Transient Stability Analysis of Grid-connected Wind Turbines with Front-end Speed Control via Information Entropy Energy Function Method

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Abstract
According to the characteristics like time-consuming and can not be quantitatively analyzed of time domain simulation in power system transient stability analysis, a direct method using information entropy combined with transient energy function method is proposed in this paper to analyze the transient stability of wind power system equipped with front-end speed controlled wind turbines (FSCWT) with synchronous generators. In which, the system kinetic energy and potential energy are used as information source to makeup information entropy function, then, a theoretical analysis of system transient stability is conducted. Based on this, simulations are carried out in IEEE 5-machine 14-bus system compared with the time domain's, which verified the consistency of information entropy energy function (IEEF) method and time domain analysis. Results show that it is more intuitively and effectively to use IEEF method for wind power system transient analysis equipped with FSCWT.

Keywords: transient stability, front-end speed controlled wind turbine (FSCWT), information entropy, energy function

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1. Introduction
With an ever-increasing wind power penetration into power system, the power system performance especially transient stability is influenced inevitably [1-5]. The wind power system transient stability under severe grid fault is sundry, which is affected by different types of wind turbines, various grid load models and unequal grid-connected capacity. The conventional doubly fed induction generator (DFIG) and directly drive synchronous generator (DDSG) with back-end inverter would consume considerable reactive power form grid side once a severe grid fault occurred [6, 7], the FSCWTs use electrically excited synchronous generators (EESG) which can achieve excellent performance like thermal power generators, relatively speaking. As the FSCWT is of good grid adaptability and fault ride through capacity, has been applied gradually. Therefore, to make study on transient stability of FSCWT with grid-connected is important and of great significance to the stable operation of the connected grid.

There have many scholars investigated on transient stability of wind power system, in paper [8, 9], the mathematical models of constant speed wind turbine (CSWT), DFIG and DDSG are established respectively and the critical fault clearing time (CCT) under transient fault is determined for transient stability analysis in time domain by simulation, the generator parameter effect of turbine transient stability is discussed, additionally. Reference [10] analyzed the power system transient stability with an unconstant voltage-dependent load into energy analysis incorporating comprehensive load characteristics and achieved a desired result while reference [11] considered a dynamic load model for transient function construction in transient analysis.

As the time domain simulation method emphasis on qualitative analysis, the energy function method can achieve a qualitative analysis. In reference [12], a combination of coincide probability function with transient energy function method is made and a quantitative method for transient stability analysis is proposed from the probability perspective. In this paper, the transient stability of power grid with FSCWT connected in system energy aspect is analyzed by
making a combination of information entropy and energy function method proposed in [13] and a time domain simulation is used to verify the correctness of proposed method.

2. Modeling of FSCSG and Power grid

The structure of FSCWT can be seen in Figure 1, which is composed by a wind wheel, a main gearbox, a hydro-dynamically controlled gearbox WinDrive and an EESG as shown in figure 2.

\[ n_t = n_g = n_b = (1 + \alpha_2) n_a \alpha_1 + n_r \alpha_2 / \alpha_3 \] (1)

therefore, the torque balance equation can be written as:

\[
\begin{align*}
M_t &= \alpha M_g \\
M_r &= -M_r / \alpha_3 \\
M_i &= M_q + M_j = M_g + M_G
\end{align*}
\] (2)

where \( M_t \), \( M_g \) and \( M_G \) stand for the turbine torque, the pump wheel torque and the generator input torque, respectively. \( M_r \), \( M_q \) and \( M_j \) separately stand for the torque of sun gear, ring gear and the planetary gear, which satisfy:

\[ M_t : M_q : M_j = 1 : \alpha_2 : (1 + \alpha_2) \] (3)

The generator input power can be derived from equation (1), (2) and equation (3):

\[ P_G = M_g \omega_g = M_r \omega_R \beta_1 - M_g \omega_g + M_g \omega_g \beta_2 \beta_3 \] (4)

where \( \beta_1 \), \( \beta_3 \) stand for the transmission efficiency from wind rotor to the planetary carrier and the transmission efficiency from center wheel to ring gear, seperately while \( \beta_2 \) is the efficiency of hydro-dynamic torque converter.

