Study on Multi-array Shaker Random Vibration Test Control

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Abstract

Some basic algorithms and theories are discussed for 9-shaker random vibration test control in this paper. A correct algorithm RMS is presented without considering influence of cross spectrum. The algorithm can make the control spectrum of convergence stably and fastly. In order to solve the problem of multiple-point input and output synthetically weighting control, Finite Markov Chains and Algorithmic is set forth for the first time to form the main control matrix. Finally through Matlab simulation and the results show that the method can meet the engineering demand.

Keywords: mul-sub array, vibration control, random vibration, synthetic control

1. Introduction

Shaking table has been considered as a very important experimental devices in engineering research, it can well reproduce the dynamic process of earthquakes in the lab and get a large amount of data, and also shorten the test cycle, the earthquake simulation shaking table is means and method to multiple-point input and output vibration test, recently, more and more attention from scholars [1, 2]. In order to response signal, sensors are placed on test specimen. Compared with set of reference signal, modification of drive signal through feedback, make it meet precision requirement [3]. Because of the different parts of the specimen vibration response different, hence, the different parts of the specimen in real more than the number of measuring point vibration table, the result bring about frequency response matrix is not square, this has brought great difficulties to feedback control [4, 5].

In the early 1980’s, Small wood DO came up with multiple-point input and output random vibration control method [6], this method was used to do triaxial vibration test of Hill air force base in 1989, Small wood DO Put forward mainly and secondary control theory in 1999. That is to say, using proper control algorithm, which all point matrix composed of important points in the priority in convergence.

Some basic algorithms and theories are discussed for multi-shaker random vibration test control in this paper. A correct algorithm RMS is presented without considering influence of cross spectrum. Finite Markov Chains and Algorithmic is set forth for the first time to form the multiple-point input and output synthetically weighting control. Using Matlab/Simulink simulation, validation of this method has important theory and application value.

2. The Basic Principle of Time Domain Randomization [7-10]

Random wave driving signal generated By IFFT in the past, its generated random waves are Special kind of periodic signal, however, most of the signal is non periodic in practical engineering, in this paper, random drive signal is produced through time domain randomization, as shown in Figure 1.

Where, $x[n]$: original sequence, $x_{i}[n]$: reorder sequence after a half sine function, $x_{i}[m]$: random wave driving signal.

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3. Basic Formula

The earthquake simulation shaking table array system is a typical multiple input multiple output system, input/output system spectrum matrix can be expressed such that:

\[ S_{yy} = G S_{uu} G^H \] (1)

Where:
\[ S_{yy} : 9 \times 9 \] response spectrum matrix
\[ S_{uu} : 9 \times 9 \] drive spectrum matrix
\[ G : \text{frequency response matrix} \]
\[ G^H : \text{conjugate transpose matrix of } G \]

The ideal driving spectrum In the specification:

\[ S_{dd}^u = A R A^H \] (2)

Where:
\[ R : \text{reference spectrum matrix} \]
\[ A : \text{system compensation matrix}( A = G^{-1} ) \] as shown in figure 2
\[ A^H : \text{conjugate transpose matrix of } A \]

According to Cholesky decomposition \( R \) and \( S_{dd}^u \) can be rewritten as:

\[ R = L L^H \quad S_{dd}^u = U U^H \] (3)

Where, \( U \) : drive signal (3) Plug in (2) function can be described as:

\[ U U^H = A L L^H A^H = A L (A L)^H \] (4)

According to the Cholesky uniqueness of decomposition, \( U \) can be rewritten as: \( U = A L \), without considering the lack of the phase of \( R \), suppose that:

\[ U = A L X \] (5)

(3) Plug in (2) function \( S_{yy} \) can be described as:

\[ S_{yy} = G S_{uu} G = (G A L X)^H L^H (G A)^H = LL^H = R \] (6)
Equation (6) is the basic formula of Array system for random vibration control system.

