State Estimation for Electrolytic Capacitor with Parameter Fitting

Wang Lei*, Xu Chunmei, Du Huiqing, Meng Linghui
School of Electrical Engineering, Beijing Jiaotong University
100044, Beijing, China, +80-010-51687082
*Corresponding author, e-mail: leiwang@bjtu.edu.cn

Abstract
Metro supply system composed of PWM rectifier, i.e. supply system with bi-directional power flow, contributes much to improve supply quality. However, the electrolytic capacitors, working as DC-side supporting capacitor, are relatively weakest in the whole system, so that state estimation for electrolytic capacitor should be taken into consideration. In view of the difficulty in application of conventional capacitor estimation approach with voltage and current ripples into PWM rectifier, this paper proposes a novel approach, which first searches into capacitor’s aging mechanism, presents approximate relation between capacitance and equivalent serial resistance of electrolytic capacitor through mathematical fitting, and then derives the discrete iterative equations of capacitance and equivalent serial resistance from the analytical model of PWM rectifier. Finally, capacitor state estimation is combined with Miner criterion, which implies digital accumulated damage principle.

Keywords: metro supply, PWM rectifier, electrolytic capacitor, parameter fitting and iteration, accumulated damage

1. Introduction
The application of PWM rectifier into metro supply system, as is shown in Figure 1, offers the capability to feed surplus DC energy from braking vehicles into AC grid, realizing energy conservation, and offers the characteristic of controllable AC-side power factor, noticeably controlled to 1.0 [1-3]. Among all the components in PWM rectifier, DC-side capacitor acts as output filter and voltage stabilizing device, making it key to the integral system. In all existing kinds of capacitors, electrolytic capacitor is chosen mostly due to its merits of less volume, of lower cost and of better access. However, electrolytic capacitor is fragile to external factors (ambient temperature, current ripple), and its life expectancy fluctuates seriously in metro supply application, leading to serious consequent blackout accident and thus making its state estimation important for malfunction prediction during operation.

Figure 1. Metro Supply System with PWM Rectifier
In the fields of damage assessment and state estimation of electrolytic capacitor, existing achievements focus mainly on the recognition of its external representation and internal parameters. Sometimes, the loss of electrolyte is considered for life expectancy prediction [4]; in other cases, the value and variation law of equivalent serial resistance (ESR) is analyzed for damage assessment of electrolytic capacitor [5, 6]; what’s more, theory of linear damage accumulation with mechanical background is applied into aging assessment and damage estimation, with which ESR is recognized by volume of electrolyte [7]. Of all the theories above, the application effect into PWM rectifier is limited, owing to reasons as follows:

1) Considering conventional voltage ripple sampling approach: large capacity PWM rectifier, normally chosen in bi-directional metro supply, operates with heavy load (MW level), with rapidly fluctuating load because of frequent accelerating and decelerating vehicles (kA/min level), therefore, voltage ripple sampled is seriously interfered;

2) Considering conventional current ripple sampling approach: on one hand, with the adoption of extremely-low-inductance DC bus, the DC-side current ripple through capacitor of PWM rectifier is commonly impossible to be sampled with conventional approaches; on another, due to precisely controlled current loop, PWM rectifier shows little AC current ripple with aged capacitors, as is shown in Figure 2.

This paper aims mainly at state estimation of electrolytic capacitor, which is based on damaging mechanism of electrolytic capacitor and electrical parameter analytical model of PWM rectifier, derives fitted discrete iterative electrical parameter of electrolytic capacitor and combined with Miner criterion of damage accumulation.

2. The Aging Mechanism of Electrolytic Capacitor in PWM Rectifier
In Figure 3 is shown AC current of PWM rectifier with no-load, with AC grid voltage of 470V, DC grid voltage of 800V, switching frequency of 2kHz, equipment capacity of 1MW. On DC-side, the supporting capacitor is made up of 72 CD137 Aluminum electrolytic capacitors, with 3 capacitors in serial as a capacitor set, respectively. Under controlling scheme of PWM rectifier in metro supply system [1-3], the reactive current component is controlled to zero in steady state, which makes AC current to be purely active, merely acting to maintain robustness of DC voltage and to convert active AC power into DC power. With neglecting parasitic resistance and power consumption of switching devices, the AC currents under no-load could be considered to implement current ripple through DC capacitors. With no-load, current ripple through capacitors is little, which can be seen from Figure 3.

Figure 4 and Figure 5 show the simulated AC current of PWM rectifier, with no-load and with half-load, respectively. It should be noticed that result in Figure 4 testifies result induced from Figure 2. Here comes two approaches to convert ripple current in Figure 2, Figure 4 and Figure 5 into rating conditions.

