Fuzzy Adaptive Sliding Mode Control of Large Erecting Mechanism

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Abstract
Since the large erecting mechanism has nonlinearities, parameter uncertainties and external disturbance, it is difficult to realize a model-based sliding mode control design. So a fuzzy adaptive sliding mode control scheme which combined fuzzy control with sliding mode control is proposed to achieve nonlinear control of the erecting mechanism. This control scheme is mainly use the fuzzy system to approximate the equivalent control of the sliding mode controller without knowing the system dynamic model. And it also releases the trial-and-error work of establishing the fuzzy inference rules. The update laws for the fuzzy tuning parameters and the switch control parameter are derived based on the Lyapunov stability analysis. The simulation results show that compared to the PID control and conventional sliding mode control, the fuzzy adaptive sliding mode control has nicer robustness and more accurate tracking ability, and the stability of the erecting process has improved.

Keywords: sliding mode control, fuzzy control, adaptive control, nonlinearity

1. Introduction
Large erecting mechanism is widely used in heavy engineering equipments and armaments. Its actuators are electro-hydraulic systems, because compared with motor actuators the hydraulic systems have many advantages such as high power capability, fast response and large output force. However, the erecting mechanism is a complicated system which is composed of mechanism, electrical equipments and hydraulic system. So it has strong nonlinearities and uncertainties. For example, the electro-hydraulic system has nonlinearities due to the flow-pressure relationship, oil leakage, dead zone of the valve, friction, volume flow unbalance of asymmetrical cylinder. Further, in the erecting process the external load act on the cylinder is always change and the erecting mechanisms are always subjected to many kinds of disturbance and various working environments. So the conventional linear control method cannot guarantee robustness and tracking accuracy. And it is hard to realize fast and steady erecting.

Recent years many research efforts on erecting mechanism control have been made. For example, a nonlinear predictive controller based on BP neural network is proposed in 2008 [1]. In the same year, an intelligent integration control method is used to tracking the erect velocity of the erecting mechanism [2]. These control methods provide satisfactory results from the simulations. But when the parameters or environments change, they cannot adaptively compensate these changes.

Sliding Mode Control (SMC) has emerged as a powerful nonlinear control method for its robustness of parameter uncertainties and disturbances. So in recent years, it is appeared in electro-hydraulic servo system control. For example, it has been successfully implemented to compensate load variations [3], friction and internal leakage [4], and uncertain original volume [5]. Chen presented an adaptive sliding mode controller to overcome the effects of the time-varying loadings in 2005 [6]. Hong-Bo Guo applied a cascade-control algorithm based on a sliding mode to realize the trajectory tracking control in 2008 [7]. However, the conventional sliding mode control needs to know the accurate dynamic model of the control object. Since it is difficult to establish the accurate model of the actual erecting mechanism, the model based controllers are not suitable. Further, high frequency chattering in the SMC may leads to bad control performance. Combining SMC with the other control methods has proved to be a good solution. Fuzzy control has no model based requirement, and it is widely used in a lot of
engineering applications. But the fuzzy controller design needs a time-consuming trial-and-error process for establishing the fuzzy inference rules and it lacks a reasonable stability and robustness analysis theory. For the high-order or complicated system the inference rules are difficult to confirmed. So some researchers have combined the advantages of the SMC and fuzzy control to develop the fuzzy sliding mode control [8-11]. In this paper, an adaptive sliding mode control with a fuzzy compensator is developed to control a large erecting mechanism. It not only has the advantage of designing a SMC without knowing the dynamic model but also has the on-line learning ability and less computation burden.

This paper is organized as follows. In section 2, the detailed nonlinear mathematic model of the erecting mechanism is established. In section 3, the proposed fuzzy adaptive sliding mode controller is given. In section 4, the simulation model is set up and simulation results are discussed. Finally, conclusions are exhibited in section 5.

2. Erecting Mechanism Nonlinear Model

Figure 1 presents the schematic diagram of the erecting mechanism which is mainly composed of a hydraulic pump, a frame, an electro-hydraulic proportional valve, an asymmetrical cylinder, a large erect arm and an angle sensor. The erect arm can rotate around the point O driven by the piston rod of the cylinder. Meanwhile, the cylinder can rotate around the point O_1 and O_2. The angle of the erect arm is controlled as follows: Once the reference angle \( \theta_0 \) and the actual \( \theta \) are transmitted to the controller, the output current \( u \) is calculated from the control algorithm. Then, the valve spool position and direction are controlled according to the output current. Depending on the spool position, the flows as well as the direction supplied to each cylinder chamber are determined. The motion of the erect arm actuated by the cylinder is controlled by the flows.

