Ship Dynamic Positioning Systems based on Fuzzy Predictive Control

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Abstract
Dynamic Positioning Systems (DPS) is a technique of automatically maintaining the position of vessels within the specified limits by controlling thrusters. This paper introduces a fuzzy predictive control to position vessels. Firstly, the mathematical model of vessel is created and simplified. Then, we use feed-forward compensation to decouple the system. The T-S model is identified by Fuzzy C-Means clustering algorithm and Least Squares method. After that, we use Generalized Predictive Control (GPC) to control the ships in three degrees of freedom (DOF)-surge, sway and yaw. The simulation results show that the fuzzy predictive control can orient the vessels effectively.

Keywords: dynamic positioning systems (DPS), T-S model, fuzzy C-Means (FCM) Clustering, least squares, generalized predictive control (GPC)

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1. Introduction
The dynamic positioning system (DPS) is a closed loop control system which resists against the disturbance by wind, wave, and current with the help of thrusters on the vessels. Further explains which means the vessels using precision advanced instruments instead of traditional mooring positioning system to measure the displacement and direction change due to the disturbances outside. At the same time, through the automatic control system such as computers to process and calculate information, meanwhile, to control several thrusters in different directions, which can make the vessels back to the original positions. Besides that, if the anchor point with a certain speed moves along the desired trajectory, the track-keeping can be also achieved. Compared with ordinary track-keeping relying on rudders, the dynamic positioning system is able to move in the direction of the breadth, which offers the possibility of high-precision track-keeping [1-3]. Above all, the dynamic positioning technology is becoming more and more important with the development of exploitation and exploration of the oceans.

The dynamic positioning system mainly consists of three sub-systems: position measuring system, control system and thrust system [4]. Among them, the progress of control system represents the development of dynamic positioning [5]. The controller uses PID control method in early stage, but now, the most widely practical application is based on Kalman filtering LQG control method [6]. Then with the progress of computer technology, the intelligent control method is becoming popular due to the improvement of calculation and processing.

The ship dynamic positioning system has some characteristics such as nonlinear, large delay and strong coupling, which is difficult to structure the accurate mathematical model and very complicated to model it. Therefore, this paper finds a way to simplify the model and proposes fuzzy predictive control method to control it.

The paper is organized as follows. In Section 2, we build the mathematical model of DP vessels. In Section 3, the definition of decoupling is introduced and feed-forward compensation decoupling method is used in this paper. Section 4 is about creating the system of T-S model by Fuzzy C-Means clustering algorithm and Least Squares method. Then the model is transferred into state-space model. Section 5 introduces Generalized Predictive Control method. We use it based on the model built in Section 4 to position the vessels accurately in three degrees of freedom (DOF)-surge, sway and yaw. Section 6 is the simulation results we tested on our model. Finally, conclusions are made in Section 7.
2. Mathematical Model of Vessels

2.1. The Reference Frames

For the vessels, the main purpose of dynamic positioning system is to control the vessels stay in the original position through three DOF—surge, sway and yaw. To build the system, the reference frames used are illustrated in Figure 1.

The Earth-fixed reference frame is denoted as $O - X_EY_EZ_E$. The origin can be randomly in principle. Then the position and direction of vessel can be measured according to this point.

The Body-fixed reference frame is denoted as $O_b - X_bY_bZ_b$. We usually select the center of gravity of vessel as the origin.

The parallel reference frame is denoted as $O - XYZ$. The origin is the same as the one in earth-fixed frame. All the coordinates are parallel to them in body-fixed frame. Because of the relationship between parallel frame and body-fixed frame, the speed and acceleration are equal and have the same direction in two frames. Therefore, the motion equations deduced in the body-fixed frame can be used in the parallel frame [7].

