Passenger Flow Forecast Algorithm for Urban Rail Transit

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Abstract
To exactly forecast the urban rail transit passenger flow, a multi-level model combining neural network and Kalman filter was proposed. Firstly, ELAN neural network model was introduced to implement a preliminary forecast of the passenger flow. Then the Kalman filter was used to correct the preliminary forecast results, so as to further improve the accuracy. Finally, in order to validate the proposed model, the passenger flow in Shanghai subway transport hub was observed and simulated. Experimental results showed that the proposed multi-level model reduced error by about 0.8% and had better actual effect compared with any single algorithm.

Keywords: rail transit, passenger flow forecast, ELAN neural network, kalman filter; system simulation

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1. Introduction
With the acceleration of urbanization construction in China, a large number of people flood into cities, which causes a rapid population growth, especially in some mega cities such as Beijing, Shanghai and Guangzhou. These cities have advanced economy, and the vehicle amounts far exceed the capacity of urban roads. Direct consequence caused by this situation is transit congestion, increasing travel cost, and serious resource wastes. Rail transit has become the most effective method to solve the urban traffic problems due to its large carrying capacity (40-60 thousand passengers an hour), less resource consumption per unit passenger volume (with electric driver system), and high punctuality rate (it has own individual track) [1].

According to statistics, average daily subway passenger flow in each big city of China has reached millions. In order to satisfy the travel demand of the large number of passengers, it is necessary to establish an efficient scheduling strategy for rail system to guarantee a high operational efficiency of rail transit. The passenger flow is the main factor dominating the scheduling policy of rail transit [2].

Passenger flow forecast algorithm directly affects the operational efficiency of rail transit, so much research was carried out by many researchers, and various algorithms were adopted to establish corresponding models. Lu S and Guo XC proposed a rail transit passenger flow forecast method based on four-step method [3]. This method mainly adopts management measure to analyze the living standard, travel characters, and development situations of traffic, so as to forecast the rail transit passenger flow and the distribution from a macroscopic perspective. Zhang DQ and Wang LN used BP neural network to forecast rail transit passenger flow [4], and took historical passenger flow data and some influence factors as training sample. Lu MX and Ye YZ, et al. applied ELAN neural network in transit passenger flow forecast, and the average error was reduced by about 2% compared with traditional BP neural network [5]. Chen DW and Xiao WZ, et al. proposed a passenger flow forecast model for urban rail transit based on iterative feedback constraint, and modified four-step method was selected as the algorithm [6].

The composition of rail transit passenger flow is complex, so previous models did not exhibit high accuracy and reliability in flowing aspects: firstly, there were not sufficient and complete experimental data, and only algorithm structure and frame were proposed theoretically. Secondly, there still exists isolation between theory and practice, how to apply
theoretical foundation to experiments was unknown, and experimental results were unconvincing.

In order to improve the forecast accuracy for rail transit passenger flow and meet actual demand, in present paper, on the basis of previous algorithms [7-13], a passenger flow forecast algorithm combining ELAN neural network and Kalman filter was developed [14].

2. Research Method

In order to forecast the passenger flow in rail transit, a mathematical model based on ELAN neural network was developed, it is a one-step prediction method. Some main influence factors for passenger flow were listed, which were taken as the training samples of the neural network. Furthermore, in order to improve the accuracy of the forecast model, Kalman filter algorithm was introduced into the model to correct the passenger flow output of the neural network [15].

2.1. Establishment of Neural Network Model

Neural network is a massive parallel distributed processing nonlinear system, and has high nonlinear computational capability, self-learning ability, self-organizing capability, associative memory ability, and parallel processing capability. The information of neural network is stored in weight coefficient in distributed pattern, and has collective computational capacity and self-learning ability. The rail transit passenger flow is a non stationary random process, and the relative time exhibits nonlinear feature, so neural network algorithm can be well applied to forecast passenger flow [16, 17].

2.1.1. ELAN Neural Network Model

For ELAN neural network model, an undertaking layer is added to the hidden layer of ordinary feedforward network as a step delay operator, so as to achieve the purpose of memory. In this way, the system owns the capacity to adapt to the time-varying characteristics, and can directly reflect the characteristics of dynamic process. Figure 1 shows the structure of ELAN neural network, it can be noticed that undertaking layers are added to input layers compared with ordinary feedforward network, and the inputs are related to the outputs of hidden layer.

