Game Analysis of Cooperative Probability between Alliance Partners

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Abstract
Aiming at the cooperation between the alliances partners, this paper takes advantage of the game theory to establish two models of the single game and repetitive game. Through the analysis of these two types of models, it is concluded that the decision of choosing to keep faith or break one’s promise is related to the winning probability and litigation costs in the every stage of a single game. Meanwhile, the choice that both sides will select trustworthiness or dishonesty in the transaction and the chances of cooperation later are concerned with the marketing discount factors as well as the income and the loss of a cooperation in a repetitive game. The basic conclusion of this paper can provide a foundation for alliance partners to make decisions in their cooperative process.

Keywords: cooperation between the alliance partners, single game, repetitive game, make decisions

1. Introduction
Competition and cooperation is one mode of behaviors for enterprises to obtain the long-term competitive advantages. The strategic management theory of competition-oriented traditional enterprises believes that only when enterprises compete fully, they can maximize efficiency. However, with the increasing social development and gradual diverse needs, it is difficult to predict the market. Therefore, through the cooperation form of alliances, enterprises obtain external resources to enhance their competitive advantages, which have become an important way. For an enterprise, there are some factors extremely important. For example, how to choose the mode of cooperation alliances, such as a single transaction alliance, repetitive business alliances; how to make the right decisions under the different modes of cooperation at all stages, such as the choice of alliance partners in the previous stage of cooperation, the selection of trustworthy promises in the previous and middle stages, the options of prosecute security and the choice of the long-term cooperation at a later stage. These factors relate to the company's own survival and development, so it is necessary to establish the game model for analysis.

Admittedly, there are many scholars having studied the relationship between competition and cooperation as well as many similar documents at present. But most of them only list some external factors for cooperation without analysing from mutual behaviors between potential partners. They neither answer in particular why to cooperate, whether to cooperate and how to cooperate. Part of articles develops appropriate incentive mechanisms to promote cooperation, according to the study of group-punishment mechanism and incentive mechanism. But few articles discuss different cooperative types between two alliance partners to analyse and then provide support for decision. Aiming at the trade cooperation between the two coalition partners, this paper which is based on the perspective of a single game and repetitive game is through the establishment of their own game model to analyze a variety of factors, and quantify each factor. Then by the use of mathematical analysis method, this paper determines the size and range of the factors, thereby providing a new method of quantitative analysis and statistical observation, which provides a basis of theoretical analysis for enterprises' decision-making in the previous, middle and latter stages of alliance process.
2. Single Game Model
2.1. Model Establishment

Suppose coalition partners A, B sides to conduct a transaction. In this transaction, there are some problems, such as issues of mutual trust, the problem of the loss side’s prosecution, the probability of winning and problems of mutual revenue. The grasp and decision of these problems' factors play a key role in the development of an enterprise. Now analyse this transaction process. The specific game model is as follows. If both sides are neither trustworthy at start, the income of both is 0. Then co-transaction suspends; If both sides are mutually trustworthy, both sides in the transaction acquire the interests of equality, \((m > n > 0)\); if B breaks its promise, deceiving A, then B is to get greater benefit \(m\), and A to get a smaller benefit \(n\). But A can protect its own interests by the means of appeal. Income can be redistributed. Assuming that the probability of winning is \(p\), the prevailing party obtained more benefits \(m\), while the losing party received less benefits \(n\), and paid damages \(s\) in addition. The model of the single-transaction game’s specific process is shown in Figure 1.

\[\begin{align*}
\text{Break one’s faith} & \quad (n,m) \\
\text{Abandon prosecution} & \\
\text{Keep faith} & \\
\text{B} & \quad (m,n-s) \\
\text{A} & \quad \text{win} \\
\text{Break one’s faith} & \quad (0,0) \\
\text{Keep faith} & \quad ((m+n)/2,(m+n)/2) \\
\text{Fail} & \\
\text{B} & \quad (n-s,m)
\end{align*}\]

Figure 1. Single Transaction Game’s Model Diagram

2.2. Model Calculations

Firstly, define several types of numerical values in the single-game model:

The income if A, B keep faith:
\[V_a = V_b = \frac{m + n}{2}\]

The income if A, B break their promise:
\[W_a = W_b = 0\]

Expected revenue if A prosecutes:
\[U_a = pm + (1-p)(n-s)\]

the income if A breaks its promise:
\[U_b = p(n-s) + (1-p)m\]

There will be several situations in the course of this single game:

First: expected income of A’s prosecution is greater than dishonest gain of A, ie, \(U_a > W_a\)

Then,
\[pm + (1-p)(n-s) > 0 \Rightarrow s < \frac{pm}{1-p} + n\]  \hspace{1cm} (1)

Second: B’s trustworthy income are greater than expected return of B’s dishonest, ie, \(V_b > U_b\).