![Figure 1. The Reference Frames [7]](image)

The heading angle is $\psi$, and then the transformational relation between the parallel reference frame and the earth-fixed frame is:

$$
\begin{bmatrix}
X \\
Y
\end{bmatrix}
= 
\begin{bmatrix}
\cos \psi & \sin \psi \\
-\sin \psi & \cos \psi
\end{bmatrix}
\begin{bmatrix}
X_E \\
Y_E
\end{bmatrix} 
$$

(1)

2.2 Kinematic Model of Vessels

The vectors defining the earth-fixed vessel position and orientation, and the body-fixed translation and rotation velocities can be given. The position vector and the velocity vector are reduced to $\eta = [x, y, \psi]^T$ and $\nu = [u, v, \dot{\psi}]^T$, respectively, if only surge, sway and yaw are considered, such that the kinematics is given as:

$$
\dot{\eta} = R(\psi) \nu
$$

(2)

Where the rotation matrix $R(\psi)$ is:

$$
R(\psi) = 
\begin{bmatrix}
\cos \psi & -\sin \psi & 0 \\
\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{bmatrix}
$$

(3)

Notice $\psi$ in $R(\psi)$ is nonsingular, and $R^{-1}(\psi) = R^T(\psi)$.

2.3. Dynamic Model of Vessels

For the dynamic positioning system, the vessels are usually working at a low-frequency motion not only in position control but also in tracking control. Therefore, we only consider the low-frequency vessel model in this paper and through deducing it can be written as:
\[ Mv + Dv = \tau \]  
\[ \tau = Bu \]  
Where \( \tau = Bu \) is the total external force, \( B \) is the control matrix describing propellers, \( u \) is the input of control, \( M \) is the system inertia matrix, \( D \) is the damping matrix [8]. In this paper:

\[
M = \begin{bmatrix}
1.1274 & 0 & 0 \\
0 & 1.8902 & -0.0744 \\
0 & -0.0744 & 0.1278
\end{bmatrix}
\]  
\[ (5a) \]

\[
D = \begin{bmatrix}
0.0358 & 0 & 0 \\
0 & 0.1183 & -0.0124 \\
0 & -0.0041 & 0.0308
\end{bmatrix}
\]  
\[ (5b) \]

Noticing this model is nonlinear so we can simplify it as a linear system. The state-space is below.

\[
\begin{cases}
\dot{x} = Ax + Bu \\
y = Cx
\end{cases}
\]  
\[ (6) \]

Where \( x = [\eta, \nu]^T \), \( y \) is the output of system.

\[
A = \begin{bmatrix}
0_{3\times3} & I_{3\times3} \\
0_{3\times3} & -M^{-1}D
\end{bmatrix} \quad B = \begin{bmatrix}
0_{3\times3}
\end{bmatrix} \quad C = \begin{bmatrix}
I_{3\times3}, 0_{3\times3}
\end{bmatrix}
\]  
\[ (7) \]

If we deduce the equations (6) (7), we notice that there is a coupling system between sway and yaw which means the outputs of sway and yaw are related to the two inputs of two controllers [9].

### 3. Decoupling of Multivariable Coupling System

Most industrial processes have multiple variables in nature, and this fact increases as result of the high demand on product quality and the required energy integration. Multi-input multi-output (MIMO) systems consist of several measurement and control signals, which often present complicated couplings between them. To cope with this problem, control engineers create some ways to decouple the MIMO systems. In this paper, we use one of the methods called feed-forward compensation decoupling.

We introduce a general method firstly. Figure 2 shows the diagonal decoupling control system, where \( G(s), N(s) \) are the process matrix and the decouple matrix respectively. \( N(s) \) is designed to minimize the process interactions in such a way that the controller sees the apparent process \( G(s) \cdot N(s) \) as a set of completely independent processes.