2.1. Electrically Excited Synchronous Generator Model

As the FSCWT uses an EESG, the excitation system should be taken into account of dynamic model, the rotor motion equation and stator voltage equation can be described as:

\[
\begin{align*}
\frac{d\delta}{dt} &= \omega_a \omega \\
\frac{d\omega}{dt} &= \frac{1}{M} (P_m - P_e - D\omega)
\end{align*}
\] (5)
and

\[
T_\text{do} \frac{dE_q'}{dt} = \frac{x_d}{x_d'} \left( x_d - x_d' \right) U \cos \delta + U_i \tag{6}
\]

where \( D \) is the damping coefficient, \( \delta \) is rotor angular, \( \omega_s = 2\pi f_0 \) is the reference frequency. \( M \) is inertia time constant of EESG, \( \omega \) is the relative angular speed in p.u.. \( P_m \) and \( P_e \) stand for the mechanical power and the electromagnetic power, respectively. \( U \) is stator voltage and \( U_i \) is the field voltage, \( E_q' \) stands for transient voltage. \( x_d \) and \( x'_d \) are reactance and transient reactance of \( d \)-axis.

The output power equation is:

\[
\begin{bmatrix}
P_e \\
Q_e
\end{bmatrix} = \begin{bmatrix}
\frac{UE_q}{x_d} \sin \delta + \frac{1}{x_q} U^2 \cos \delta \\
\frac{U}{x_d'} (E_q' - U \cos \delta) - \frac{U^2 \sin 2\delta}{x_q}
\end{bmatrix} \tag{7}
\]

where \( Q_e \) is the output reactive power, \( E_q \) and \( x_q \) stand for induced potential and reactance of \( q \)-axis.

### 2.2. Grid Model

For general power system, it can be described by DAE as follows [16]:

\[
\begin{align*}
\dot{x} &= f(x, y) \\
0 &= g(x, y)
\end{align*} \tag{8}
\]

where \( x \) is a continuous variable about time and represents for continuous dynamic process of generator, \( y \) stands for algebraic variables can mutated like node voltage, phase angle and etc., and \( u \) is control variable. An N node power grid can be expressed as:

\[
\begin{bmatrix}
Y_{11} & Y_{12} & \cdots & Y_{1n} \\
Y_{21} & Y_{22} & \cdots & Y_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
Y_{n1} & Y_{n2} & \cdots & Y_{nn}
\end{bmatrix} \begin{bmatrix}
U_1 \\
U_2 \\
\vdots \\
U_n
\end{bmatrix} = \begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_n
\end{bmatrix} \tag{9}
\]

where \( Y_{ik} = G_{ik} + jB_{ik} \) stands for self admittance of node \( i \), \( Y_{ik} = G_{ik} + jB_{ik} \) is the mutual admittance between node \( i \) and node \( k \). \( U_i \) is the node voltage vector of node \( i \) to the ground, \( I_i \) is the current vector of node \( i \) inflows to grid. Therefore, the power injected into node \( i \) can be described as:

\[
\begin{align*}
P_i &= \sum_{k=1}^{n} [U_i U_k B_{ik} \sin(\theta_i - \theta_k) + U_i U_k G_{ik} \cos(\theta_i - \theta_k)] \\
Q_i &= \sum_{k=1}^{n} [U_i U_k G_{ik} \sin(\theta_i - \theta_k) - U_i U_k B_{ik} \cos(\theta_i - \theta_k)] \tag{10}
\end{align*}
\]

For easy to analysis, second order models are used for the rest synchronous generators in system, the generator model can be converted to center of inertia (COI) coordinate in consideration of relative rotor angular and angular speed as follows:
\[
\begin{align*}
\frac{d\ddot{\phi}_i}{dt} &= \omega_i\dot{\phi}_i \\
\frac{d\ddot{\omega}_i}{dt} &= P_m - P_w - D_i\dot{\phi}_i - \frac{M_i}{M_r}P_{\text{COI}}
\end{align*}
\] 