In Figure 1, building the reference frame XOY where O is the origin. Let \( OO_1 = l_1 \), \( OO_2 = l_2 \), \( O_1 O_2 = l_3 \), \( O_3 O_4 = l_4 \), \( \angle O_1 O_2 O_3 = \alpha \), \( \angle XO_3 = \gamma \) where \( O_3 \) is the erect arm center of gravity, and \( \angle O_1 O_0 = \theta_0 \) at zero second. The cylinder of the erecting mechanism is an asymmetrical cylinder, and its dynamic characteristics are different from the traditional symmetrical cylinder. The nonlinear model is composed of three equations: force balance equation, valve flow equation, and flow continuity equation of the cylinder.

![Figure 1. Schematic Diagram of the Erecting Mechanism](image)

2.1. Force Balance Equation

Applying Newton’s second law to the piston, the force balance equation of the cylinder can be obtained as follows:

\[
A_1P_1 - A_2P_2 = m\ddot{x}_p + B_c \dot{x}_p + KX_p + F_L
\]  

(1)

Where \( A_1, A_2 \) are the effective area of the both sides of the piston, \( m \) is the equivalent mass of the cylinder, \( B_c \) is the equivalent viscous damping coefficient, \( K \) is the load spring gradient, \( F_L \)
is the output force from the cylinder, $P_1$ and $P_2$ denote the supply and return pressure, respectively.

Based on the rotation differential equation of the erect arm and the law of sines, we can receive the following equation:

$$F_L = \frac{J\ddot{\theta} + Gl_4\cos(\gamma + \theta)}{l_4 \sin(\theta + \theta_0) / (l_4 + X_p)}$$  \hspace{1cm} (2)$$

Where $J$ is the erect arm moment of inertia, $\theta$ is the erect angle, and $G$ is the gravity of the erect arm.

If the piston is in the steady state, the Equation (1) can be reduced to:

$$A_i (P_1 - nP_2) = F_{L0}$$  \hspace{1cm} (3)$$

Where $n = A_2/A_1$, and $F_{L0}$ represent the steady load force act on the piston. So the load pressure can be defined as:

$$P_L = P_1 - nP_2$$  \hspace{1cm} (4)$$

### 2.2. Valve Flow Equation

Suppose that the spool valve is a symmetric valve, and the flow areas of the valve be proportional to the spool displacement $x_v$. Then the flow of oil across the spool valve can be written as:

$$Q_1 = \begin{cases} 
C_d w x_v \sqrt{2(P_s - P_1)} / \rho & x_v \geq 0 \\
C_d w x_v \sqrt{2P_1} / \rho & x_v < 0 
\end{cases}$$  \hspace{1cm} (5)$$

$$Q_2 = \begin{cases} 
C_d w x_v \sqrt{2P_2} / \rho & x_v \geq 0 \\
C_d w x_v \sqrt{2(P_s - P_2)} / \rho & x_v < 0 
\end{cases}$$  \hspace{1cm} (6)$$

Where $Q_1$ and $Q_2$ represent the flow of the two ports, $P_s$ is the supply pressure, $C_d$ is the discharge coefficient, $w$ is the spool valve area gradient, and $\rho$ is the oil mass density. The spool displacement $x_v$ is proportional to the control signal $u$ that is $x_v = k_p u$, where $k_p$ is the proportional coefficient.

Velocity of the piston rod is $v = Q_1/A_1 = Q_2/A_2$, so $Q_2/Q_1 = n$. And from the Equation (4), (5) and (6), we have:

$$P_1 = \begin{cases} 
(n^3P_s + P_L) / (n^3 + 1) & x_v \geq 0 \\
(nP_s + P_L) / (n^3 + 1) & x_v < 0 
\end{cases}$$  \hspace{1cm} (7)$$

$$P_2 = \begin{cases} 
(n^3P_s - P_L) / (n^3 + 1) & x_v \geq 0 \\
(P_s - n^3P_L) / (n^3 + 1) & x_v < 0 
\end{cases}$$  \hspace{1cm} (8)$$