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According to the demands, we have:

\[
\begin{bmatrix}
G_{t1}(s) & G_{t2}(s) \\
G_{t1}(s) & G_{t2}(s)
\end{bmatrix}
\begin{bmatrix}
N_{t1}(s) \\
N_{t2}(s)
\end{bmatrix}
= 
\begin{bmatrix}
G_{t1}(s) & 0 \\
0 & G_{t2}(s)
\end{bmatrix}
\begin{bmatrix}
U_{t1}(s) \\
U_{t2}(s)
\end{bmatrix}
\tag{8}
\]

Then, the expression of relationship between the inputs and the outputs is obtained as follows:

\[
\begin{bmatrix}
Y_{t1}(s) \\
Y_{t2}(s)
\end{bmatrix} = 
\begin{bmatrix}
G_{t1}(s) & 0 \\
0 & G_{t2}(s)
\end{bmatrix}
\begin{bmatrix}
U_{t1}(s) \\
U_{t2}(s)
\end{bmatrix}
\tag{9}
\]

From equation (9), we can see the outputs are only influenced by the related one input which is the meaning of decoupling. The decouple matrix defined as:

\[
\begin{bmatrix}
N_{t1}(s) & N_{t2}(s) \\
N_{t1}(s) & N_{t2}(s)
\end{bmatrix} = 
\begin{bmatrix}
\frac{G_{t1}(s)G_{t2}(s)}{G_{t1}(s)G_{t2}(s) - G_{t1}(s)G_{t2}(s)} & -\frac{G_{t2}(s)G_{t1}(s)}{G_{t1}(s)G_{t2}(s) - G_{t1}(s)G_{t2}(s)} \\
\frac{G_{t1}(s)G_{t2}(s)}{G_{t1}(s)G_{t2}(s) - G_{t1}(s)G_{t2}(s)} & \frac{G_{t2}(s)G_{t1}(s)}{G_{t1}(s)G_{t2}(s) - G_{t1}(s)G_{t2}(s)}
\end{bmatrix}
\tag{10}
\]

Figure 3 shows the feed-forward compensation decoupling control system which is one of the most general methods in decoupling.

![Figure 3. Feed-Forward Compensation Decoupling Control System Block-Diagram](image)

Equations (11) are the premise conditions that we meet the demands.

\[
\begin{align*}
U_{t1}G_{t1}(s) + U_{t1}N_{t2}(s)G_{t2}(s) &= 0 \\
U_{t2}G_{t2}(s) + U_{t2}N_{t1}(s)G_{t1}(s) &= 0
\end{align*}
\tag{11}
\]

Deducing the equations, the transfer functions of feed-forward compensation are obtained.

\[
\begin{align*}
N_{t1}(s) &= -\frac{G_{t1}(s)}{G_{t2}(s)} \\
N_{t2}(s) &= -\frac{G_{t2}(s)}{G_{t1}(s)}
\end{align*}
\tag{12}
\]

Thus, we can decouple the system into single input single output (SISO) [10].
4. T-S Fuzzy Model of Nonlinear System and Its Identification

4.1. Description of T-S Fuzzy Model

T-S fuzzy model is presented by Takagi and Sugeno at 1985. The innovation in this model is that the premise is language variable, but the consequence is a linear function, which creates a new method to solve the control problems in complicated systems. Compared with Mamdani fuzzy model, the premise in T-S model is related tightly to the parameters in consequence. Therefore, one fuzzy rule in T-S model can be seen as a linear combination of many rules in one set. It has also an advantage of T-S model that using less fuzzy rules can describe a dynamic system.

T-S fuzzy model is suitable for big, complex nonlinear system. It is an effective method to model and identify systems by using measured input-output data. So far, researchers have done a lot of works on SISO system and MISO system, and we can see MIMO system as several MISO systems.

To obtain the T-S model, we need to identify the parameters in premise and in consequence. For identification of premise rules, we use Fuzzy C-Means clustering method. For identification of parameters in consequences, we adopt Least Squares method [11].