![Figure 1. Structure of ELAN Neural Network](image)

The operational rule of ELAN neural network consists of following equations:

\[ H_i(k) = \left[ (C(k), X(k)) + B^i \right] W^{1,2} \]  \hspace{1cm} (1)

\[ H_o(K) = f_1(H_o(K) + B^2) \]  \hspace{1cm} (2)

\[ Y(k) = f_1(H_o(k) W^{2,3} + B^3) \]  \hspace{1cm} (3)
\[ C(k) = H_o(k - 1) \]  

(4)

Where \( Y(k) \) is network output, \( X(k) \) is network input, \( W^{i,i+1} \) is the weight matrix connecting the \( i \)th layer and the \((i+1)\)th layer neurons, \( B_i \) is the input offset of the \( i \)th layer neurons, \( C(k) \) is the input of the undertaking layer, \( H_i(k) \) and \( H_o(k) \) are the input and output of the hidden layer. \([C(k), X(k)]\) in Equation (1) denotes the linkage of \( C(k) \) and \( X(k) \).

2.1.2. Training of Neural Network

Back-Propagation Algorithm has been applied widely due to the advantages of simple algorithm and high operation speed. To train neural network using BP algorithm, following information is necessary,

\[ \{X, T, n'_i, A^i, A^\cdots A^M\} \]  

(5)

Where \( X \) is network input, \( T \) is expected output, \( n'_i \) is the local domain induced by the \( i \)th neuron in the \( j \)th layer, \( A_i \) is the output of the \( i \)th neuron, \( M \) is the layer number of neural network, and \( A^M \) is the network output. The update process of weight matrix and input offset is listed as follows:

1) Forward propagation of input

\[ A^0 = X \]  

(6)

\[ A^{i+1} = f^{i+1}(W^{i,i+1}A^i + B^{i+1}) \]  

(7)

2) Back propagation

\[ s^M = -2F^{\hat{M}}(n^M)(T - A^M) \]  

(8)

\[ s^n = F^n(n^n)(W^{n,n+1})^T s^{n+1} \]  

(9)

3) Update of weight and deviation value

\[ W^{n,n+1}(K + 1) = W^{n,n+1}(K) - \eta s^n(A^{n-1})^T \]  

(10)

\[ B^n(K + 1) = B^n(K) - \eta s^n \]  

(11)

\[ \hat{F}^n(n^n) = \begin{bmatrix} f^n(n^n) & 0 & \cdots & 0 \\ 0 & f^n(n^n) & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & f^n(n^n) \end{bmatrix} \]  

(12)

Where \( \eta \) denotes learning rate, it may be a constant, and can take different value in every sample training according to certain rules.

It is important to note that the selection of initial weights matrix may cause the converged local minimum points not to be global minimum points.

2.1.3. Selection of Training Sample

To train the neural network in a supervised way, it is necessary to select the input and output of training sample firstly. The input of training sample is various external environmental factors which affect rail transit passenger flow. The output of training sample is the observations of rail transit passenger flow. Table 1 lists the input samples selected by this algorithm, including...
weather factor, national policy, and main holidays, etc. Moreover, in order to facilitate the processing of the above-mentioned information using neural networks and computer, above information was digitized before being input into neural network as training samples, and the obtained results are shown in the second column of Table 1.

Table 1. Main Influence Factors for Rail Transit Passenger Flow and Their Digitized Results

<table>
<thead>
<tr>
<th>Factors</th>
<th>Digitized results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Days of the week</td>
<td>1~7</td>
</tr>
<tr>
<td>Date</td>
<td>Six-digit date</td>
</tr>
<tr>
<td>National policy</td>
<td>The average retail price of gasoline; 2 Large-scale activities</td>
</tr>
<tr>
<td>Holidays</td>
<td>Rest days n, the n days before rest and the n days after rest, the days of Spring Festival holiday×1.5</td>
</tr>
<tr>
<td>Weather</td>
<td>Rainfall: 0<del>4, Wind force: 0</del>12, Temperature: the average of actual values</td>
</tr>
</tbody>
</table>

2.2. Kalman Filter Model

Kalman filter is a recursive filter proposed by Kalman for time-variant linear system. In present algorithm, Kalman filter was applied to correct the output results of neural network, so as to further improve the accuracy of forecast results [18]. Kalman filter consists of following forecast and updating equations,

\[
\hat{x}_k = A\hat{x}_{k-1} + B_k u_k \\
P_k = AP_{k-1}A^T + Q_k \\
K_k = P_k H^T (H P_k H^T + R_k)^{-1} \\
\hat{x}_k = \hat{x}_k + K_k (z_k - H\hat{x}_k) \\
P_k = (I - K_k H) P_k
\]

Where Equation (13) and Equation (14) are forecast equations, Equation (15) through Equation (17) are updating equations, A is a state transition model applied to \( x_k \), B is a input control model applied to controller vector \( u_k \), \( Q_k \) is noise covariance matrix, and \( R_k \) is observed noise covariance matrix.

About the detailed derivation process of Kalman filter model and the meaning of every parameter, please see literature [19].