Defining the load flux $Q_L = Q_1$, so:

$$Q_L = \begin{cases} 
C_d w x_v \sqrt{2P_s - 1/P_1} (P_s - P_1) & x_v \geq 0 \\
C_d w x_v \sqrt{2P_s - 1/P_1} (nP_s + P_L) & x_v < 0 
\end{cases}$$  \hspace{1cm} (9)$$
2.3. Flow Continuity Equation of the Cylinder

Assume that the pressure in each piston chamber is the same everywhere and the temperature and density are constant. The flow continuity equation of the piston chambers can be expressed as:

\[ Q_i = A_i \dot{X}_i + C_{in} (P_i - P_{\Delta}) + \frac{V_0}{\beta_e} \dot{P}_i \]  \hspace{1cm} (10)

\[ Q_i = A_i \dot{X}_i + C_{in} (P_i - P_{\Delta}) - C_{out} P_2 + \frac{V_0}{\beta_e} \dot{P}_2 \]  \hspace{1cm} (11)

Where \( C_{in} \) is the internal leakage coefficient of the cylinder, \( C_{out} \) is the external leakage coefficient of the cylinder, \( \beta_e \) is the fluid bulk modulus, and \( V_0 \) is the initial volume of the chamber.

Equation (7), (8), (10) and (11) can be combined to simplify as:

\[ \frac{Q_i + nQ_i}{(1 + n^2)} = A_i \dot{X}_i + C_{in} P_i \frac{V_0}{\beta_e (1 + n^2)} \dot{P}_i + b P_i \]  \hspace{1cm} (12)

with

\[ C_i = \frac{1 + n}{1 + n^2} C_{in} + \frac{n^3}{(1 + n^2)(1 + n^3)} C_{out} \]

\[ b = \frac{1 + n}{1 + n^2} b C_{in} - \frac{n}{1 + n^2} b C_{out} \]

\[ h_1 = \frac{n}{1 + n^3}, \quad h_2 = \frac{n^2}{1 + n^3} \quad x_r \geq 0 \]

\[ h_1 = \frac{n - 1}{1 + n^3}, \quad h_2 = \frac{1}{1 + n^3} \quad x_r < 0 \]

So the load flux can be written as:

\[ Q_i = \frac{Q_i + nQ_i}{(1 + n^2)} = \frac{Q_i + n^2 Q_i}{(1 + n^2)} = Q_i \]  \hspace{1cm} (13)

That is we defined in the previous section 2 and the Equation (12) is converted into:

\[ Q_i = Q_i A_i \dot{X}_i + C_{in} P_i \frac{V_0}{\beta_e (1 + n^2)} \dot{P}_i + b P_i \]  \hspace{1cm} (14)

Define the state variables as \( x_1, x_2, x_3 = [X, \dot{X}, \ddot{X}] \), where \( x_1, x_2, x_3 \) represent displacement, velocity, and acceleration of the piston rod, respectively. The system model including Equation (1), (9) and (13) can be expressed in a state-space form as:

\[ \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} a_1 x_2 + a_2 x_1 + a_3 g(x) u + a_4 + d \end{bmatrix} \]  \hspace{1cm} (15)

with

\[ a_1 = \frac{\beta_e (1 + n^2)}{m V_0} (A_i^2 + C_i B_i), \quad a_2 = \frac{B_i}{m} (A_i^2 + C_i B_i) \frac{C_i \beta_e (1 + n^2)}{V_0}, \quad a_3 = \frac{A_i \beta_e (1 + n^2) C_i w k_p}{m V_0} \sqrt{\frac{2}{\rho (1 + n^2)}}, \]

\[ a_4 = \frac{F_m}{m} + \frac{C_i \beta_e (1 + n^2)}{m V_0} F_L + \frac{A_i^2 \beta_e (1 + n^2)}{m V_0} b P_i \],

\[ g(x) = \begin{cases} \sqrt{P_1 - P_{\Delta}} & x_r \geq 0 \\ \sqrt{u_0 P_1 + P_{\Delta}} & x_r < 0 \end{cases} \]

3. Fuzzy Adaptive Sliding Mode Controller

In this section, a fuzzy adaptive sliding mode controller is proposed to realize the nonlinear control of the erecting mechanism.
3.1. Model-based Sliding Mode Controller Design