4.2. Fuzzy C-Means Clustering Method

The Fuzzy C-Means clustering algorithm is a good choice of all the clustering methods based on the objective function. In this method, firstly, we can choose some clustering centres randomly, then, we revise them through repeated iterative operation while let them close to all the data points. After that, the fuzzy vectors’ membership function relationship can be obtained by those new clustering centres. The membership function relationship can be divided automatically by this method, which can let fuzzy partition of variables more reasonable without any experiences on clustering and data collection. The main process is written below [12].

Set a group of data $x_k$, $1 \leq k \leq N$, and divide them into $C$ (number) fuzzy sets, $\mu_{ik}$ represents for the membership functions of the $k$th vector in the $i$th fuzzy set. The minimum objective function optimized by iterative operation is written as:

$$J = \sum_{k=1}^{N} \sum_{i=1}^{C} (\mu_{ik})^\nu \|x_k - C_i\|^2$$

(13)

$$\sum_{i=1}^{C} \mu_{ik} = 1, 1 \leq i \leq C, 1 \leq k \leq N$$

(14)

Where $\nu$ is weight value (fuzzy factor), $1 \leq \nu < \infty$.

And define $U$ as the matrix of $\mu_{ik}$. $C_i$ is the centre of the $i$th fuzzy set.

The requirements to obtain objective function are:

$$C_i = \frac{1}{\sum_{k=1}^{N} (\mu_{ik})^\nu} \sum_{k=1}^{N} (\mu_{ik})^\nu x_k$$

(15)

$$\mu_{ik} = \frac{1}{\sum_{j=1}^{C} (\frac{\|x_k - C_i\|}{\|x_k - C_j\|})^{2/\nu}}$$

(16)

Define the distance between the $k$th data and the $i$th fuzzy set:
\[ d_{ik} = \left\| x_k - C_i \right\| \]  \hspace{1cm} (17)

Above all, the main steps of fuzzy C-means clustering are:

1. Set the initial value of the number of clustering \( C \), the initial membership matrix \( U \), \( v \) and the initial step number.
2. Modify \( \mu_{ij}^{(m+1)} \) and \( C_i^{(m+1)} \) according to function (15) and function (16).
3. Set \( \varepsilon \), if \( U^{(m+1)} - U^{(m)} \leq \varepsilon \), end, else, return to step (2).

4.3. Least Squares Method

We use Least Squares method to identify the parameters in consequence [11]. The output is:

\[ y = p_i' + p_1' x_1 + p_2' x_2 + \ldots + p_r' x_r \]  \hspace{1cm} (18)

It also can be written as: \( y = \phi \theta \)
Where:

\[ \theta = [p_{10}, \ldots, p_{a0}, p_{11}, \ldots, p_{a1}, p_{1r}, \ldots, p_{ar}]^T \]  \hspace{1cm} (19)

\[ \phi = [y_1, \ldots, y_a; x_1, \ldots, x_r] \]  \hspace{1cm} (20)

\[ y = [y_1, y_2, \ldots, y_N]^T \]  \hspace{1cm} (21)

According to algorithm:

\[ \theta = (\phi^T \phi)^{-1} \phi^T y \]  \hspace{1cm} (22)

Least Squares method is an estimation method which can obtain the mathematical model best fitting to experimental data on minimum variance. The estimation result has the best statistical characteristics such as unbiased, consistent (convergence) and effective.

4.4. Format of T-S Fuzzy Model

Suppose identification system is \( P(U, Y) \), \( U \) is input, \( Y \) is output. And MIMO system can be divided into \( q \) (number) subsystems. So one implication can be written as:

\[ R_i: \text{ If } x_i \text{ is } A_{i1}, x_2 \text{ is } A_{i2}, \ldots, x_r \text{ is } A_{ir} \hspace{1cm} \text{Then } y_i = p_i' + p_{i1}' x_1 + p_{i2}' x_2 + \ldots + p_{ir}' x_r \]  \hspace{1cm} (23)

