The Calculation of Voltage Collapse Point by Introducing Branch Current

Yi Tao*1, Wang Cheng-min1, Xie Ning1, Gao Yang2
1Department of Electric Engineering, Shanghai Jiao tong University, Minhang District, Shanghai 200240, China
2Department of Electric Engineering, Shenyang Institute of Engineering, Shenyang 110136, China
*Corresponding author, e-mail: yitao4965@126.com

Abstract

The solutions of power flow can not be obtained easily near the voltage collapse point because the Jacobian matrix is singular. Through analysis of voltage instability region, find the voltage instability region margin, and that is to find the collapse conditions of static voltage stability. We form the equation that reflect the voltage collapse conditions and establish the extended power flow equations with the hybrid equations as mentioned above, the static voltage collapse point can be calculated by solving the extended equations. The advantage of this method is that the power flow calculation can be convergence smoothly near the collapse point because of the extended power flow equations when the Jacobian matrix tends to be singular. The convergence can be achievable just because the joining of the equation reflecting the voltage collapse conditions has changed the structure of the Jacobian matrix. The simulation results show that this method is very effective. It can solve the voltage collapse point of single-node instability, two-node instability or even all-node instability at the same time.

Keywords: power flow calculation, voltage stability, branch current, collapse point

1. Introduction

Modern power system operates more and more close to the stability limit for the considerations of economy and other aspects, especially in the electricity market environment, voltage collapse occasionally occurs. Therefore, it is necessary to calculate the static voltage collapse point to observe the system running state, try to avoid voltage collapse accident due to the system running near the voltage collapse point.

Voltage collapse is an inherently nonlinear phenomenon that relates to collapse point from the viewpoint of nonlinear dynamic systems. Substantial research has been conducted to help understand and analyze the mechanism of this instability based on static voltage stability theory [1, 2]. As a framework of security assessment, voltage stability assessment has been studied to avoid voltage collapse of the worst scenario [3, 4]. The conventional methods on voltage stability may be classified as follows:
1) Direct method [5, 6];
2) Continuous Power Flow Calculation [7, 8];
3) Nonlinear Programming method [9, 10];

Method 1) evaluates the static voltage collapse points directly, but they do not give the margin to it. In that sense, the method is not acceptable from a stand point of system operation and planning. Method 2) evaluates PU or QU curves with the extended power flow calculation, so obtains the maximum load points as the voltage collapse points. It is one of robust methods that give the static voltage stability point. Also, the method easily gives the load margin to it. Some scholars improve the continuous power flow calculation by considering the dynamic characteristics of the power system components to enhance the power flow equations convergence near the voltage collapse point. Method 3) calculates the voltage collapse point with nonlinear programming method which converts the collapse point conditions to optimize load. Kuhn-Tucker optimal conditions can solve this problem, but the dimensions are at least two times the ordinary power flow equations, and the equations can not be decoupled, so the amount of calculation is too large. Method 4) directly forms the extended power flow equations
according to the characteristics of Jacobian matrix singular near the collapse point. It can get more precise collapse point, but the shortcoming is also convergence difficulties. In summary, the existing main problem of solving voltage collapse point algorithm lies in:
1) The power flow calculation convergence is difficult near the collapse point;
2) The physical meaning of additional equations is not obvious, sometimes too complicated;
3) The amount of calculation is too large.

In recent years, voltage stability assessment becomes more important due to the uncertainties of the power system liberalization and distributed generation. As power systems are inclined to be more complicated, voltage stability assessment requires an efficient method that estimates the static voltage collapse point.

In this paper, it has established the hybrid electric network analysis model composed with branch current and node voltage on the basis of \( \pi \) equivalent circuit which is made up of impedance branch as chain branch and grounded branch as tree branch. Define the voltage instability mode and add an equation reflecting the characteristic of voltage collapse point to the hybrid electric network analysis model mentioned above, then form the extended equations. The static voltage collapse point can be obtained by solving the extended equations. Power flow calculation can be convergence smoothly at the collapse point where Jacobian matrix should has been singular. It is mainly because the structure of Jacobian matrix has been changed when the equation represents the character of voltage collapse point is introduced. The results of system simulate calculation show that this method is very effective.

2. Extended Power Network Equations

The line (or transformer) of electric power system can be simulated by the \( \pi \) equivalent circuit model, as shown in Figure 1, called loop. Per loop is composed of three branches, an impedance branch and two grounded branch.

