On Electricity Spot Price Properties by t-innovation GARCH Model

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Abstract
The modeling of heteroskedasticities and kurtosises of electricity prices is crucial to forecast the future distribution of electricity prices, to understand the behavior of derivatives pricing and to quantify the risk in electricity markets. A GARCH model with t-innovations, which is solved by maximum likelihood estimation, is proposed. The model can explicitly address the relationship with system loads, seasonalities, heteroskedasticities, and kurtosises of electricity prices. The empirical analysis based on the historical data of the PJM electricity market shows that the system load squares have a significant effect on the average daily electricity prices, there exist volatility clustering and weekly, semi-monthly, monthly, bimonthly, quarterly and semi-annual periods, and the variances and kurtosises of electricity prices manifest clearly time-varying characteristics. The model holds parsimonious scale of estimated parameters, less computational costs, easy to select the orders and high practical application value.

Keywords: electricity price, t-innovation GARCH model, heteroskedasticity, kurtosis

1. Introduction
Deregulated electricity markets continue to confound international financial economists. With the rapid growth of derivative securities, the modeling and management of price risk have become important topics both in power industry and in academic community. The pricing in deregulated electricity markets is based on marginal generation costs of the last power plant that is required to cover the demand. Electricity prices are influenced not only by the objective factors such as weather condition, system load, production/operation costs, available generation capacity and transmission network congestion, but also by the subjective factors such as market trading rules, participants’ bidding strategies and their psychological reactions to price changes [1]. All of these factors make an accurate price forecasting become a complex issue. The price forecasting models can be classified in long- and short-term ones. The long-term price forecasting can be achieved by simulating the competitive rules, mainly including game theory and simulation models. With the statistical analysis for a large number of historical data, the mathematical model to reflect price continuous changes can be established and short-term price forecasting can be obtained, mainly including time series analysis (TSA), artificial neural network (ANN) and hybrid prediction approaches [2].

Because of the adaptive ability to uncertain fuzzy systems, ANN has been widely used for nonlinear multivariate problems [3]. In [4, 5], the electricity spot prices in California, Iran and Spain electricity markets were predicted by using three layer feedforward ANNs based on the training methods of back propagation and/or Levenberg-Marquardt algorithms. In [6, 7], the performances of Gaussian radial basis function and traditional ANNs were compared, indicting that Gaussian radial basis function ANN is more suitable for short-term electricity price forecasts because of its faster learning speed and better approximation capability. In [8-12], short-term electricity price forecasting approaches using combination of fuzzy logic, Kalman filter, support vector machine and ANN were proposed respectively, the results show that significantly improved prediction performance can be achieved by using the hybrid forecasting methods. However, the lower learning speed and parameters not to be easily adjusted have impeded ANNs’ application in practice.

TSA has the advantage of analytical tractability. The continuous changes of time series can be accurately reflected with a relatively small amount of historical data. Autoregressive
moving average (ARMA) and autoregressive moving average with exogenous variables (ARMAX) models are two commonly used methods. In [13], an ARMAX model with load as an exogenous explanatory variable was used to predict the next 24-hour spot prices in the Pennsylvania-New Jersey-Maryland (PJM) electricity markets. Considering the non-constant means and variances for most electricity price series, a spot price forecasting method based on autoregressive integrated moving average (ARIMA) model was proposed in [14]. Cuaresma et al. [15] have noted that each hour of days is also an important factor to influence electricity spot prices, and an ARMA-based period-decoupled price forecasting model was proposed, showing greatly improved prediction accuracy for the price spikes. The electricity price forecasting methods using combination of ARIMA with predicted errors improvement and/or wavelet transfer function were respectively proposed in [16] and [17]. A period-decoupled electricity price forecasting method based on transfer function model, taking the effect of load on electricity price and the non-stationary properties of price series into account, was presented in [18], and further improved the prediction accuracy. However, with the assumption that the electricity price series distribution is normal with constant variance, these models in [13-18] can not effectively deal with the heteroscedasticity. Moreover, the more estimated parameters and computational costs have also impeded their heavy use in practice.