\( R_i \) is the \( i^{th} \) fuzzy rule; \( x_i \) is the \( i^{th} \) input variable; \( A_{ij} \) is the \( j^{th} \) fuzzy subset of \( x_i \); \( y_i \) is output of the \( i^{th} \) fuzzy rule; \( p_{ij}' \) is a parameter. Let

\[ v_i = \frac{\xi_i}{\sum_{j=1}^{c} \xi_i} \]  \hspace{1cm} (24)

And the output can be written as:
\[
y = \sum_{i=1}^{c} \xi_i y_i = \sum_{i=1}^{c} v_i y_i = \sum_{i=1}^{c} v_i (p_i^0 + p_i^1 x_i + p_i^2 x_2 + \ldots + p_i^r x_r)
\]

(25)

Notice, \( \xi_i = \mu_i \wedge \mu_2 \wedge \ldots \wedge \mu_v \), \( \mu_i \) is calculated by Fuzzy C-means clustering, \( \wedge \) is the minimizing operation.

\[
f_v = \sum_{i=1}^{c} v_i p_i
\]

(26)

\[
y^f = f_{j0} + f_{j1} x_1 + f_{j2} x_2 + \ldots + f_{jr} x_r
\]

(27)

The output of linear model can be expressed as:

\[
Y(t) = \begin{bmatrix}
y^1(t) \\
y^2(t) \\
\vdots \\
y^n(t)
\end{bmatrix} = F \cdot x + \zeta
\]

(28)

Above equation is CARIMA model for generalized predictive control (GPC) [11].

5. The Description of Generalized Predictive Control

Generalized predictive control (GPC) is one of the important adaptive control algorithms. Besides keeping characteristics such as online identification, output prediction and minimum output variance of minimum variance self-correcting control, it also absorbs rolling optimization strategy in DMC and MAC, which let the GPC have self-adaptive and predictive functions. Based on the parameter model, GPC introduces prediction length and control length which lets the system design easily. Moreover, GPC has good control performance and robustness. GPC is based on CARIMA model:

\[
A(z^{-1})y(k) = z^{-d}B(z^{-1})u(k) + C(z^{-1})e(k) / \Delta
\]

(29)

\[
A(z^{-1}) = 1 + a_1 z^{-1} + \ldots + a_n z^{-n} \\
B(z^{-1}) = b_0 + b_1 z^{-1} + \ldots + b_n z^{-n} \\
C(z^{-1}) = 1 + c_1 z^{-1} + \ldots + c_n z^{-n}
\]

(30)

Suppose time delay of system \( d = 1 \), we have:

\[
A(z^{-1})y(k) = B(z^{-1})u(k-1) + C(z^{-1})e(k) / \Delta
\]

(31)

If \( d > 1 \), we just let \( b_0 = b_1 = \ldots b_{d-2} = 0 \), and formula (31) can be written as:
Let output error on time \((k + j)\) is:

\[ \hat{y}(k + j) = y(k + j) - \hat{y}(k + j/k) , j \geq 1 \]  

Then the variance of prediction error is

\[ J = E\{\hat{y}^2(k + j/k)\} \]  

The minimum optimal prediction formula is obtained by

\[ C(z^{-1})y(k+j/k) = G_j(z^{-1})y(k) + F_j(z^{-1})\Delta u(k+j-1) \]  

Formula (35) is also the optimal predictive output formula. Where \(F_j(z^{-1}), G_j(z^{-1})\) are satisfied by Diophantine formulas:

\[
\begin{align*}
C(z^{-1}) &= \mathcal{A}(z^{-1})E_j(z^{-1}) + z^{-j}G_j(z^{-1}) \\
F_j(z^{-1}) &= B(z^{-1})E_j(z^{-1}) \\
E_j(z^{-1}) &= 1 + e_{j,1}z^{-1} + \ldots + e_{j,n}z^{-n_j} \\
G_j(z^{-1}) &= g_{j,0} + g_{j,1}z^{-1} + \ldots + g_{j,n}z^{-n_y} \\
F_j(z^{-1}) &= f_{j,0} + f_{j,1}z^{-1} + \ldots + f_{j,n}z^{-n_i}
\end{align*}
\]