![Figure 1. The \( \pi \) Equivalent Circuit](image)

Where: \( i, j \) are two nodes on both sides of branch \( l \), \( s_i = p_i + j q_i \), \( s_j = p_j + j q_j \) are node power injections of \( i \) and \( j \), node voltages are as follows: \( u_i = e_i + j f_i \), \( u_j = e_j + j f_j \), impedance branch current is \( i_l = i_l^0 + ji_l^r \), \( R_l + jX_l \) is the impedance of branch \( l \), \( jB_i \) and \( jB_j \) are the grounded susceptance of node \( i \) and \( j \). The grounded conductivity is ignored for simple calculation in this paper.

For a grounded branch, the current trend is in two ways including grounded capacitor branch and load branch. The current flows in the load branch are not only the current of the circuit itself but also the adjacent loop current. The analytical method of load branch is the same principle with node voltage. Node \( i \), for example, the voltage of equivalent voltage source of the load branch is:

\[
  u_i = \frac{p_i - jq_i}{\sum_{l=i} p_l - jq_l - j\sum_{l=i} B_l}
\]
Where: \( p_i - jq_i \) is the load of node \( i \), \( \sum_{i=1}^{n} i_{i} \) is the sum of the injection current of node \( i \), \( \sum_{i=1}^{n} B_i \) is the sum of the capacitance to ground of node \( i \), \( ju_i \sum_{i=1}^{n} B_i \) is the sum of the capacitance current of node \( i \), \( \sum_{i=1}^{n} i_{i} - ju_i \sum_{i=1}^{n} B_i \) represents the load branch current of node \( i \). Derive from Equation (1):

\[
\sum_{i=1}^{n} i_{i} - \sum_{i=1}^{n} i_{i} B_i = p_i - jq_i
\]

The above equation is written in the plural form:

\[
\sum_{i=1}^{n} i_{i} - \sum_{i=1}^{n} i_{i} B_i = p_i - jq_i
\]

Separate the real and imaginary parts:

\[
e \sum_{i=1}^{n} i_{i} + f \sum_{i=1}^{n} i_{i} = p_i
\]

\[
e \sum_{i=1}^{n} i_{i} - f \sum_{i=1}^{n} i_{i} = -q_i
\]

The same to node \( j \):

\[
e \sum_{i=1}^{n} i_{i} + f \sum_{i=1}^{n} i_{i} = p_j
\]

\[
e \sum_{i=1}^{n} i_{i} - f \sum_{i=1}^{n} i_{i} = -q_j
\]

Impedance branch equation:

\[
i_i (R_i + jX_i) = -(\dot{u}_i - \dot{u}_j)
\]

Derive from Equation (6):

\[
i_i R_i - i_j X_j + e_i - e_j = 0
\]

\[
i_i X_j + i_j R_j + f_i - f_j = 0
\]

The formula (4), (5) and (7) constitute the hybrid electricity network equations in Cartesian coordinates in which the branch currents and node voltages are state variables.

3. Voltage Instability Region (Unstable Round)

Suppose: \( x_i = \sum_{i=1}^{n} i_{i} \); \( y_i = \sum_{i=1}^{n} i_{i} \); \( G_i = \sum_{i=1}^{n} B_i \). To the PQ node, by the formula (4), the node voltage can be derived as:

\[
\begin{align*}
e_i &= (p_i - f_j y_i) / x_i \\
f_i &= \left[ 2G_i p_i x_i - x_i (x_i^2 + y_i^2) \right] \pm x_i \sqrt{(x_i^2 + y_i^2)^2 + 4G_i q_i (x_i^2 + y_i^2) - 4G_i^2 p_i^2} / 2G_i (x_i^2 + y_i^2)
\end{align*}
\]
The Equation (8) is the node voltage analytical expression represented by the branch current. If the Equation (8) has solutions the following condition must be met:

\[ x_i^2 + y_i^2 \geq -2G_i q_i + 2G_i \sqrt{p_i^2 + q_i^2} \] (9)

Physical meaning of Equation (9) is: 1) When the square of the amplitude of the injection current of nodes is out of the circle of which the center is \(2G_i q_i\) and the radius is \(2G_i \sqrt{q_i^2 + p_i^2}\), that is only ‘>’ condition has been met in the Equation (9), the high and low solutions of Equation (8) exist, and the system is stable. 2) When the square of the amplitude of the injection current of nodes is in the circle, that is only ‘<’ condition has been met in the Equation (9), no solutions of equation (8) exist, so the system is unstable. 3) Equation (8) has a unique solution and the solution is on the circle when '=' condition has been met in the Equation (9), so the unique solution is the stable margin of system. System voltage collapse point can be found if calculate the power flow equations under this conditions.