Up to now electrical energy cannot be stored economically and therefore demand for electrical energy has an untempered effect on electricity prices. In particular, electricity price exhibits considerably richer structure than load curve and has the following characteristics: mean reversion, multiple seasonalities, stochastic volatility and extreme behavior with fast-reverting spikes. Therefore understanding the pricing dynamics of electricity is of vital importance for all market players’ survival under deregulated environment. In this paper, a multicycle GARCH model with t-innovations (hereinafter refer to as “t-innovation GARCH”) is proposed, in which the heteroscedasticities, kurtosises/fat-tails and multiple seasonalities of electricity prices are described by time-varying variances, time-varying degrees of freedom and sinusoidal functions. The proposed model holds the advantages of less computational costs and parsimonious scale of estimated parameters. Moreover, the time trend, multiple seasonalities, conditional heteroscedasticities, conditional kurtosises and relationship among loads and spot prices can be explicitly taken into account. The empirical analysis based on the historical data of the PJM electricity market shows that system load squares have a significant effect on the average daily electricity prices, there exist volatility clustering and weekly, semi-monthly, monthly, bimonthly, quarterly and semi-annual periods, and the variances and kurtosises of electricity prices manifest clearly time-varying characteristics.

2. Model and Solution Method
2.1. T-innovation GARCH Model

Electricity price forecasting model can be viewed as a multi-input single-output system, in which the output variable is the electricity price and the input variables are the impact factors of electricity price such as fuel prices, seasonality, climate, load and bidding strategies of market participants. Moreover, in this paper, the system will be delineated by a t-innovation GARCH model. Considering the market clearing prices and system loads are publicly available in each market all over the world. Therefore, the system loads at periods $t$, $t-1$, ... and electricity prices at periods $t-1$, $t-2$, ... are selected as the input variables. Assuming that $p_t$, $d^2_t$, $e_t$ and $z_t$ denote the spot price, system load square, residual and standardized residual at period $t$, respectively, then the t-innovation GARCH model can be formulated as follows:

$$p_t = f(t) + \gamma(B)d^2_t + \phi(B)p_t + \kappa(B)e_t$$

$$e_t = \sqrt{h_t}z_t, \quad I_t \sim D(0, h_t), z_t | I_{t-1} \sim D(0, 1)$$

$$h_t = \beta_0 + \sum_{i=1}^{A} \beta_1 h_{t-i} + \sum_{i=1}^{S} \beta_2 e^2_{t-i}$$
On Electricity Spot Price Properties by t-innovation GARCH Model (Wang Jin)

\[ f(t) = a_0 + a_1 t + a_2 d_{wd} + \sum_{i=1}^{m} a_{i1} \sin \left( \frac{2\pi t}{365} + a_{i2} \right) \]  
(1c)

\[ \gamma(B) = \gamma_0 + \gamma_1 B + \gamma_2 B^2 + \ldots + \gamma_u B^u \]  
(1d)

\[ \varphi(B) = \varphi_1 B + \varphi_2 B^2 + \varphi_3 B^3 + \ldots + \varphi_p B^p \]  
(1e)

\[ \kappa(B) = 1 + \kappa_1 B + \kappa_2 B^2 + \ldots + \kappa_q B^q \]  
(1f)

where \( B \) is the backshift operator, \( m \) denotes the number of changing cycles of electricity price series per year, \( h_t \) denotes conditional variance of \( e_t \), \( I_{t-1} \) denotes an available information set till period \( t-1 \), \( u \) and \( p \) and \( q \) represent respectively the lagged orders of \( d_t^2 \), \( p_t \) and \( e_t \) in the mean equation, \( r_s \) and \( s_h \) denote the lagged orders of \( h_t \) and \( e_t^2 \) in the conditional variance equation, \( f(t) \) denotes the time trend and seasonal changes, \( d_{wd} \) is a dummy variable that takes a value of 1 if the observation is in weekday and zero otherwise, \( \alpha = (\alpha_0, \alpha_1, \alpha_2, \ldots, \alpha_m) \), \( \gamma = (\gamma_L, \gamma_U) \), \( \varphi = (\varphi_1, \varphi_2, \ldots, \varphi_p) \), \( \kappa = (\kappa_1, \kappa_2, \ldots, \kappa_q) \) and \( \theta = (\theta_1, \theta_2, \ldots, \theta_u) \) and \( \beta = (\beta_0, \beta_1, \ldots, \beta_f, \beta_s, \beta_h) \) are the estimated parameters.