In the Figure 1, the erect angle $\theta$ and the displacement $X_p$ of the piston rod have the following relationship:

$$X_p = \sqrt{l_1^2 + l_2^2 - 2l_1l_2 \cos(\theta + \theta_0)} - l_3 \tag{16}$$

Set the desired angle is $\theta_d$ and from the Equation (16) the reference displacement of the cylinder is $X_{pd}$. Define the tracking error $e = x_1 - X_{pd}$ and the sliding surface.

$$s = c^2 e + 2c\dot{e} + \ddot{e} \tag{17}$$

Where $c>0$ and $x_1$ is the actual displacement of the piston rod.

The time derivative of $s$ is:

$$\dot{s} = c^2 \dot{e} + 2c\ddot{e} + \dot{x}_1 - \ddot{X}_{pd} \tag{18}$$

Let $\dot{s} = 0$, and substituting (14) into the (18), the equivalent control of the SMC is:

$$u_{eq} = [\ddot{X}_{pd} - c\ddot{x} - c\dddot{x} - a_1x_1 - a_2x_1 - d - a_4] / (a_1g(x_1)) \tag{19}$$

Based on the SMC theory, the system stability can be guaranteed at the condition $s\dot{s} < 0$. So the control law can be chosen as:

$$u = [\ddot{X}_{pd} - c\ddot{x} - c\dddot{x} - a_1x_1 - a_2x_1 - d - a_4 - u_s] / (a_1g(x_1)) \tag{20}$$

Where $u_s$ is switch control, and $u_s=\eta\text{sgn}(s)$, $\eta>\text{max}(d)$. The control law can guarantee the system output error convergence.

3.2. Fuzzy Adaptive Sliding Mode Control

The large erecting mechanism has nonlinear time-varying dynamics and parameters uncertainties. So it is difficult to estimate an accurate dynamic model. And the control law presented in section 3.1 is hard to derive. The fuzzy system has high approximate ability, so we can realize the equivalent control approximate by the fuzzy system. Here, an adaptive fuzzy strategy is introduced to realize nonlinear control. And the adaptive rules of the fuzzy sliding controller can be derived from Lyapunov stability theorem.

$s$ and $\dot{s}$ are denote the relative distance and velocity to the sliding surface $s = 0$, respectively. So we can use the $s$ and $\dot{s}$ to evaluate the equivalent control $u_t$. And using the $s$ and $\dot{s}$ as the input and $u_t$ as the output of the fuzzy approximate system. The generalized fuzzy logic system consists of a set of linguistic rules as follows:

$$R_j: \text{IF } S \text{ is } A_{1j} \text{ and } \dot{S} \text{ is } A_{2j}, \text{ then } U_t \text{ is } \theta_j$$

Where $S$, $\dot{S}$ and $U_t$ are the fuzzy variables of the $s$, $\dot{s}$ and $u_t$, respectively. $A_{1j}$, $A_{2j}$, and $\theta_j$ are the corresponding fuzzy sets, respectively.

The center of gravity method is employed to defuzzify the fuzzy output variable for obtaining the control voltage of the valve. The equivalent control voltage is derived from the fuzzy inference and defuzzification operation as:

$$u_f = \sum_{j=1}^{m} \theta_j [u_{A_{1j}}(s)u_{A_{2j}}(\dot{s})] / \sum_{j=1}^{m} [u_{A_{1j}}(s)u_{A_{2j}}(\dot{s})] = (\theta_1, \theta_2, \cdots, \theta_m)^T \tilde{\xi}(s, \dot{s}) \tag{21}$$
Where \( u_{A1} \) and \( u_{A2} \) are the Gauss membership function, \( m \) is the number of rules, and 
\[
\xi(s, \dot{s}) = [\xi_1, \xi_2, \ldots, \xi_m], \quad \xi_i = \frac{u_{A_i}(s) u_{A_i}(\dot{s})}{\sum_{j=1}^{m} [u_{A_j}(s) u_{A_j}(\dot{s})]}
\]
\( \theta_i \) is the consequent unknown parameter which can be adjusted by an adaptive rule.
So we needn’t the trial-and-error process for finding appropriate fuzzy rules. Based on the fuzzy approximate theory, it has a best fuzzy system \( u^*_f = u_f(s, \dot{s}, \theta^*) \) to approximate the equivalent control, and it can be expressed as \( u_{eq} = u_f^* + \varepsilon \), where \( \varepsilon \) is the approximate error, \(|\varepsilon| < E\).