Node types also include PU node and balance node in power system analysis, the voltage of balance node is considered to be known, and so do not need to be calculated. The voltage expression of PU node is similar with PQ:

\[
\begin{align*}
    e_i &= (p_i - f_i y_i) / x_i \\
    f_i &= p_i y_i \pm \sqrt{p_i^2 y_i^2 - (x_i^2 + y_i^2)(p_i^2 - V_i^2 x_i^2)} / x_i^2 + y_i^2
\end{align*}
\] (10)

Also,

\[ x_i^2 + y_i^2 \geq \frac{p_i^2}{U_i^2} \] (11)

The Equation (10) has solutions if the Equation (11) has been met, and there is a voltage instability circle, too. The circle’s center is origin and the radius is \(p_i / U_i\), the unstable region is in the circle, stability margin is on the circle.

4. Voltage Collapse Point
4.1. The Type of Collapse Point

The analysis above shows that the power system is in a voltage collapse state when the equality of Equation (9) or (11) holds, and the voltage collapse point is closely related to the active and reactive power of PQ node and active and voltage of PU node. The type of voltage collapse points is multiple.

Assuming the quantity of the electric power system node is N, PQ, PU and balance node number is \(N_p, N_g, N_s\) respectively. The voltage collapse point is defined as the single-node instability mode when ‘=’ condition is met in the Equation (9) or (11) of a node, and its type number is \(C_{N-N_s}\). The voltage collapse point is defined as the two-node instability mode when ‘=’ condition is met in the Equation (9) or (11) of two node simultaneously, and its type number is \(C_{N-N_s}^2\). And so on, the voltage collapse point is defined as the all-node instability mode when ‘=’ condition is met in the Equation (9) or (11) of all node simultaneously, and its type number is \(C_{N-N_s}^N\), that is only one type. So, the types of static voltage stability are

\[ \sum_{k=1}^{N-N_s} C_{N-N_s}^k \]
4.2. Static Voltage Stability Margin

Assuming, the current operating point of power system represent by the initial value of $P_{L0}, Q_{L0}$ (active and reactive power) of PQ nodes and $P_{G0}, V_{G0}$ (active and voltage) of PU nodes. The changes of PQ and PU nodes can be represented as below when the running state of system changes:

\[
\begin{align*}
\Delta P_{Li} &= P_{L0} + \lambda_{Li} \Delta P_{Li} \\
\Delta Q_{Li} &= Q_{L0} + \lambda_{qi} \Delta Q_{Li}
\end{align*}
\]

or

\[
\begin{align*}
\Delta P_{Gi} &= P_{G0} + \lambda_{Gi} \Delta P_{Gi} \\
\Delta U_{Gi} &= U_{G0} + \lambda_{Gi} \Delta U_{Gi}
\end{align*}
\]

Where $\lambda$ represents the changing magnitude and direction of PQ and PU nodes, and assume that the change is equal proportion. So the equations above can be written as:

\[
\begin{align*}
\Delta P_{Li} &= P_{L0} + \lambda_{Li} \Delta P_{Li} \\
\Delta Q_{Li} &= Q_{L0} + \lambda_{qi} \Delta Q_{Li}
\end{align*}
\]

or

\[
\begin{align*}
\Delta P_{Gi} &= P_{G0} + \lambda_{Gi} \Delta P_{Gi} \\
\Delta U_{Gi} &= U_{G0} + \lambda_{Gi} \Delta U_{Gi}
\end{align*}
\]

By the analysis previous, voltage instability will occur in parts of the nodes when the node injection power grows. Assume that voltage instability phenomenon occurs in the node $i$ firstly. Then:

\[
(x_i^2 + y_i^2)^2 + 4G_i q_i (x_i^2 + y_i^2) - 4G_i^2 p_i^2 = 0
\]  
(12)

The load is considered as variable, and define load growth mode and the introduction of parameter $\lambda$:

\[
(x_i^2 + y_i^2)^2 + 4G_i \lambda q_i (x_i^2 + y_i^2) - 4G_i^2 \lambda^2 p_i^2 = 0
\]  
(13)