2.2. Parameters Calibration

Before parameters calibration, assumption on the distribution of innovations needs to be made. Assuming that the probability density function (PDF) for the standardized residual \( z_t \), a white noise process with zero mean and constant variance equal to 1, is consistent with a student-t distribution, then the conditional PDF of \( e_t \) can be expressed as [19]:

\[
g(e_t | I_{t-1}) = \frac{1}{\sqrt{h_t}} \frac{1}{\Gamma(\eta_t/2)} \left( \frac{1}{(\eta_t - 2)^{\eta_t/2}} \right)^{1/2} \left( 1 + \frac{e_t^2}{(\eta_t - 2)} \right)^{-\eta_t/2} \]  
(2)

\[ \eta_t = \eta_L + \frac{U_{\eta} - L_{\eta}}{1 + \exp(-\lambda_{\eta})} \]  
(2a)

\[ \lambda_{\eta} = \delta_0 + \sum_{i=1}^{r_{\eta}} \delta_i e_{t-i} + \sum_{i=1}^{s_h} \delta_{2i} e_{t-i}^2 + \sum_{i=1}^{v_h} \delta_{3i} e_{t-i}^3 \]  
(2b)

where \( \Gamma \) is a Gamma function, \( \eta_t \) is the conditional degree of freedom corresponding to the distribution of \( z_t \), \( U_{\eta} \) and \( L_{\eta} \) denote the upper and lower limits of \( \eta_t \), \( r_{\eta} \), \( s_h \) and \( v_h \) are respectively the lagged orders of the innovations, residual squares and degrees of freedom in the conditional freedom degree equation, \( \delta = (\delta_0, \delta_1, \ldots, \delta_{r_{\eta}}, \delta_{2i}, \delta_{3i}) \) are the parameters to be estimated.

Let \( \xi = (\alpha, \rho, \theta, \gamma, \beta, \delta) \), then the log-likelihood function for all observations corresponding to \( e_t \) is given by:
L(ξ) = \sum_{i=1}^{n} l_i(ξ) = \sum_{i=1}^{n} \left[ -\frac{1}{2} \ln(2\pi) - \ln(n_i) + \frac{1}{2} \ln(n_i) + \frac{1}{2} \ln(\Gamma(n_i/2)) - \ln(\Gamma(n_i/2)) - \frac{n_i + 1}{2} \ln(n_i) - \frac{z_i^2}{n_i(n_i - 2)} \right] \tag{3}

where \( l_i(ξ) = \ln g(ξ_i | I_i, 1) \) is the log-likelihood function for one observation at period \( t \). By maximizing the \( L(ξ) \), the estimated values of parameters \( ξ, \hat{ξ} \), can be obtained. It is important to note that the log-likelihood function \( L(ξ) \) is highly nonlinear. Therefore the starting values of parameters \( ξ \) must be selected with care. In order to improve the accuracy of estimation, a successive approximation method, namely using the parameters estimated from simpler models as starting values for more complex one, is used in this paper.

2.3. Model Checking

Under large sample, the distribution of the maximum likelihood estimation \( \hat{ξ} \) can be approximated by normal distribution:

\[ \hat{ξ} \sim N(ξ_0, H(ξ_0)^{-1}) \tag{4} \]

where \( ξ_0 \) is the truth values of the estimated parameters \( ξ \), \( H \) is a Hessian matrix. A consistent estimate of \( H(ξ_0) \) can be obtained by evaluating \( \frac{\partial^2 L(ξ)}{\partial ξ \partial ξ^T} \) at \( \hat{ξ} \). After calculating the variance of \( \hat{ξ} \), the significance of estimated parameters can be tested using t-statistics.

The Nyblom-statistic, holding the advantage that its asymptotic distribution only depends on the number of estimated parameters, is used to test the constancy of the proposed model [20]. The Nyblom-statistic \( W_N \) can be expressed as:

\[ W_N = \frac{1}{n} \sum_{i=1}^{n} S_i^T V^{-1} S_i \tag{5} \]

Where

\[ S_i = \hat{ξ}' \left[ \frac{\partial l(ξ)}{\partial ξ} \right]_{\hat{ξ}} \]
\[ V = \left[ \frac{\partial^2 L(ξ)}{\partial ξ \partial ξ^T} \right]_{\hat{ξ}} \]

The Nyblom-statistic can be also used to test the constancy of a single estimated parameter. The Nyblom-statistic \( W_{N,k} \) corresponding to the \( k \)th estimated parameter is given by:

\[ W_{N,k} = \frac{1}{n} \sum_{i=1}^{n} S_{ik}^2 \]
\[ V_{kk} \]

where \( S_{ik} \) is the \( k \)th element of \( S_i \), \( V_{kk} \) is the \( k \)th diagonal element of \( V \).

