}, \quad A_p = \begin{bmatrix} -1/(\tau_m + 3T_d) & 0 & 0 & 0 \\ 1/T_d & -1/T_d & 0 & 0 \\ c/T_d & 1/T_d - c/T_d & 0 & 0 \\ 0 & 0 & -1/T_d & 0 \end{bmatrix}
\]

\[
B_{p1} = \begin{bmatrix} 0 & 0 & 0 \\ (X_d^* - X_q^*)/T_d & 0 & 0 \\ 0 & (X_d^* - X_q^*)/T_d & 0 \\ 0 & 0 & (X_q^* - X_d^*)/T_d \end{bmatrix}, \quad B_{p2} = \frac{K_{peus}(X_d - X_i)}{R_f(\tau_m + 3T_d)}, \quad C_p = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}
\]

### Table 1. Parameters used in Excitation System Model

<table>
<thead>
<tr>
<th>Labels</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field resistance Rf (pu)</td>
<td>0.0037</td>
</tr>
<tr>
<td>d-axis synchronous reactance xd(pu)</td>
<td>1.52</td>
</tr>
<tr>
<td>q-axis synchronous reactance xq(pu)</td>
<td>0.996</td>
</tr>
<tr>
<td>d-axis transient reactance xd(pu)</td>
<td>0.152</td>
</tr>
<tr>
<td>q-axis transient reactance xq(pu)</td>
<td>0.116</td>
</tr>
<tr>
<td>d-axis subtransient reactance x’d(pu)</td>
<td>0.192</td>
</tr>
<tr>
<td>q-axis subtransient reactance x’q(pu)</td>
<td>0.208</td>
</tr>
<tr>
<td>d-axis open circuit transient time constant T’d(s)</td>
<td>0.022</td>
</tr>
<tr>
<td>q-axis open circuit subtransient time constant T’q(s)</td>
<td>0.011</td>
</tr>
<tr>
<td>Rotor Inertia H(kgm²)</td>
<td>109.0223</td>
</tr>
<tr>
<td>Equivalent excitation Gain Ka</td>
<td>40</td>
</tr>
<tr>
<td>Equivalent excitation time constant Ta (s)</td>
<td>0.1</td>
</tr>
<tr>
<td>Generator Inertia time constant H (s)</td>
<td>4.25</td>
</tr>
<tr>
<td>Line reactance Xe (pu)</td>
<td>0.5</td>
</tr>
</tbody>
</table>

In this paper, we used the parameters in Table 1 [15] for controller design, the weighting functions \(W_1, W_2\) and \(W_3\) used for \(H_\infty\) excitation controller are:

\[
W_1 = \frac{0.02(s + 500)}{s + 0.002}, \quad W_2 = 1.0 \times 10^{-5}, \quad W_3 = \frac{0.02(s + 10)}{s + 80}
\]

The controller can be obtained by solving the LMI equation [16]:

\[
K_{C_1} = \frac{-338787993.9183(s + 1.561)(s + 0.9816)(s + 0.7692)}{(s + 1.707e+005)(s + 2.929)(s + 1.197)(s + 0.03153)}
\]

As we can see from above, the zeros and poles of \(K_{C_1}\) are all in the left half-panel of complex frequency domain, which indicated the closed-loop system is stable.

#### 3.2. Design of PSS

The traditional PSS design method is based on the singular value theory or Glover McFarlane’s loop shaping method with defined structure by choosing suitable weighting functions. Usually, the weighting functions used for \(H_\infty\) loop shaping method for PSS design is selected by experience trial and error. In this paper, the design of \(H_\infty\) PSS was attributed to the mixed sensitively problem [16], the structure can be seen in Figure 5.

![Figure 5. Structure of Mixed Sensitivety Problem](image-url)
Where \( V_1(s) \), \( V_2(s) \) and \( V_3(s) \) are the weighting functions used for \( H_\infty \) PSS design and the weighting function \( V(s) \) is used for configuration the poles of system.