2.1. R.M.S Conversion of Current Ripple

With no-load, considering energy balance between AC and DC sides, AC current with no-load should be converted by Equation (1).

\[ I_{rip} = \frac{\sqrt{3} \cdot u_{UV} \cdot I_{U0}}{U_{dc} \cdot Num} \]  

(1)

In Equation (1), \( I_{rip} \) is R.M.S.(Root-Mean-Square value) of current ripple, \( u_{UV} \) is the R.M.S. of AC line voltage, \( I_{U0} \) is R.M.S. of AC line current with no-load, \( U_{dc} \) is the average of DC voltage of PWM rectifier, Num is the quantity of paralleled capacitor-sets.

2.2. Frequency Conversion of Current Ripple

For CD137 Aluminum electrolytic capacitors, rating current ripple tolerance is defined under 120Hz, therefore current ripple [8] under other frequencies should be converted to that under 120Hz, by Equation (2):

\[ I'_{rip} = K_{rip} \cdot I_{rip} \]  

(2)

In Equation (2), \( K_{rip} \) is conversion coefficient. The value of \( K_{rip} \) is given in Table 1. \( I_{rip} \) and \( I'_{rip} \) is current before and after conversion, respectively.
Table 1. Conversion Coefficient for CD137

<table>
<thead>
<tr>
<th>Freq.</th>
<th>50.60</th>
<th>120</th>
<th>300</th>
<th>1K</th>
<th>≥ 10k</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_{\text{rip}} )</td>
<td>0.8</td>
<td>1.0</td>
<td>1.1</td>
<td>1.3</td>
<td>1.4</td>
</tr>
</tbody>
</table>

With switching frequency of 2 kHz, \( K_{\text{rip}} \) is 1.3, then the current ripple converted with no-load and half-load is 0.8819A and 17.89A, respectively. Under permitted current ripple of 18.7A, it is obvious that the aging of DC side capacitor is caused mainly by the DC side current ripple due to the operation of switching devices in PWM rectifier.

During the aging of electrolytic capacitor, the loss of electrolyte leads to damage of electrolytic capacitor, even to malfunction of it. The main reason of electrolyte loss is heat dissipation generated by current ripple through the capacitor and by ESR of the capacitor, given in Equation (3):

\[
\Delta T = I_{\text{rip}}^2 \cdot R_{\text{ESR}} \cdot R_{hL}
\]  

(3)

In Equation (3), \( I_{\text{rip}}^2 \cdot R_{\text{ESR}} \) is thermal power, \( R_{\text{ESR}} \) is ESR of the capacitor, \( R_{hL} \) is thermal resistance. \( R_{hL} \) is defined as is in Equation (4):

\[
R_{hL} = (S \cdot D)^{-1}
\]  

(4)

Where \( S \) is surface area of capacitor, and \( D \) is dissipation factor, which is often \( 1.2 \sim 3.0 \times 10^{-1} \text{W/cm}^2 \cdot ^\circ \text{C} \).

The loss of electrolyte is companied with the increase of ESR and decrease of capacitance. Without the consideration of the interaction between ESR and capacitance, with achievements of Venet and Lahyani [8], variation law of ESR with temperature can be given in Equation (5), where \( k \) is shape coefficient, defined by thermal resistance, \( T \) is temperature of capacitor, \( R_i \) is ESR after time \( t \), \( R_0 \mid _{t = 0} \) is initial ESR under temperature \( T_{\text{base}} \), \( E \) is temperature constant during aging process, often given through experimental results.

\[
R_i = (R_0 \mid _{t = 0})\cdot [1 - kte^{-\left(4700/T_{\text{base}} + 273\right)}]^{-1} \cdot e^{\left(4700/T_{\text{base}} + 273\right)}/E
\]  

(5)

With:

\[
\frac{R_i}{R_0} = \left(\frac{V_i}{V_0}\right)^2
\]  

(6)

It gives:

\[
V_i = V_0 \cdot \sqrt{1 -kte^{-\left(4700/T_{\text{base}} + 273\right)}}
\]  

(7)

3. The Parameter Fitting Approach of Capacitor in PWM Rectifier

1) The parameter fitting during capacitor pre-charging before the operation of PWM rectifier