Cramer-Von Mises statistic can be used to test if the distribution of innovations is consistent with student-t distribution. Let \( F_n(z) \) denote the cumulative distribution function (CDF) of student-t, \( F(z) \) denote the CDF of the innovations. Then Cramer-Von Mises statistic can be formulated as:
2.4. Forecasting Accuracy Evaluation

Generally speaking, the electricity price forecasting model is one with time-varying parameters, and its parameters should be modified by the new available data in order to improve the forecasting accuracy. In this paper, the mean absolute percentage error (MAPE) is used to evaluate the forecasting accuracy. It can be calculated as following:

$$\text{MAPE} = \frac{1}{n} \sum_{t=1}^{n} \left| \frac{\hat{p}_t - p_t}{p_t} \right|$$

(8)

where $\hat{p}_t$ and $p_t$ respectively refer to the forecasted and actual realized electricity prices at period $t$, $n$ is the period number to be forecasted.

3. Empirical Results

The PJM electricity market is organized as a day-ahead one. Participants submit their buying and selling bid curves for each of the next 24 hours. Then the market operator aggregates bids for each hour and determines market clearing prices and volumes for each hour of the following day. In this paper, a total of 1197 observations of average daily electricity spot prices in dollars per megawatt hour ($/MWh) and average daily loads in gigawatt (Gw) are employed to validate the performance of the t-innovation GARCH model. The sample period begins on 1 Jun., 2007 and ends on 9 Sep., 2010. Table 1 presents some descriptive statistics for the average daily electricity spot price and system load series. It can be seen from Table 1 that electricity prices and system loads are quite volatile, highly non-normal, clearly skewed rightward, and with median well below the mean. In fact the nulls of normality of electricity price and system load series are rejected with the Jarque-Bera test. This is typical of electricity spot prices in a competitive market.

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Price($/MWh)</th>
<th>Load(GW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>53.52041</td>
<td>81.19221</td>
</tr>
<tr>
<td>Median</td>
<td>49.97068</td>
<td>79.89221</td>
</tr>
<tr>
<td>Maximum</td>
<td>189.6557</td>
<td>115.7839</td>
</tr>
<tr>
<td>Minimum</td>
<td>24.87494</td>
<td>58.34586</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>20.20158</td>
<td>10.50560</td>
</tr>
<tr>
<td>Skewness</td>
<td>1.420081</td>
<td>0.375318</td>
</tr>
<tr>
<td>Jarque-Bera(p-value)</td>
<td>1046.748</td>
<td>36.78506</td>
</tr>
</tbody>
</table>

Analyzing the correlation coefficient, partial correlation coefficient and time trend chart of the sample data, the values of $m, p, q, u, r_h, s_h, r_b, s_b, v_b$ in the t-innovation GARCH model can be identified. In our situation, they are equal to 52, 1, 3, 1, 1, 1, 1, 1, 1 respectively. Table 2 shows the results of maximum likelihood estimation. Investigating the estimated results in Table 2, the following conclusions can be derived:

1) The MAPE 6.005% of our t-innovation GARCH model is approximately equal to that of the models of references [13-18], but the number of estimated parameters is only 27, which is less than the models of references [13-18]. To some extent this reduces the model complexity, improves the computing speed and strengthens the practical application ability of the model.
2) The t-statistics for $a_{t_i}, a_{t_j}, \hat{\alpha}$ (2, 4, 6, 12, 24, 52) are significant at the 95% confidence level. This shows that there exists weekly, semi-monthly, monthly, bimonthly, quarterly and semi-annual cycles in the sample periods. The amplitude of the peak for the weekly cycle is larger than others.

3) The t-statistic for $a_2$ is significant at the 99% confidence level. This shows that the impacts of system loads on the average daily electricity spot prices for weekday and weekend are more different.