2.2.1. Observation Noise Variance

The Kalman filter assumes that the observation noise satisfies the Gaussian distribution,

\[
f(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2\sigma^2} (x - \mu)^2}
\]

The solution of observation noise variance can be assumed to be the estimation of unknown parameters in Gaussian distribution. According to some basic knowledge of probability, maximum likelihood estimate was adopted to estimate the unknown parameters. The maximum likelihood function of Gaussian distribution is:

\[
\text{max} L(\mu, \sigma^2) = \max \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2\sigma^2} (x_i - \mu)^2}
\]
In order to facilitate the solution of the algorithm, the maximum likelihood function was transformed to logarithmic form,

$$\ln L = -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln\sigma^2 - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i - \mu)^2$$

(20)

Get the first order and second order ordinary differential of Equation (20), and set it equal zero, then get the extreme point,

$$\frac{\partial}{\partial \mu} \ln L = \frac{1}{\sigma^2}\left[\sum_{i=1}^{n}x_i - n\mu\right] = 0$$

(21)

$$\frac{\partial}{\partial \sigma^2} \ln L = -\frac{n}{2\sigma^2} + \frac{1}{2(\sigma^2)^2}\sum_{i=1}^{n}(x_i - \mu)^2 = 0$$

(22)

By solving Equation (21) and (22), the maximum likelihood values of Gaussian distribution were estimated as:

$$\hat{\mu} = \bar{X}$$

(23)

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2$$

(24)

The average observation noise distribution function was zero, so we set $O_i$ as the passenger flow statistic on the ith day, set $P_i$ as the forecasted passenger flow on the ith day, and set the forecasted error as the average of the distribution function,

$$\hat{\mu} = \sum_{i=1}^{n}(O_i - P_i)$$

(25)

$$X_i = O_i - P_i$$

(26)

By substituting Equation (25) and Equation (26) into Equation (24), we can get the variance of observation noise distribution function. It is important to note that the obtained variance using above method is not a constant, instead, it varies as new statistics are continually added.

### 2.2.2. Determination of other Parameters

Within $n$ days, the expected ratio of observed passenger flow and forecasted one in rail transit is:

$$E = \frac{1}{n}\sum_{i=1}^{n}\frac{O_i}{P_i}$$

(27)

Gain $A$ can be described in following form:

$$A = E$$

(28)

The value of $A$ varies as new statistics added.

Set $P_0$ as the initial value of estimation error covariance $P$. The value of $P_0$ is not critical, because any value except zero can make filter to convergence. Here, we set $P_0$ equal 1. As the statistics of daily passenger flow in rail transit is performed by machine, statistical error can be gained from the observed data. In general situation, the statistical error of passenger

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flow is assumed to be nearly zero, and can be neglected. Here, we set the variance of incentive process noise $Q$ as zero.

3. Results and Discussion

In order to verify the correctness and usefulness of the proposed model, we took the rail transit of Shanghai as research object. Firstly, the historical statistical data was taken as training sample, and was input into neural network, so as to enable the network to forecast passenger flow. Then, the neural network was applied to forecast the passenger flow data in a period of time, and Kalman filter was adopted to modify the forecasted results. Finally, the error between forecasted results and actual results was calculated, and the forecasting accuracy was evaluated.

3.1. Sample Data Input

Data of the total 123 days from July to October in 2012 whereas taken as samples data. In these four months, there are season changes, important holidays, and gasoline price fluctuations, etc., which improved the flexibility of network to different situations. Some data of these four months were not issued through official channels, so they were directly discarded to guarantee the normal operation of the algorithm. The valid sample number was 117.

According to the method shown in Table 1, we took weather, date and economy situation as the training sample input, and digitized these inputs, only to get the training sample input of the neural network.

3.2. Sample Data Output

Figure 2 shows the change trend of passenger flow within sampling time, and they are the expected output of neural network training. It can be seen that rail transit passenger flow has several characteristics: 1) The change of passenger flow follows a one-week cycle. 2) As time goes on, passenger flow exhibits an increasing trend, because factors such as city development, higher gas prices, etc., induce more citizen to choose rail transit as main travel tool. 3) There is an obvious decline of passenger flow on September 29, because this day is near the National Day holiday. This phenomenon just shows the influence of important holiday on rail transit passenger flow.

3.3. Model Simulation

A function of toolbox of MATLAB was applied to the molding and simulation of ELAN neural network [20-22]. Momentum BP algorithm with variable learning rate and different parameters were selected to train the network, and training results are shown in Table 2.