Applying the fuzzy system to approximate \( u_f^* \), we set:
\[
\hat{u}_f(s, \dot{s}, \hat{\theta}) = \hat{\theta}^T \xi
\]
(22)

And the approximate error:
\[
\hat{u}_f = \hat{u}_f - u_{eq} = \hat{u}_f - u_f^* - \varepsilon
\]
(23)

Define \( \hat{\theta} = \hat{\theta} - \theta^* \), and the Equation (22) can be written as:
\[
\hat{u}_f = \hat{\theta}^T \xi - \varepsilon
\]
(24)

The Equation (19) can be simplified as:
\[
u_{eq} = [\ddot{x}_m + \ddot{\varepsilon} - \ddot{s} - a_1 x_2 - a_2 x_3 - a_3 - d] / (a_2 g(x_s)) = [\ddot{x}_1 - \ddot{s} - a_1 x_2 - a_2 x_3 - a_3 - d] / (a_2 g(x_s)) = [a_2 g(x_s) - \ddot{s}] / (a_2 g(x_s))
\]
(25)

From the Equation (19) and (25), we can obtain:
\[
\dot{s} = a_3 g(x_s)(u - u_{eq})
\]
(26)

Define the Lyapunov function:
\[
V_1 = \frac{1}{2} \dot{s}^2 + \frac{a_3 g(x_s)}{2 \eta} \hat{\theta}^T \hat{\theta}, \quad \eta > 0
\]
(27)

The time derivative of \( V_1 \) can be derived as:
\[
\dot{V}_1 = s a_3 g(x_s)(\dot{u}_f + u - u_{eq}) + \frac{a_3 g(x_s)}{\eta} \hat{\theta}^T \hat{\theta} = a_3 g(x_s) \hat{\theta}^T (s \dot{\xi} + \frac{1}{\eta} \hat{\theta}) + s a_3 g(x_s)(u - \varepsilon)
\]
(28)

To make sure \( \dot{V}_1 \leq 0 \), the update law and switch control can be chosen as:
\[
\hat{\theta} = \hat{\theta} = -\eta s \dot{\xi}, u_s = -E \text{sgn}(s)
\]
(29)

Substituting Equation (29) into (28), obtain:
\[
\dot{V}_1 = -E a_3 g(x_s)|s| - E s a_3 g(x_s) \leq -E a_3 g(x_s)|s| + |s| |a_3 g(x_s)| 
\]
(30)

\[\text{As } |s| a_3 g(x_s) \leq 0, \text{ the update law and switch control can be chosen as: } \]

\[
\hat{\theta} = \hat{\theta} = -\eta s \dot{\xi}, u_s = -E \text{sgn}(s)
\]

Substituting Equation (29) into (28), obtain:
\[
\dot{V}_1 = -E a_3 g(x_s)|s| - E s a_3 g(x_s) \leq -E a_3 g(x_s)|s| + |s| |a_3 g(x_s)| 
\]
(30)
Parameter $E$ of the switch control is difficult to evaluate, and its value effects the stability of the control system. So we use the estimate $\hat{E}$ to instead of the $E$, and the estimate error is $\dot{\hat{E}} = \hat{E} - E$. Hence, the switch control can be written as:

$$u_s = -\hat{E} \text{sgn}(s)$$  \hfill (31)

Define the Lyapunov function as:

$$V = \frac{1}{2} \dot{s}^2 + \frac{a_1 g(x_c) \cdot \dot{\theta} \cdot \ddot{\theta}}{2\eta_1} + \frac{a_2 g(x_c) \cdot \dot{\hat{E}}^2}{2\eta_2}$$  \hfill (32)

Where the parameter $\eta_2 > 0$.