<table>
<thead>
<tr>
<th>parameters</th>
<th>estimated</th>
<th>Std. Err.</th>
<th>t statistics</th>
<th>p-value</th>
<th>Nyblom statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>0.7896</td>
<td>0.3436</td>
<td>2.298</td>
<td>0.0215</td>
<td>0.1215</td>
</tr>
<tr>
<td>$a_2$</td>
<td>-1.8039</td>
<td>0.3178</td>
<td>-5.677</td>
<td>0.0000</td>
<td>0.1992</td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>0.0063</td>
<td>0.0002</td>
<td>36.738</td>
<td>0.0000</td>
<td>0.1834</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>-0.0061</td>
<td>0.0002</td>
<td>-35.857</td>
<td>0.0000</td>
<td>0.1200</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>0.9820</td>
<td>0.0035</td>
<td>281.29</td>
<td>0.0000</td>
<td>0.1537</td>
</tr>
<tr>
<td>$a_{12}$</td>
<td>0.3155</td>
<td>0.0830</td>
<td>3.802</td>
<td>0.0001</td>
<td>0.0369</td>
</tr>
<tr>
<td>$a_{22}$</td>
<td>277.59</td>
<td>4.7365</td>
<td>58.607</td>
<td>0.0000</td>
<td>0.0257</td>
</tr>
<tr>
<td>$a_{14}$</td>
<td>-0.1373</td>
<td>0.0408</td>
<td>-3.636</td>
<td>0.0008</td>
<td>0.0929</td>
</tr>
<tr>
<td>$a_{24}$</td>
<td>-323.35</td>
<td>4.6496</td>
<td>-69.544</td>
<td>0.0000</td>
<td>0.0284</td>
</tr>
<tr>
<td>$a_{16}$</td>
<td>0.0886</td>
<td>0.0417</td>
<td>2.125</td>
<td>0.0336</td>
<td>0.2719</td>
</tr>
<tr>
<td>$a_{26}$</td>
<td>-18.198</td>
<td>4.8807</td>
<td>-3.728</td>
<td>0.0002</td>
<td>0.0809</td>
</tr>
<tr>
<td>$a_{112}$</td>
<td>-0.0996</td>
<td>0.0510</td>
<td>-1.950</td>
<td>0.0511</td>
<td>0.0936</td>
</tr>
<tr>
<td>$a_{212}$</td>
<td>17.334</td>
<td>2.5088</td>
<td>6.909</td>
<td>0.0000</td>
<td>0.2947</td>
</tr>
<tr>
<td>$a_{124}$</td>
<td>-0.1744</td>
<td>0.0797</td>
<td>-2.190</td>
<td>0.0286</td>
<td>0.1081</td>
</tr>
<tr>
<td>$a_{224}$</td>
<td>-81.445</td>
<td>1.0991</td>
<td>-74.102</td>
<td>0.0000</td>
<td>0.0740</td>
</tr>
<tr>
<td>$a_{152}$</td>
<td>0.9759</td>
<td>0.1489</td>
<td>6.554</td>
<td>0.0000</td>
<td>0.0784</td>
</tr>
<tr>
<td>$a_{252}$</td>
<td>-94.757</td>
<td>0.1964</td>
<td>-482.38</td>
<td>0.0000</td>
<td>0.3484</td>
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<tr>
<td>$\kappa_1$</td>
<td>-0.2657</td>
<td>0.0293</td>
<td>-9.081</td>
<td>0.0000</td>
<td>0.3268</td>
</tr>
<tr>
<td>$\kappa_2$</td>
<td>-0.2383</td>
<td>0.0312</td>
<td>-7.641</td>
<td>0.0000</td>
<td>0.2640</td>
</tr>
<tr>
<td>$\kappa_3$</td>
<td>-0.1781</td>
<td>0.0275</td>
<td>-6.497</td>
<td>0.0000</td>
<td>0.4698</td>
</tr>
<tr>
<td>$b_0$</td>
<td>0.2532</td>
<td>0.1118</td>
<td>2.264</td>
<td>0.0236</td>
<td>0.1864</td>
</tr>
<tr>
<td>$b_{11}$</td>
<td>0.8228</td>
<td>0.0270</td>
<td>30.436</td>
<td>0.0000</td>
<td>0.5669</td>
</tr>
<tr>
<td>$b_{21}$</td>
<td>0.2140</td>
<td>0.0388</td>
<td>5.515</td>
<td>0.0000</td>
<td>0.5727</td>
</tr>
<tr>
<td>$d_0$</td>
<td>-1.2305</td>
<td>0.4080</td>
<td>-3.016</td>
<td>0.0026</td>
<td>0.1779</td>
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<tr>
<td>$d_{11}$</td>
<td>0.1850</td>
<td>0.0465</td>
<td>3.981</td>
<td>0.0001</td>
<td>0.0736</td>
</tr>
<tr>
<td>$d_{21}$</td>
<td>-0.0046</td>
<td>0.0013</td>
<td>-3.605</td>
<td>0.0003</td>
<td>0.0756</td>
</tr>
<tr>
<td>$d_{31}$</td>
<td>0.4441</td>
<td>0.1592</td>
<td>2.790</td>
<td>0.0053</td>
<td>0.2525</td>
</tr>
</tbody>
</table>