(11)

where \( P_m = E_i^2 Q_i + \sum_{j=1}^{n} [E_E E_i \sin(\hat{f}_i - \hat{f}_j) + E_E E_i \cos(\hat{f}_i - \hat{f}_j)] \), in which \( \hat{\phi}_i \) and \( \dot{\phi}_i \) represent for rotor angular and angular speed and \( \ddot{\omega}_i = \omega_i - \omega_{\text{COI}} \), where \( \omega_{\text{COI}} \) is the weighted mean of \( \omega_i \).

3. Transient Stability Analysis Based on Entropy Function

3.1. Configuration of Energy Function

In wind power system including FSCWTs, define the kinetic energy of synchronous generator \( i \) is \( V_k \), by using equation (5), we can get

\[ V_k = \frac{1}{2} M_i \omega_i^2 \] 

(12)

and the system kinetic energy is:

\[ V_k = \sum_{i=1}^{n} V_k = \frac{1}{2} \sum_{i=1}^{n} M_i \omega_i^2 \] 

(13)

where \( \omega_i \) is the difference of rotor angular speed and synchronous angular speed of SG \( i \). When the fault is cleared, the transient kinetic energy of SG \( i \) is:

\[ V_k |_{t=0} = \frac{1}{2} M_i \omega_i^2 = \int_{\delta_i}^{\delta_i} M_i \frac{d\omega_i}{dt} d\delta_i = \int_{\delta_i}^{\delta_i} (P_{mg} - P_{w}) d\delta_i \] 

(14)

When the system is steady, \( V_k = 0 \) as \( \omega_i = 0 \). By using \( \delta_i \) as the potential energy reference point, the potential energy of SG \( i \) can be written as:

\[ V_p = \int_{\delta_i}^{\delta_i} (P_{mg} - P_{w}) d\delta_i \] 

(15)

From equation (14) and (15), the energy function of generator \( i \) at any moment is:

\[ V_i = V_k |_{t=0} = \frac{1}{2} M_i \omega_i^2 + \int_{\delta_i}^{\delta_i} (P_{mg} - P_{w}) d\delta_i \] 

(16)

The system transient kinetic energy, transient potential energy and transient energy shown as equation (12), (15) and 16 can be converted to COI coordinate:

\[ V_k = \frac{1}{2} M_i \omega_i^2 \] 

(17)

\[ V_p = \int_{\delta_i}^{\delta_i} - (P_{mg} - P_{w} - \frac{M_i}{M_r}P_{COI}) d\theta_i \] 

(18)

\[ V_f = \frac{1}{2} M_i \dot{\omega}_i^2 + \int_{\delta_i}^{\delta_i} - (P_{mg} - P_{w} - \frac{M_i}{M_r}P_{COI}) d\theta_i \] 

(19)
Where, $\theta_i = \delta_i - \delta_{COI}$, where $\delta_{COI}$ is the weighted mean of $\delta_i$, $M_r$ is the inertial time constant sum of all generators in system. The procedure for system transient energy calculation can be seen in Figure 3.

$$\delta_{COI} = \frac{\sum \delta_i}{\sum M_r}$$

3.2. Configuration of Information Entropy Function

We have defined information strictly in terms of the probabilities of events. Therefore, let us suppose that we have a set of probabilities (a probability distribution). $P = \{p_1, p_2, \ldots, p_n\}$. We define the entropy of the distribution $P$ by [18]:

$$H(P) = \sum_{i=1}^{n} p_i \ln \left( \frac{1}{p_i} \right)$$  \hspace{1cm} (20)

As is shown above, $H(P)$ can be used for describing average uncertainty of probability system, where $p_i$ is the information source. Each part of the defined system remains a stable state in certain rules as there are correlations of each part in power system. Once a severe fault occurred, the stable state would be broken: the system would become chaos if the system tends to instability and the system information entropy would increase, otherwise would decrease. If we have a continuous rather than discrete probability distribution $P(x)$:

Figure 3. Steps of transient analysis using PEBS method
In this paper, the power system kinetic energy $V_k$ and potential energy $V_p$ is used as information source $p_1$ and $p_2$ to construct information entropy function $H(P)$, thus, to any SG i in system, we can define:

$$p_1 = \frac{|V_k|}{|V_k| + |V_p|}, \quad p_2 = \frac{|V_p|}{|V_k| + |V_p|}$$

To a stable system, there exist $p_1 \in [0,1], \ p_2 \in [0,1]$. According to Gibbs inequality, we know that: $0 \leq H(P) \leq n \ln 2$, the kinetic energy and potential energy satisfy $V_k = V_p = 0$ and $H(P) = 0$ when the system is convergent. Once a severe fault is cleared, for each generator, the probability is equal for system to remain stable or unstable and $p_1 = p_2 = 1/2, \ H(P) = n \ln 2$, accordingly.

During the transient fault, the potential energy boundary surface (PEBS) method [17], which can be described like that: the system would start from the stable equilibrium point of stable status, if the trajectory is inside of potential energy boundary, then the system goes stable due to damping effect, or unstable if outside of potential energy boundary. The transient stability analysis process using IEEF method can be generalized as follows:

1) calculate the power flow before system fault.
2) tracking system trajectory under sustained fault and calculate $V_{ki}$ and $V_{pi}$ real-time.
3) calculate $H(P)$ by using $V_{ki}$ and $V_{pi}$.
4) if $H(P) = n \ln 2$, stop calculating, return $H(P)$ and calculation time $t$.

Figure 4. IEEE 5-machine 14-bus system

4. Simulations and Case Studies

From equation (17) we can know that when system fault occurred, the kinetic energy incremental of generators losing synchronization are more greater than the ones keeping in synchronization, so, the information entropy keeps increasing. Otherwise, the information entropy would fluctuate. To a $n$ machine system, form equation (19) and (20) we can determine that $V_{ki}$ and $V_{pi}$ would keep increasing if the system lose synchronization and goes unstable, then the system probability is the biggest with information entropy $H(P) = n \ln 2$. 
In this paper, the IEEE 5-machine 14-bus system is used for simulation analysis which scheme can be seen in Figure 4, firstly. In COI coordinate, as the kinetic energy increases of an unstable, the system transient stability can be determined by information entropy after a severe fault. In order to shown the relationship of transient stability and system information entropy, the IEEE 3-machine 9-bus system and the IEEE 10-machine 39-bus system is studied, behind.

**Figure 5.** Information entropy of IEEE 5-machine 14-bus

**Figure 6.** Power angle of IEEE 5-machine 14-bus

Figure 5 showns the system information entropy $H(P)$ with fault time lasted for 518ms and 519ms of IEEE 5-machine 14-bus sytem during a three phase fault. And Figure 6 showns the traditional time domain simulation with a same fault, it can be seen that the IEEF method has a good consistency with time domain simulation.

**Figure 7.** Information entropy of IEEE 3-machine 9-bus

Figure 7 showns the information entropy of the IEEE 3-machine 9-bus system with fault clear time 393ms (the green line) and 395ms (the purple line) and Figure 8 showns the information entropy of the IEEE 10-machine 39-bus system with fault clear time 230ms (the green line) and 232ms (the purple line). It can be seen that, if information entropy closer to $\ln 2$, the system goes to unstable, otherwise remains a stable state with information entropy varies.
5. Conclusions

The transient stability is crucial to the stable operation of wind power system. In this paper, the energy function method is combined with information entropy, and the procedure of using this proposed method is illustrated. Numerical simulation verifies the validity effectiveness and correctness of the analysis result in comparison with time domain simulation, which shows that the IEEF method is valid for transient stability analysis in power system including FSCWTs and the IEEF method is more intuitive in reflecting the system transient behavior.

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