By taking the time derivative of the Lyapunov function $V$, we obtain:

$$\dot{V} = -\hat{E} a_2 g(x_c) |s| - \varepsilon s a_2 g(x_c) + \frac{a_1 g(x_c)}{\eta_2} \hat{E} (\hat{E} - E) \dot{\hat{E}}$$  \hfill (33)

The update law for $\hat{E}$ can be chosen as:

$$\dot{\hat{E}} = \eta_2 |s|$$  \hfill (34)

Substituting Equation (34) into (33), obtain:

$$\dot{V} = -\hat{E} a_2 g(x_c) |s| - \varepsilon s a_2 g(x_c) + \frac{a_1 g(x_c)}{\eta_2} (\hat{E} - E) \dot{\hat{E}}$$

$$\leq |s| |s| a_2 g(x_c) - \varepsilon s a_2 g(x_c) = -(E - |s|) |s| a_1 g(x_c) \leq 0$$  \hfill (35)

That means this control system stability can be guaranteed by using the update laws shown in Equation (29) and (34). Based on Barbaret’s lemma, the convergence of the output error can be guaranteed by using the adaptive fuzzy sliding mode control.

4. Simulation Results

In order to investigate the effectiveness of the proposed fuzzy adaptive sliding mode controller, the compared simulations are presented in this section. From the Equation (16), we can obtain the desired displacement curve of the piston rod $X_{PD}(t)$. For the actual erecting mechanism, the main original parameters are presented in Table 1. In practice, some parameters have uncertainties or hard to confirm because of the external disturbances and environment changes.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbols</th>
<th>Values</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply pressure</td>
<td>$P_s$</td>
<td>18</td>
<td>MPa</td>
</tr>
<tr>
<td>Discharge coefficient of the spool valve</td>
<td>$C_d$</td>
<td>0.62</td>
<td>—</td>
</tr>
<tr>
<td>Area gradient of the spool valve</td>
<td>$w$</td>
<td>2.51×10^{-2}</td>
<td>m</td>
</tr>
<tr>
<td>Bulk modulus of the oil</td>
<td>$\beta_0$</td>
<td>7.5×10^{9}</td>
<td>Pa</td>
</tr>
<tr>
<td>Length of the piston rod</td>
<td>$l$</td>
<td>1.5935</td>
<td>m</td>
</tr>
<tr>
<td>Equivalent viscous damping coefficient</td>
<td>$B_{eq}$</td>
<td>800</td>
<td>N/m/s</td>
</tr>
<tr>
<td>Mass density of the oil</td>
<td>$\rho$</td>
<td>868</td>
<td>Kg/m³</td>
</tr>
<tr>
<td>Equivalent mass of the cylinder</td>
<td>$m$</td>
<td>178.31</td>
<td>Kg</td>
</tr>
<tr>
<td>Inwards leakage coefficient of the cylinder</td>
<td>$C_n$</td>
<td>2.41×10^{-11}</td>
<td>m³/s*Pa</td>
</tr>
<tr>
<td>Outwards leakage coefficient of the cylinder</td>
<td>$C_o$</td>
<td>7.1×10^{-13}</td>
<td>m³/s*Pa</td>
</tr>
<tr>
<td>Equivalent mass of the erect arm</td>
<td>$M_e$</td>
<td>1155.98</td>
<td>Kg</td>
</tr>
<tr>
<td>Moment of inertia of the erect arm</td>
<td>$J$</td>
<td>10023</td>
<td>Kg*m²</td>
</tr>
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<td>Section area of piston-side/rod-side</td>
<td>$A/A_2$</td>
<td>0.0175/0.0133</td>
<td>m²</td>
</tr>
<tr>
<td>Internal volume of the chamber</td>
<td>$V_0$</td>
<td>1.5×10^{3}</td>
<td>m³</td>
</tr>
<tr>
<td>Length of OO₁/OO₂/O₁O₂/OO₃</td>
<td>$l/l_1/l_2/l_3$</td>
<td>1.132/1.62/1.032/3.5</td>
<td>m</td>
</tr>
<tr>
<td>Angle of $\angle O_2/O_1/O_2/O_2/O_1$ at 0s</td>
<td>$\theta/\gamma$</td>
<td>0.6816/0.1047</td>
<td>rad</td>
</tr>
</tbody>
</table>

Fuzzy Adaptive Sliding Mode Control of Large Erecting Mechanism (Liang Li)
In order to show the influence of the uncertain parameters, and to test the performance of the proposed control scheme. In this paper, we suppose the damping coefficient $B_c$, bulk modulus $\beta_e$, mass of the erect arm $M$ and the external disturbance $d$ have change as the following expression:

$$
B_c = B_{c0} + 0.04 B_{c0} \sin(0.1\pi t), \beta_e = \beta_{e0} + 0.01 \beta_{e0} \sin(0.1\pi t), M = M_0 + 0.09 M_0 \sin(0.1\pi t), d = 2000 \sin(0.1\pi t).
$$