Equation (13) is the characteristic equation of system voltage collapse point with variable parameter $\lambda$, and the characteristic equation means the collapse conditions. So add Equation (13) to Equation (4) or (5) to establish the extended power flow equations for calculating the voltage collapse point. Equation (14) is needed if voltage instability occurs in two nodes simultaneously with the growth of the power injection:

\[
\prod_{i=1}^{2} \left[(x_i^2 + y_i^2)^2 + 4G_i \lambda q_i (x_i^2 + y_i^2) - 4G_i^2 \lambda^2 p_i^2 \right] = 0
\]  
(14)

Similarly, for $N$ nodes voltage instability mode, the equation is:

\[
\prod_{i=1}^{N} \left[(x_i^2 + y_i^2)^2 + 4G_i \lambda q_i (x_i^2 + y_i^2) - 4G_i^2 \lambda^2 p_i^2 \right] = 0
\]  
(15)

Where, the symbol $\prod$ means multiplying. $N$ equals the total number of nodes if the voltage of all nodes are instability as the load changes, and the voltage instability is called all-node voltage instability. But actually, the voltage collapse phenomenon may generally occur only in a few nodes simultaneously, so the Equation (15) can be simplified according to the actual situation. The correctional equation of extended power network equations is:

\[
\begin{bmatrix} J & b \\ a & c \end{bmatrix} \begin{bmatrix} \Delta U \\ \Delta \lambda \end{bmatrix} = \begin{bmatrix} \Delta S \\ \Delta W \end{bmatrix}
\]  
(16)

Where, $J$ represents Jacobian matrix of the power network equation; $b$ represents load changing mode; $\Delta U$ represents node voltage deviation; $\Delta S$ represents the deviation of nodes
power or voltage amplitude; $\Delta W$ represents the deviation of Equation (15). Equation (17) can be derived by Equation (7):

$$
\begin{align*}
\begin{bmatrix}
i^o_i \\
i'_i
\end{bmatrix} &= \frac{-X_y(f_i - f_j) - R_y(e_i - e_j)}{R_y^2 + X_y^2} \\
& + \frac{X_y(e_i - e_j) - R_y(f_i - f_j)}{R_y^2 + X_y^2}
\end{align*}
$$

(17)

Standard node voltage power network equations can be obtained if substitute the Equation (17) into the Equation (4) or (5), so $J$ matrix is the same with the Jacobian matrix of traditional power network. Sub-matrix $a$ is expressed as:

$$
a = \prod_{i=1}^{N} \left[ \frac{2(x_i^2 + y_i^2)\partial(x_i^2 + y_i^2) + 4G_i\lambda p_i \partial(x_i^2 + y_i^2)}{\partial U_i} \right]
$$

(18)

And the last line of the diagonal elements of the Jacobian matrix is:

$$
c = \prod_{i=1}^{N} \left[ 4G_i x_i^2 y_i^2 \partial x_i^2 y_i^2 - 8G_i^2 \lambda p_i^2 \right]
$$

(19)

Equation (20) represents the (4) or (5) and (13) extended equations, PQ nodes, for example:

$$
\begin{align*}
e_i \sum_{l=1}^{n} i^o_l + f_i \sum_{l=1}^{n} i'_l &= \lambda_i p_i \\
e_i \sum_{l=1}^{n} i'_l - f_i \sum_{l=1}^{n} i^o_l - (e_i^2 + f_i^2) \sum_{l=1}^{n} B_i &= -\lambda_i q_i \\
(x_i^2 + y_i^2)^2 + 4G_i \lambda q_i(x_i^2 + y_i^2) - 4G_i^2 \lambda_i^2 p_i^2 &= 0
\end{align*}
$$

(20)

The PU node has the similar conclusion. When system is running near voltage collapse point $\Delta W$ is smaller, and $\lambda$ is smaller, too. So the load increases slowly according to the first formula of Equation (20), it is conducive to the flow convergence. Difficult convergence phenomenon sometimes occurs in the vicinity of the collapse point of the flow equations, mainly because the nodes in the corresponding row lose diagonally dominant. The formula (8) and (10) shows that the node voltage can be expressed as Equation (21) or (22) near the voltage collapse point.

$$
\begin{align*}
e_i &= (\lambda p_i - f_i y_i)/x_i \\
f_i &= \frac{2B_i \lambda p_i y_i + x_i(x_i^2 + y_i^2)}{2B_i(x_i^2 + y_i^2)}
\end{align*}
$$

(21)

or

$$
\begin{align*}
e_i &= (\lambda p_i - f_i y_i)/x_i \\
f_i &= \frac{\lambda p_i y_i}{x_i^2 + y_i^2}
\end{align*}
$$

(22)

The convergence characteristics of the power flow equations can be improved if replaces the corresponding node injection power equation with Equation (21) or (22). Specific practices is: determine which node in the network is more close to the voltage collapse margin by Equation (9) or (11) in the iterative process, and then replace the corresponding node.
injection power equation with Equation (21) or (22). The problem that the power flow equations do not converge due to Jacobian matrix singular near the collapse point will be solved.