Maximum Log-likelihood -3319.20
Cramer-Von Mises Statistics 0.259435

4) The t-statistics for $\gamma_0$ and $\gamma_1$ are significant at the 99% confidence level, indicating that $d_t^2$ has a marked impact on the average daily electricity spot prices. However, when $d_t^2$ is incorporated in the mean equation, the sign of $a_2$ changes from positive to negative. This shows that there exists some substitution effect between $d_{wkdd}$ and $d_t^2$.

5) The t-statistic for $b_{11}$ in the conditional variance equation is positive and significant at the 99% confidence interval, indicating that the volatility of electricity prices is strongly
persistent. The impacts of prior period volatility on current period volatility show a gradually weakening trend, because the value of $b_{11}$ is less than 1. As shown in Figure 1, there clearly exists volatility clustering, demonstrating that high conditional variance is followed by high conditional variance.

![Figure 1. Conditional heteroskedasticities of price series](image)

6) The t-statistic of $b_{21}$ in the conditional variance equation is positive and significant at the 99% confidence interval, indicating that the volatility of electricity prices will be strengthened by external shocks. The sum of $h_{11}$ and $b_{21}$ is close to 1, indicating that there may exist an integrated GARCH effect for the average daily electricity prices. The impacts of volatility of prior periods and external shocks on current period volatility have longer persistence.

7) The t-statistics of $d_{11}$ and $d_{21}$ are significant at the 99% confidence interval, indicating that the conditional degrees of freedom of student-t distribution manifest obviously time-varying features. The innovations and their squares have a significant impact on the conditional degrees of freedom.

8) The t-statistic of $d_{31}$ is significant at the 99% confidence interval, indicating that the conditional freedom degrees are more strongly persistent. The impacts of the freedom degrees of prior periods on the one of current period show a gradually weakening trend. There exists obvious volatility clustering appearance, but the influencing strength is weaker than the conditional variance ($d_{31} < b_{11}$). It can be seen from Figure 2, the conditional freedom degrees are mainly between 2 and 8, indicating that there exist obvious kurtosises and fat-tails in the electricity price series.

9) The Cramer-Von Mises statistic 0.259 is less than the critical limit 0.333 at the 99% confidence level, indicating that the student-t distribution is fully consistent with the actual distribution of innovations, as shown in Figure 3.

10) The Nyblom-statistics of all estimated parameters are less than the critical limit at the 99% confidence level, but the one for the whole model, 9.4704, is slightly larger than the critical limit at the 99% confidence level, indicating that there exists some instability for the above model. When removed $d_{wkd}$ or replace $d_i^2$ with $d_i$ from the mean equation, the Nyblom-statistic of the whole model will be less than the critical limit at the 99% confidence level, but the MAPE will increase about 0.3%. One possible explanation is that only using $d_{wkd}$ and $d_i^2$ is not accurately described the relationship of loads and electricity prices. So how to more reasonably
address the influencing factor of system loads will be the main problem to be solved in future research work.

![Graph](image)

**Figure 2. Probability density of degrees of freedom**

![Graph](image)

**Figure 3. Probability density of the innovations**

### 4. Conclusion

With comprehensive consideration of the changing rules and influencing factors of the electricity spot prices, a GARCH model with t-innovations is proposed, in which the multiple seasonalities, heteroskedasticities and kurtoses/fat-tails of electricity prices are described by sinusoidal functions, time-varying variances and time-varying degrees of freedom respectively. The proposed model holds the advantages of less computational costs and parsimonious scale of estimated parameters. Moreover, the heteroscedasticities, time trend, kurtoses/fat-tails, multiple seasonalities and relationship among loads and spot prices can be fully taken into account. The empirical analysis based on the historical data of the PJM electricity market from 1 Jun., 2007 to 9 Sep., 2010 shows that the system load squares have a significant effect on the average daily electricity spot prices, there exist volatility clustering and weekly, semi-monthly, monthly, bimonthly, quarterly and semi-annual periods, and the variances and kurtoses of electricity prices manifest clearly time-varying features. How to more reasonably address the
relationship among loads and spot prices and further improve the goodness of fit of the proposed model is a relevant subject to future research work.

References