In metro supply application, DC side capacitor of PWM rectifier should be pre-charged before operation, to eliminate the surge of capacitor current, resulting from the voltage surge across DC side capacitors soon after PWM rectifier operates with control scheme with voltage-loop [3]. During pre-charging of capacitor, parameter fitting is carried out with equivalent model of PWM rectifier, as is shown in Figure 6. It should be noted that with the adoption of extremely-low-inductance DC bus, conventional RLC model of capacitor could be reduced to RC model.
With KCL and KCV equation, it gives:

$$C \cdot \frac{d}{dt} (U_C - i_C \cdot R_{ESR}) = i_C, i_C > 0$$  \hspace{1cm} (8)$$

$$i_C = \sum_{j=A, B, C} i_j \cdot f_{sign}(i_j) + i_{dc}' \cdot i_{dc}' > 0$$  \hspace{1cm} (9)$$

$$U_{dc}' = U_C + L \cdot \frac{d}{dt} i_{dc}', i_{dc}' > 0$$  \hspace{1cm} (10)$$

In Equation (9), it gives:

$$f_{sign}(x) = \begin{cases} 1, & x > 0 \\ 0, & x \leq 0 \end{cases}$$  \hspace{1cm} (11)$$

In Equation (8) through (10), $i_C$ is charging current through capacitor, $i_{A, \text{Charge}}$, $i_{B, \text{Charge}}$, and $i_{C, \text{Charge}}$ are the AC currents during pre-charging process, $i_{dc}'$ is the DC current during pre-charging process, $U_C$ is DC voltage across capacitor during pre-charging process, $R_{ESR}$ is ESR, $C$ is capacitance, $U_{dc}'$ is DC grid voltage from feeder line outside. With $U_{dc}'$ lower than $U_C$, where $i_{dc}' = 0$, Equation (10) is not tenable. After simplification and discretization, Equation (8) gives:

$$C \cdot \frac{U_C(k) - U_C(k-1)}{T_s} - R_{ESR} \cdot C \cdot \frac{i_C(k) - i_C(k-1)}{T_s} - i_C = 0$$  \hspace{1cm} (12)$$

$T_s$ is sampling time interval. After discretization, Equation (9) and Equation (10) turn into:

$$i_C(k) = \sum_{j=A, B, C} i_j(k) \cdot f_{sign}(i_j(k)) + \frac{i_{dc}'(k) - i_{dc}'(k-1)}{T_s}$$  \hspace{1cm} (13)$$

$$U_C = \frac{U_{dc}(k) - U_{dc}(k-1)}{T_s} + L \cdot \frac{i_{dc}'(k) - i_{dc}'(k-1)}{T_s}$$  \hspace{1cm} (14)$$

With AC and DC grid current transducers, $i_{A, \text{Charge}}$, $i_{B, \text{Charge}}$, $i_{C, \text{Charge}}$ and $i_{dc}'$ are detectable, therefore $i_C$ is observable; With DC grid voltage transducer, $U_{dc}'$ is detectable.
therefore $U_C$ is observable. Therefore, Equation (12) forms a two-element-constraint equation with no solution. However, with Equation (5) and Equation (7), another two-element-constraint equation could be derived through mathematical fitting approach, approximately expressed in Equation (15), where $\lambda_1$ and $\lambda_2$ are fitting coefficients.

$$\frac{C}{C_0} = \lambda_1 \cdot \left[ \frac{(1 + \lambda_2) R_{ESR} e^{\frac{U_m}{|\theta|}}}{(R_{base} + \lambda_2) e^{\frac{U_m}{|\theta|}}} \right]^{\frac{1}{T}} \tag{15}$$

Combining Equation (12) and Equation (15), it sees Equation (16), where $i_C$ is derived with Equation (13), and $U_C$ is derived with Equation (14). Equation (13), (14), (16) form Discrete Iterative Equation Set of C and $R_{ESR}$ of PWM rectifier during pre-charging stage.

$$\begin{aligned}
C(k) &= C_0 \left\{ \frac{i_C(k) + 4 \frac{U_C(1) - U_C(k-1)}{T} \frac{\dot{i}_C(k) - \dot{i}_C(k-1)}{T} \frac{2 \lambda_1}{1+\lambda_2} C_0^2 (R_{base} e^{\frac{U_m}{|\theta|}}) e^{\frac{U_m}{|\theta|}}}{1+\lambda_2} \right\} \\
R_{ESR}(k) &= \left\{ \frac{i_C(k) + 4 \frac{U_C(1) - U_C(k-1)}{T} \frac{\dot{i}_C(k) - \dot{i}_C(k-1)}{T} \frac{2 \lambda_1}{1+\lambda_2} C_0^2 (R_{base} e^{\frac{U_m}{|\theta|}}) e^{\frac{U_m}{|\theta|}}}{1+\lambda_2} \right\}
\end{aligned} \tag{16}$$