While the optimal prediction error is

\[ y^*(k+j/k) = E_j(z^{-1})e(k+j) \]  

According to theory, we can obtain predictive output model:

\[ y(k+j) = \]

\[ E_j e(k+j) + \frac{F_j}{C} \Delta u(k+j-1) + \frac{G_j}{C} y(k) \]  

6. Simulation

6.1. Simulation Results of Decoupling

According to the description of decoupling methods before, we make a simulation of coupling system and decoupling system using feed-forward compensation method based on MATLAB. If we deduce the equations (6) (7), we conclude that there is a coupling system between sway and yaw but surge is a SISO system. We model the DPS vessel with PID controllers between three DOF. We set \( r_1 = 20, r_2 = 15, r_3 = 10 \), the results are showed in Figure 4(a). If we set \( r_1 = 20, r_2 = 80, r_3 = 10 \), the results are showed in Figure 4(b).
Figure 4(a). The Results of DPS Vessel
\( r_1 = 20, r_2 = 15, r_3 = 10 \)

Figure 4(b). The Results of DPS Vessel
\( r_1 = 20, r_2 = 80, r_3 = 10 \)

Figure 5(a). The Results of DPS Vessel
\( r_2 = 15, r_3 = 10 \)

Figure 5(b). The Results of DPS Vessel
\( r_2 = 80, r_3 = 10 \)

Figure 6(a). The Output of Surge

Figure 6(b). The Output of Surge Controller

Figure 7(a). The Output of Sway

Figure 7(b). The Output of Sway Controller

Figure 8(a). The Output of Yaw

Figure 8(b). The Output of Yaw Controller

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Then, we use feed-forward compensation method to decouple the system. We set $r_2=15, r_3=10$ and $r_2=80, r_3=10$, the results are showed in Figure 5(a) and 5(b) respectively.

As we can see in Figure 4(a) and 4(b), the output of yaw is influenced a little bit when the input of sway changes, but the output of surge stay the same all the time. It is proved that the sway and yaw are coupled which means the outputs of sway and yaw are related to the two inputs of both controllers. However, we notice that the difference between two results is very small because the model of vessel is simplified from a complicated system before.

And we can see in Figure 5(a) and 5(b), the outputs of sway and yaw which are almost same when the inputs change. It can be concluded that feed-forward compensation method is effectively to decouple the system.

### 6.2. Simulation Results of DPS Vessel

After simulation of decoupling, we turn on modeling the DPS vessel in MATLAB to confirm the effectiveness of generalized predictive control based on T-S model.

For the parameters of DPS, we suppose $\eta=[60,60,10]^T$ and the initial value is $\eta=[0,0,0]^T$. We translate the state-space model into differential equations and divide each part of it into 2 clustering. The input of system is random signal, sampling time is 0.05, prediction length is 14 and control length is 6. After setting necessary initial parameters, we can obtain T-S model and CARIMA model. The results are shown in Figure 6, Figure 7 and Figure 8.

From those results, the output response curve of system can track target value tightly without obvious error. Therefore, GPC based on T-S model can achieve anticipated goal.

### 7. Conclusion

This paper presents a fuzzy predictive control to position the vessels which has dynamic positioning systems. The mathematical model of vessel is described firstly which is nonlinear and complicated. Then, it is simplified into linear model and is decoupled through feed-forward compensation decoupling method. Through simulation results, we can see this decoupling method performing well. After that, the Generalized Predictive Control method is applied to the T-S model of vessel which is identified by Fuzzy C-Means clustering algorithm and Least Squares method. The simulation results show that the vessel can be oriented by the instructions accurately. Therefore, it can be concluded that the fuzzy predictive control effectively controls the vessel and meets our demands.

This paper only makes the simulation on the model with 3 DOF and without disturbances; therefore, the vessel model with all DOF and with disturbances can be researched in the future.

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### References