In addition, when the grounded admittance equals 0 the power network equations will be expressed as follows:

\[ e_i \sum_{i=1}^{n} i_i^m + f_i \sum_{i=1}^{n} i_i^n = \lambda p_i \]
\[ e_i \sum_{i=1}^{n} i_i^n - f_i \sum_{i=1}^{n} i_i^m = -\lambda q_i \]

(23)

then node voltage is:

\[ e_i = (\lambda p_i - f_i y_i) / x_i \]
\[ f_i = \lambda p_i y_i + \lambda q_i x_i / x_i^2 + y_i^2 \]

(24)

So the node voltage instability problem does not exist to the nodes which grounded admittance is 0.

5. Case Study

(1) Test the proposed method by IEEE-30 bus system. Precision of convergence is set to 10^-5. The type of node is: 1 represents generator node, 2 represents the load node, 3 represents balanced node. Firstly, verify the single-node voltage instability mode, it is assumed that node 28 is a hypothetical voltage collapse node, increase the load, the calculated results are shown in Table 1.

Table 1. The Calculated Results of Power Flow Under One-node Voltage Collapse mode

<table>
<thead>
<tr>
<th>No.</th>
<th>Node Voltage Amplitude</th>
<th>Node Voltage Phase Angle</th>
<th>Active Power Injection</th>
<th>Reactive Power Injection</th>
<th>Node Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0913</td>
<td>-56.9983</td>
<td>0.1793</td>
<td>0.5048</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
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<td>-22.2392</td>
<td>0.1691</td>
<td>6.5130</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
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<td>-0.0351</td>
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<td>2</td>
</tr>
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<td>2</td>
</tr>
<tr>
<td>5</td>
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<td>-35.0127</td>
<td>-0.1064</td>
<td>-0.0190</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
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<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
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<td>-43.3999</td>
<td>-0.0241</td>
<td>-0.0120</td>
<td>2</td>
</tr>
<tr>
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<td>-58.1818</td>
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</tr>
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</tr>
<tr>
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<td>0.0500</td>
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</tr>
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<td>0</td>
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</table>
After 4 iterations, \( \lambda \) is 1.8581. When consider the two-node voltage instability mode (7 and 28), the calculated results are shown in Table 3.

Table 2. The Calculated Results of Power Flow Under Two-nodes Voltage Collapse Mode

<table>
<thead>
<tr>
<th>No.</th>
<th>Node Voltage Amplitude</th>
<th>Node Voltage Phase Angle</th>
<th>Active Power Injection</th>
<th>Reactive Power Injection</th>
<th>Node Type</th>
</tr>
</thead>
<tbody>
<tr>
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Table 3. The Calculated Results of Power Flow Under New England 39 Nodes Voltage Collapse Mode

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After 7 iterations, \( \lambda \) is 1.6619, node 7 is the main reason in voltage instability. The above calculation results show that the proposed method can be used to analyze power system
voltage instability, and then do the voltage security assessment. In fact, the conditions of different load types, different load growth modes, generator output constraints are all have been analyzed. The results show that the method can calculate voltage collapse point in a large range in the process of load change.

6. Conclusion
Established the hybrid power network equations composed of node voltage and branch current. Analyzed the characteristic of voltage collapse point conditions and extracted the equation from it to form the extended power flow equations. Then, the power flow calculation has been smoothly carried out near the static voltage collapse point. The following conclusions have been obtained by simulative calculation:
1) The voltage collapse point can be represented by the branch current and node voltage, its physical meaning is more intuitive, the expression is simple, so it is a suitable algorithm to calculate voltage collapse point;
2) The calculation method proposed in this paper forms the extended electricity network equations, and it can significantly improve the convergence characteristics of the power flow equations near the voltage collapse point;
3) Retains the advantages of small amount calculation and convenient storage that the other continuous power flow methods have.

Acknowledgement
This work was supported by the Youth Science Fund Project of National Natural Science Foundation of China (Grant No. 51307108).

References