Figure 2. Compensation Matrix of Control System Diagram

4. Control Algorithm

4.1. Proportion of RMS Correction Algorithms

We can see that They are consistent between Response spectrum matrix of array and Reference spectrum matrix from function (5). You must first the frequency response matrix for identification if you want to get compensation matrix $\hat{A}$ in the practical engineering, Zhi fang-Fu study show that according to $\varepsilon_j$ the estimation model under the condition of low level white noise [11, 12], spectrum matrix can be described as:

$$\hat{G} = S_{\omega \omega} S_{\omega \omega}^{-1}$$

Where:
- $\hat{G}$ : Low level response spectrum matrix
- $S_{\omega \omega}$ : Cross spectrum matrix of Incentive and response
- $S_{\omega \omega}$ : incentives of Since the spectrum matrix

Suppose $\hat{A} = \hat{G}$, plug it into the formula (5), drive signal can be written as:

$$U = \hat{A}LX$$

Due to the existence of the error, so:

$$G \hat{A} = I + E \neq I$$

Where:
- $I$ : unit matrix
- $E$ : error matrix

Formula (9) into the formula (1): The actual response spectrum matrix can be described as:

$$S_{yy} = GS_{\omega \omega} G = (G \hat{A})(LX)(L^*) (G \hat{A})^* = (I + E)L^* (I + E)^*$$

Feedback loop for error correction must be added because $S_{yy} \neq R$. 9-sub array reference spectrum can be written as:

$$L = \text{diag}(l_1, l_2, \ldots, l_9)$$

Then the system output spectrum matrix $S_{yy}$ can be described as:

$$S_{yy} = (I + E)\text{diag}(L_1^2, L_2^2, \ldots, L_9^2)(I + E)^H$$
Where:

\( L \): diagonal matrix

Modification \( L \) can be written as:

\[
L' = \text{diag}(L'_1, L'_2, \ldots, L'_n) = \text{diag}(\Delta_1, \Delta_2, \ldots, \Delta_n)
\] (13)

Formula (13) into the formula (10): The output reference spectrum can be described as:

\[
R = (I + E)L^H(I + E)^H
\]

\[
= (I + E)\text{diag}(L_1^2 / \Delta_1^2, L_2^2 / \Delta_2^2 \ldots, L_n^2 / \Delta_n^2)(I + E)^H
\] (14)

We put the formula (12) and (14) for unfold and tidy and ignore the second order dimensionless, \( \Delta_i^2 \) (\( i = 1, 2 \ldots, 9 \)) can be written as:

\[
\Delta_1^2 = S_{yy1} / R_1
\]
\[
\Delta_2^2 = S_{yy2} / R_2
\]
\[
\Delta_3^2 = S_{yy3} / R
\] (15)

9-sub array of correction algorithm of RMS can be described as:

\[
\Delta_1 = \sqrt{\frac{S_{yy1}}{R_1}}, \Delta_2 = \sqrt{\frac{S_{yy2}}{R_2}}, \ldots, \Delta_9 = \sqrt{\frac{S_{yy9}}{R}}
\]

\[
L_1^{(k+1)} = L_1^{(k)} / \Delta_1, L_2^{(k+1)} = L_2^{(k)} / \Delta_2 \ldots L_9^{(k+1)} = L_9^{(k)} / \Delta_9
\] (16)

Where:

\( k \) : the number of iterations.

4.2. Based on a Finite Markov Chain Evolution Algorithm is the selection of Main Control Condition [13, 14]

Maximum, minimum and weighted average of the three kinds of control mode are widely used in the multi-dimensional multi-point input and output random vibration test. Maximum control can ensure specimens security in the test process; The minimum control can make all of the measuring point tested in the process of test; weighted average of the control points for weighted comprehensive control according to the measuring point to the importance of the test.