Then,
\[
\frac{m+n}{2} > p(n-s) + (1-p)m \rightarrow s > \frac{1-2p}{2p}(m-n)
\]

(2)

Third: B’s trustworthy income are smaller than expected return of B’s dishonest, ie, \( V_b < U_b \).

Then,

\[
\frac{m+n}{2} < p(n-s) + (1-p)m \rightarrow s < \frac{1-2p}{2p}(m-n)
\]

(3)

Fourth: expected income of A’s prosecution is greater than A’s revenue not to prosecute, ie, \( U_a > n \).

Then,

\[
p m + (1-p)(n-s) > n \rightarrow s < \frac{p}{1-p}(m-n)
\]

(4)

Combine with (2), (4), in order to make (2), (4) established,

Then,

\[
\frac{1-2p}{2p}(m-n) < \frac{p}{1-p}(m-n) \rightarrow p > \frac{1}{3}
\]

(5)

ie,

\[
\frac{1-2p}{2p}(m-n) < s < \frac{p}{1-p}(m-n), \text{ and } p > \frac{1}{3}
\]

(6)

Combined with (3) and (4), it can be seen when \( p < \frac{1}{3} \), only (4) need to be set up;

when \( p > \frac{1}{3} \), only (3) need to be set up.

2.3. Result Analysis

From (1), it can be concluded that when litigation costs is less than a certain value, A will choose to trust B in the initial stage of cooperation. Because no matter whether the litigation fails, A’s revenue gain is always greater than uncooperative income.

From (2), it can be known that a litigation cost is not as small as possible. When s is lower than a certain level, B will not be trustworthy. Penalties for acts of dishonesty would require a litigation cost is no less than a certain limit.

From (4), it shows that the cost of litigation s needs to control above some extent. That is, A’s prosecution must meet the constraint of costs. When the winning probability p and income distribution (m-n) are in certain circumstances, if litigation costs s are too high, then A will abandon litigation. If litigation is uncertain of winning, the cost s is a risk for A. When the litigation cost s is in certain circumstances, if increasing A’s win probability p, and trading income inequities (m-n) grows, then A will choose to prosecute. Only in this way can dishonest acts of B be punished and A’s own interests be safeguarded.

From (5), it is clear that the cost of litigation s must satisfy certain conditions, in other words, s is in the area, moreover winning probability p must be greater than 1/3. Only when these terms are met, do A have the motive to sue B dishonest, and B is also motivated to be trustworthy. At last mutually beneficial cooperation can be achieved.
From (6), it is evident that when winning probability $p$ is less than $1/3$, B will not keep its word. Now it has nothing to do with winning probability $p$ and litigation cost $s$; when winning probability $p$ is greater than $1/3$, A is bound to prosecution and fluctuating range in the litigation costs expands as $p$ increases.

Simplifying the model of a single game mainly analyse the range of the successful probability $p$ and litigation costs $s$ to judge decisions that the two companies make in all stages of a single game. Create a function graph of the relationship between $s$ and $p$, as shown in Figure 2.

![Figure 2. Function Model Diagram About Winning Probability and Litigation Costs](image)

Summing up, taking advantage of the scope of this figure and combining their own states with cooperative situations, enterprises would be clear to make decisions of alliance probability. That is, an enterprise should be on the base of calculation of litigation costs to choose corporate partners and make trustworthy decisions in the initial stage of cooperation. In addition, two companies may exist formal or informal agreements, which may not be effectively implemented in cooperation. There is only the simple relationship of marketing transactions. Some companies will make full use of informational asymmetrical phenomenon during the transaction progress. They will maximize the behavior of misappropriating other corporates' interests in order to reach the margin where cooperative enterprise can not check. In this case, enterprises should increase costs of mutual transaction and analyse winning probability, while controlling a range of litigation costs to enhance the stability of cooperation.

3. Repetitive Game Model
3.1. Model Establishment
Suppose that coalition partners A, B sides play the repetitive transaction game, dittoing once a month. At the end of each stage of the game, each firm will get the next trading opportunities with the probability of $\delta$. $\delta(0 \leq \delta \leq 1)$ is the discount factor for long-term cooperation, in other words, the enterprise's patience index is the probability of going on the game. Such a transaction involves mutual trust issues, questions of the companies’ gains and losses, probability problems of companies'existence on the market, etc. These factors determine whether two corporates can start the cooperation and repeat cooperation. Now analyze the process of such transactions, and the specific game model is as follows. Suppose that A adopts trigger strategy. A at first trusts B and chooses to cooperate while B has the choice to be trustworthy or dishonest. If B coope rates trustworthily, each side gain the income $R$. Moreover, they can continue to cooperate. If not, B gain the income $R + r$. Here $r (r > 0)$ refers to the interests that B encroaches A. However, A will punish B through interrupting the trade. Then B will gain nothing afterwards. The first model of transaction game in the repetitive transaction is shown in Table 1.