During actual operation, accurate mathematical fitting is hardly possible, which is affected by accuracy of transducers and external interferences. It is suggested to calculate optimized fitting value of C and $R_{ESR}$ through weighted average of continuous M points, as is shown in Equation (17), where $N_i$ is the $N_i$-th weighted average.

$$\begin{aligned}
C_{N_i} &= \sum_{j=1}^{M} \left( \frac{i_C(j) \cdot C(j)}{M} \right) \\
R_{ESR,N_i}^{\frac{1}{2}} &= \sum_{j=1}^{M} \left( \frac{i_C(j) \cdot R_{ESR}^{\frac{1}{2}}(j)}{M} \right) \tag{17}
\end{aligned}$$

2) The parameter fitting during rectifying operation of PWM rectifier.
In Figure 7 is shown the equivalent model of rectifying PWM rectifier. With KCL and KCV equation, it gives:

$$i_C(k) = \sum_{j=A}^{C} \frac{i_{CJ}(k) \cdot S_j(k) - i_{C_j}(k)}{T} \tag{18}$$

$$U_C(k) = U_{sw}(k) + L \cdot \frac{i_{sw}(k) - i_{sw}(k-1)}{T} \tag{19}$$

Where $S_j$ is the switching function of phase $j$, which is valued to be 1 when upper IGBT is triggered, to be 0 when lower IGBT is triggered It is obvious that $\int_{sign}$ is a special case of $S_j$.  

---

**TEKOMNIKA** Vol. 11, No. 8, August 2013: 4461 – 4469
during natural rectifying (i.e. pre-charging stage). It should be noted that the observability of $R_L$ in Figure 8 depends on $U_{dc}$ and $i_{dc}$, therefore, $R_L$ is not adopted during iteration.

In Equation (16), $i_c$ is derived with Equation (18), and $U_C$ is derived with Equation (19). Equation (16), (18), (19) form Discrete Iterative Equation Set of $C$ and $R_{ESR}$ of PWM rectifier during rectifying stage.

In Figure 8 is shown the equivalent model of inverting PWM rectifier. With KCL and KCV equation, it gives:

\[ i_c(k) = \sum_{j=A}^{N} i_j(k) \cdot S_j(k) + i_{de}(k) \]  \hspace{1cm} (21)

\[ U_C(k) = U_{dc}(k) - L \cdot \frac{i_{de}(k) - i_{de}(k-1)}{T_s} \]  \hspace{1cm} (22)

In Equation (16), $i_c$ is derived with Equation (21), and $U_C$ is derived with Equation (22). Equation (16), (21), (22) form Discrete Iterative Equation Set of $C$ and $R_{ESR}$ of PWM rectifier during inverting stage.
4. The State Estimation of Capacitor based on Fitted Parameters and Miner Criterion

According to theory of damage accumulation principle, aging results from accumulated damage, which is expressed by Miner M.A. in 1945, dealing mainly with linear accumulation of damage caused by periodic stress [9, 10], known as Miner criterion. However, during its direct application into state estimation of electrolytic capacitor in PWM rectifier, Miner criterion is found in dilemma. On the one hand, it is hard for Miner criterion to deal with non-linear problems; on another, the mechanism and inducement of electric damage accumulation is more complex and hard to grasp through experiments and simulations. Therefore, this paper corrects Miner criterion with fitted parameters, and then applied such corrected criterion into state estimation of electrolytic capacitor.

Given that $D$ symbolizes accumulated damage, the damage accumulated after the $N_i$-th iteration, symbolized by $\Delta D_{N_i}$ is given as:

$$
\Delta D_{N_i} = \mu_i \left| \frac{\Delta R_{ESR}}{R_0} \right|_{N_i} + \mu_2 \left| \frac{\Delta C}{C_0} \right|_{N_i}
$$

Figure 9. Evaluated and Measured Capacitance of CD137 through Time

Where $\left| \Delta R_{ESR} \right|_{N_i}$ and $\left| \Delta C \right|_{N_i}$ is ESR and capacitance difference, respectively, and $R_0$ is initial ESR, $C_0$ is initial capacitance. $\mu_i$ and $\mu_2$ are weighted coefficients, where it gives:

$$
\mu_i + \mu_2 = 1
$$

According to Miner criterion, overall damage accumulated $D$ is then given as:

$$
D = \sum_{N_i} \Delta D_{N_i}
$$

For CD137, $D \geq 0.3$ normally implies an aged capacitor. It should be noted that due to the 3-serialed capacitor structure, a capacitor set should be changed as integrity. Figure 9 shows the change of capacitance of CD137 through time. Actual capacitance is measured by electric bridge, under frequency of 2 kHz.

References