In the SMIB system, the PSS designed via \( H_\infty \) strategy can be obtained by using \( \Delta \omega \) as input and \( \Delta u_x \) for output. Suppose that the system variables are \( x = [\Delta \omega \ \Delta \delta \ \Delta E_g' \ \Delta E_i']^T \), \( \Delta u_e \) and \( \Delta \delta \) are respectively the system input and output, then we can get the output equation and the state equation of the proposed system from (17) as below:

\[
\begin{align*}
\dot{x} &= A_x x + B_x u \\
y &= C_x x + D_x u
\end{align*}
\]

Where:

\[
A_x = \begin{bmatrix}
-K_1/2H & -K_2/2H & 0 \\
2\pi f_0 & 0 & 0 \\
0 & -K_4/T_{\phi_0} & -K_5/T_{\phi_0} \\
0 & -K_6/K_A & -K_6/K_A \\
-K_7/K_A & 1/T_A & -1/T_A
\end{bmatrix}
\]

\[
B_x = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
K_A/T_A
\end{bmatrix}
\]

\[
C_x = \begin{bmatrix}
1 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

\[
D_x = 0
\]

We can get the characteristic roots: \( \lambda_1 = -0.0205 \pm j5.3394 \), \( \lambda_2 = -17.4215 \pm j50.0488 \) of \( A_x \) by substituting the parameters in table 1. As the weak damping poles \((-0.0205 \pm j5.3394)\) is nearby imaginary axis, the system would instability when a system fault occurs. So we reconfigured the poles as \((-2.22 \pm j5.3394)\) and obtained \( V \) as:

\[
V = \frac{(s + 2.22 + j5.3394)(s + 2.22 - j5.3394)}{(s + 0.0205 + j5.3394)(s + 0.0205 - j5.3394)} = \frac{s^2 + 4.44s + 33.438}{s^2 + 0.053s + 28.51}
\]

weighting functions \( V_1(s) \) and \( V_2(s) \) are respectively:

\[
V_1(s) = 0.3/(s+50)/(s+4), \quad V_2(s) = (s+6)/(s+880)
\]

As the frequency of low-frequency oscillation in power system ranges from 0.7 to 2Hz, we can get:

\[
V(s) = \frac{0.0002s+0.01}{0.5s+1}
\]

an 18 order \( H_\infty \) controller can be obtained by solving Riccati equation [17], in order to have an easily realization, a 3rd order \( H_\infty \) controller with stable zeros and poles was get by order deduction.

\[
K_c = \frac{-624.6s^2 - 8.978s - 17860}{s^3 + 12.46s^2 + 28.62s + 355.6}
\]

4. Simulation Analysis

Figure 6(a) and Figure 6(b) are respectively the rotor speed and stator voltage of EESG with a 0.04 pu fluctuation of mechanical power sustained for 100 milliseconds at 3.0 seconds. What we can see from figure is that the rotor speed and the stator voltage would pass into a stable level quickly with single \( H_\infty \) excitation control, but the amplitude of adjustment process was greater and the oscillation time was longer than which used both \( H_\infty \) excitation control and \( H_\infty \) PSS. This indicated that the double \( H_\infty \) control was more effective to ensure that the system can remain at a stable level.

With a 20% and a 40% input mechanical power reduction, the active power of FSCWT can be seen in Figure 7(a) and Figure 7(b), the stator voltage can also be seen in Figure 7(c) and Figure 7(d) and the reactive power were shown in Figure 7(e) and Figure 7(f) respectively. What we can see from figure is that the stator voltage can remain at a stable in short times as the input mechanical power of generator reduced. What’s more, the stator voltage and reactive power of FSCWT with double \( H_\infty \) control is of a better stability performance than which used only \( H_\infty \) excitation control.
In order to verify the transient performance of VRCS with a $H_\infty$ PSS, a three phase fault was set at 1.0s with 120ms lasted. As we can see in Figure 8(a) and Figure 8(b), the excitation systems responded as quickly as the short occurred to produce reactive power so as to maintain the stator voltage at a stable level. When the fault is cleared, the stator voltage and rotor speed can remain to the original value in a short time as is shown in Figure 8(c) and Figure 8(d). Compared with the single $H_\infty$ excitation control, the double loop control with $H_\infty$ exciter and $H_\infty$ PSS not only realized fast excitation but also achieved transient stability of the proposed system.
5. Conclusion

A stable output reactive power and voltage of FSCWT has an important influence on the stability of power system with large-scale wind farms connected to the grid. The excitation system parameters will change as the operation condition changes and an approximation of the system model causes the uncertainty of the actual controlled plant model, which results in an undesirable control effect of the VRCS in a wide range. The exciter and PSS based on $H_\infty$ control theory collected the uncertainty and nonlinear into design, which is of a good robustness and radically improves the system stability. The VRCS used in this paper enabled the EESG with a fast excitation, which brought negative damping at the same time. What is gratifying is that the VRCS also take the influence of fast excitation on transient stability into account on the bases of voltage feedback control. Simulation results show that the VRCS has a strong robustness, which embodies the effectiveness of double $H_\infty$ control in fast excitation system to realize the voltage and reactive power control.

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References


Figure 8. Transient Response of EESG with $H_\infty$ PSS and $H_\infty$ excitation Controller