The proposed controller parameters are designed as $c=100$, $\eta_1=5$, $\eta_2=0.5$. In order to reduce the chattering of the control signal, the saturation function is select to replace sign function of the switch control $u_s$. We obtain:

$$
u_s = -\dot{E}_{sat}(s)
$$

Where $sat(s) = \begin{cases} 1 & s > \Delta \\ s / \Delta & |s| \leq \Delta \\ -1 & s < -\Delta \end{cases}$, and $\Delta=0.5$ is the thickness of the boundary of the sliding surface.

Here, two control methods that PID control and conventional SMC are used and compared. The control law of the PID control can be expressed as:

$$
u_1(t) = k_p e(t) + k_i \int e(t) dt + k_d \dot{e}(t)
$$

Where $k_p$ is the proportional coefficient, $k_i$ is the integral coefficient, $k_d$ is derivative coefficient, and $k_p=8$, $k_i=4$, and $k_d=0.005$, respectively.

The control law of the conventional SMC can be expressed as:

$$
u_2(t) = [k_1 e_1 + k_2 e_3 + \hat{x}_{id} - a_1 x_2 - a_2 x_3 - a_3 + \epsilon \operatorname{sgn}(s)] / (\alpha_2 g(x_i))
$$

Where $\epsilon=3000$, $k=2.2$, $k_1=1 \times 10^6$, $k_2=2000$, $s = k_1 e_1 + k_2 e_3 + e_1$, $e_1 = x_{id} - x_1$, $e_2 = \dot{x}_{id} - x_2$, $e_3 = \dot{x}_{id} - x_3$.

The simulation model is established in the software Matlab/Simulink environment. Using the above three controllers to track the desired angle curve in the condition that the system has parameters uncertainties and external disturbance. The simulation results are shown in Figure 3-7. The lines in these figures are defined as follows:

- Red line: the reference curve, Blue line: PID controller,
- Black line: fuzzy adaptive SMC, Green line: conventional SMC
Figure 2 exhibits the position tracking angle curves of the three controllers in tracking the reference trajectory. And Figure 3 presents the tracking error. It can be observed that the proposed fuzzy adaptive sliding mode controller has the best tracking performance, and the other two controllers have distinct vibration affected from the parameters uncertainties and external disturbance. The max tracking error of the proposed control method is 0.298 degree at about 47.5 second, but 0.5663 degree for the PID control, 0.5122 degree for the conventional SMC.

![Figure 2: Position Tracking Angle Curves](image1)

**Figure 2. Position Tracking Angle Curves**

Figure 4 and Figure 5 present the velocity tracking curves and tracking errors, respectively. We can see that the tracking performance of the fuzzy adaptive sliding mode control is steadier and more accurate compared to the other two controllers. And the conventional SMC has high frequency vibration, so it is difficult to realize precise control and actual engineering applying.

![Figure 3: Velocity Tracking Errors](image2)

**Figure 3. Velocity Tracking Errors**

![Figure 4: Velocity Tracking Curves](image3)

**Figure 4. Velocity Tracking Curves**

Figure 6 shows the control signal of the three controllers. As seen, the curve of the proposed controller is smoother than that of the other two controllers, since the adaptation laws can compensate the parameters uncertainties. However, the other two curves have vibration under the influence of the parameters uncertainties and external disturbances. Furthermore, in order to realize good performance, it must be increased the value of the parameter $\varepsilon$ for the conventional SMC. This leads to the control signal chattering with high frequency and make the erecting process unstable.

![Figure 5: Control Signals](image4)

**Figure 5. Control Signals**

5. Conclusion

The erecting mechanism is a complicated system and it has strong nonlinearities, parameter uncertainties and external disturbances. It is difficult to realize the model-based
sliding mode control. So a fuzzy adaptive sliding mode control method is introduced to achieve model free control. The stability of the proposed controller is guaranteed by means of Lyapunov theorem. Simulation results show that the proposed controller can compensate the parameters uncertainties and external disturbance. Compared to the PID control and the conventional SMC, the proposed control method and adaptive schemes can obtain excellent position tracking performance robust control.

References