<table>
<thead>
<tr>
<th>Company B</th>
<th>dishonest</th>
<th>dishonest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Company A</td>
<td>trustworthy</td>
<td>( R,R)</td>
</tr>
<tr>
<td>dishonest</td>
<td>( R+r,R-r)</td>
<td>(0,0)</td>
</tr>
</tbody>
</table>
3.2. Model Calculations

First of all, define several types of numerics in the model of repetitive game—B's and A's expected return if B is trustworthy and dishonest. There will be several situations in the course of the repetitive game.

First: If B choose to trust, the profit B gets is long-standing. Both sides are able to get equal benefits R from each other's honest transaction and obtain the probability of \( \delta \) to obtain the next trading income R, the probability of R to get the next transaction benefits, and so on.

Then A and B's expected returns are 
\[
E_{a1} = E_{b1} = \frac{R}{1-\delta}
\]
Scilicet:
\[
(7)
\]

Second: If B selects dishonesty, its gain is \((R + r)\) in the first stage. Here \(r>0\) refers to the interest B occupy A. But B's gain is 0 in each stage afterwards. Then B's expected returns is \(R+r+0=R+r\), and A's is \(R-r+0=R-r\)

Scilicet:
\[
E_{b2} = R + r; \quad E_{a2} = R - r
\]
(8)

3.3. Result Analysis

Through the comparative analysis with combining (1) and (2), it can be drawn two situations.

First: The condition to make A trust B at first is \(E_{a1} > E_{a2}\),

Scilicet:
\[
\frac{R}{1-\delta} > \frac{R-r}{r-R} \quad \text{because} \quad 0<\delta<1, \quad \text{scilicet} \quad \frac{r}{r-R} < 1, \quad \text{then} \quad r<R.
\]

In other words, at the premise of the loss value \(r\) is less than a gain value \(R\), as long as

\[
\delta > \frac{r}{r-R}, \quad \text{A can choose cooperation in trust at first.}
\]

Second: condition for B to be trustworthy is \(E_{b1} > E_{b2}\),

Scilicet:
\[
\frac{R}{1-\delta} > \frac{R+r}{r+R}
\]

That is, as long as the discount factor \(\delta\) is large enough and the possibility B may be larger than \(r/(R+r)\) next time continues to appear on the market, the optimal strategy for B is to choose trustworthy cooperation. Similarly, A also will choose to continue cooperation.

The repetitive game model makes the analysis from the marketing discount factor number \(\delta\) of partner enterprises, the range of a gains numeric \(R\) and losses numeric \(r\), as shown in Table 2.

Summing up, decision about the probability of several cooperations between enterprises is made. That is, enterprises are inclined to take the uncooperative attitude if they pay more attention to the immediate benefits and lack confidence and patience in the continuity of potential business. This needs to analyze the patient extent of corporates' long-term cooperation. If the economic environment for enterprises' long-term cooperation is lack of consistency or stability, the patience that enterprises expect for long-term cooperation is smaller. At this time, the business is easy to take a strategic of non-alignment, which requires to developed the effective incentive conditions, such as co-benefits, losses of missing appointments and so on. Besides, whether repetitive collaboration can be conducted relies on the numerical size of a co-benefits \(R\) between the two companies and the losses \(r\). Therefore,
in the face of choosing partners and cooperative modes, it is better for the enterprises to make the right decisions to enhance mutually beneficial cooperation in the light of comprehensive analysis of the number of opposite enterprise’s market discount factor and the estimate of the numbers of revenue and lost.

4. Conclusion
There are both one game and repetitive games during the transaction process between two alliances. In the process of these dynamic games, it will involve the problem of credibility, namely the issues of partner’s dishonesty and trustworthiness. This paper analyses many influential factors in the different processes of different situations to offer a general template for decision-making. Each enterprise takes advantage of the template to make the right decisions according to the actual situation. The decisions include whether trust should be chosen in the first stage, whether credit should be observed in the second stage, whether litigation should be taken in the third phase and whether the cooperation should be selected again after one transaction. In order to balance the unfairness between dishonesty and trustworthy persons, dishonesty punishment system with correction and specification must be introduced. Enterprises regulate the credit activities through the system so as to institutionalize the matter of punishing unethical Conduct. Thus, make sure of trustworthy gains and dishonesty punishment, such as contractual penalty clauses, legal punishment. The legislative level should guarantee the implementation of dishonesty punishment mechanism to effectively prevent the arbitrariness of individual behaviors and acts of dishonesty with the legal system of justice, fairness, openness.

References