Maximum control: \( S_c = \max(S_{c1}, S_{c2}, S_{c3}, \ldots, S_{cn}) \)

Minimum control: \( S_c = \min(S_{c1}, S_{c2}, S_{c3}, \ldots, S_{cn}) \)

Weighted average control: \( S_j = \sum_{j=1}^{n} \omega_j S_{ij}, (\sum_{j=1}^{n} \omega_j = 1) \)

Where:

\( S_{ij} \): since the spectrum signal of \( j \) control point

\( \omega_j \): weighting coefficient

The number of control points is greater than the number of vibration table in engineering, system compensating matrix using pseudo-inverse matrix instead of, cause system can’t control of driving signal stability. Based on lots of research and summarize, in this paper, select main control matrix of finite markov chain evolution algorithm, and got very good solution to this problem [15-17]. The method combined with RMS correction algorithm have good control performance and realize the request of comprehensive control.

4.3. Control System Block Diagram

Control system block diagram is illustrated in Figure 3. First according to specification give reference spectrum \( R \), second cholesky decomposition \( R \), according to select main control
matrix of finite markov chain evolution algorithm to select main control matrix and solve the compensation matrix. Then calculate the system drive signal in the frequency domain through Fourier transform into drive signal, collection each control point of the respective control spectrum after a random signal incentive specimen, integrated the reference spectrum. The process cycle according to the error requirement.

5. Matlab/Simulink Simulation Validation
In this paper, we verify the correctness of the method by comparing the spectral density and RMS error in the 9-array system by Matlab/Simulink simulation. To simplify the calculation process based on the 2-array system \( \left( \begin{array}{c} m \\ m \end{array} \right) \) of Beijing University of Technology as an example. Random vibration test specification specified reference spectrum from the spectrum curve is illustrated in Figure 4, Weighted average control as an example validate control spectrum curve is illustrated in Figure 5. Control error are shown Table 1.

Table 1. Three Point Maximum Control Error

<table>
<thead>
<tr>
<th></th>
<th>Spectrum control 1</th>
<th>Spectrum control 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>±3dB tolerance/%</td>
<td>1.5</td>
<td>8.4</td>
</tr>
<tr>
<td>±6dB tolerance/%</td>
<td>0.0</td>
<td>0.81</td>
</tr>
<tr>
<td>RMS value /g</td>
<td>0.740</td>
<td>0.790</td>
</tr>
<tr>
<td>RMS error /%</td>
<td>-0.41</td>
<td>-2.30</td>
</tr>
</tbody>
</table>

From table 1, we can see that test precision satisfies the requirement of control spectrum curve of the spectral density within ±3dB and root mean square error of less than 10% of the technical index requirements.

In Matlab/Simulink simulation, control frequency band from 10Hz to 2000Hz, the reference spectrum of RMS were 0.70g and 0.74g, out of line ±6dB / Oct, respectively, the maximum value control, minimum value control and weighted average control are used.
6. The Experimental Data and Results
6.1. Experimental Data

According to the above mentioned control thought and Using application software produced by Beijing university of technology autonomy. In the paper, select a four span continuous girder bridge as the prototype bridge, combination models for 4x40m, bridge span 160m, the upper structure for the single box 3 rooms of reinforced concrete box girder, bottom structure for double reinforced concrete column type piers. To meet the needs of the experiment, the geometric scale ratio of 1:10, and through five table array simulation experiment to research carried out to verify the algorithm. Completed bridge model is illustrated in Figure 6, finite element model is Shown in Figure 7, White noise excitation in this test are illustrated in Figure 8. Measuring point arrangement of strain 72, 38 measuring point displacement and acceleration 24 points in the test. Layout of sensor are illustrated in Figure 9.
6.2. Experimental Analysis

We can see from Figure 10 that between design spectrum and fitting spectrum consistency is better, the experimental results show that Meet the requirements of the test error and prove the correctness of the above control algorithm.
7. Conclusion
(1) Without considering cross spectrum conditions, the use of RMS correction algorithm is applied to the 9-array random vibration test, the results show that the convergence of the method is rapid, meanwhile, the method has a high theoretical value and practical significance.
(2) In order to solve the problem of multiple-point input and output synthetically weighting control, Finite Markov Chains and Algorithmic is set forth for the first time to form the main control matrix.
(3) This paper prove the method can meet the requirements of the error though using the Matlab/Simulink simulation and provides the reliable theory basis for engineering practice.

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References