<table>
<thead>
<tr>
<th>Training rounds</th>
<th>Hidden neuron</th>
<th>Expected error</th>
<th>Running time</th>
<th>Actual error</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>200</td>
<td>1e-3</td>
<td>2s</td>
<td>0.0198</td>
</tr>
<tr>
<td>300</td>
<td>400</td>
<td>1e-3</td>
<td>6s</td>
<td>0.0096</td>
</tr>
<tr>
<td>600</td>
<td>200</td>
<td>1e-3</td>
<td>4s</td>
<td>0.055</td>
</tr>
<tr>
<td>600</td>
<td>400</td>
<td>1e-3</td>
<td>12s</td>
<td>0.0012</td>
</tr>
<tr>
<td>1000</td>
<td>200</td>
<td>1e-3</td>
<td>8s</td>
<td>0.007</td>
</tr>
<tr>
<td>1000</td>
<td>400</td>
<td>1e-3</td>
<td>20s</td>
<td>0.009</td>
</tr>
</tbody>
</table>

From Table 2, it can be found that the number of hidden neuron has a great effect on the running time, and excessive hidden neurons would cause higher error. The data in the fourth line were selected as training parameters. Figure 2 shows the fitting curves of the historical passenger flow after training, in which solid lines denote actual passenger flow, and dotted lines denote passenger flow output from neural network.

Then, the passenger flow in the rail transit of Shanghai during from November 1 to 5, 2012 was selected as forecast object to check test the ELAN neural network model, and the obtained results are shown in Figure 3.
3.4. Error Correction

The forecasted results were corrected using Kalman filter, and the correction algorithm has been described in detail in the previous section. Corrected results are shown in Table 3.

<table>
<thead>
<tr>
<th>Forecasted passenger flow</th>
<th>Actual passenger flow</th>
<th>Gain of filter</th>
<th>Filter variance P</th>
<th>Corrected results</th>
</tr>
</thead>
<tbody>
<tr>
<td>661.3</td>
<td>682</td>
<td>1.01</td>
<td>1.0</td>
<td>668</td>
</tr>
<tr>
<td>698.3</td>
<td>733.4</td>
<td>1.01</td>
<td>0.99</td>
<td>706.7</td>
</tr>
<tr>
<td>588.4</td>
<td>566.7</td>
<td>0.98</td>
<td>0.99</td>
<td>578.1</td>
</tr>
<tr>
<td>510.9</td>
<td>526.9</td>
<td>0.98</td>
<td>0.98</td>
<td>505.8</td>
</tr>
<tr>
<td>700.6</td>
<td>682.8</td>
<td>0.99</td>
<td>0.98</td>
<td>692.5</td>
</tr>
</tbody>
</table>

Table 4. Error Comparison

<table>
<thead>
<tr>
<th>Actual passenger flow</th>
<th>Forecasted passenger flow</th>
<th>Forecasting error</th>
<th>Corrected passenger flow</th>
<th>Error after correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>682</td>
<td>661.3</td>
<td>-3.03%</td>
<td>668</td>
<td>-2.05%</td>
</tr>
<tr>
<td>733.4</td>
<td>698.3</td>
<td>-4.8%</td>
<td>706.7</td>
<td>-3.6%</td>
</tr>
<tr>
<td>566.7</td>
<td>588.4</td>
<td>3.8%</td>
<td>578.1</td>
<td>2.01%</td>
</tr>
<tr>
<td>526.9</td>
<td>510.9</td>
<td>-3.03%</td>
<td>505.8</td>
<td>-4.0%</td>
</tr>
<tr>
<td>682.8</td>
<td>700.6</td>
<td>2.6%</td>
<td>692.5</td>
<td>1.42%</td>
</tr>
</tbody>
</table>

It can be seen from Table 3 that the gap between forecasted data and actual data was closed after the correction. In addition, Kalman filter variance P constantly tend to be zero, which proves that the initial value of P hardly affect the calculation of filter. Table 4 shows the comparison of errors before and after correction using Kalman filter.
3.5. Results Comparison

The specific application of long-term passenger flow forecast for urban rail transit remains relatively limited. Table 5 shows the results of passenger flow forecast in different algorithms in references.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Field</th>
<th>Error</th>
<th>Reference No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>ELAN neural network</td>
<td>Subway traffic</td>
<td>5.487%</td>
<td>5</td>
</tr>
<tr>
<td>ARMA</td>
<td>Subway traffic</td>
<td>&lt;20%</td>
<td>7</td>
</tr>
<tr>
<td>Kalman filter</td>
<td>Road traffic</td>
<td>23.19%</td>
<td>10</td>
</tr>
<tr>
<td>Wavelet neural network</td>
<td>Road traffic</td>
<td>18%</td>
<td>13</td>
</tr>
</tbody>
</table>

Most of the results listed above are based on short-term forecast algorithm. That is because most of researchers contributed related methods in short-term passenger flow forecast. Moreover, a few of authors proposed their theories and methods; nevertheless, they did not illustrate experiments and their results [3, 4], [6], [8, 9]. From the conclusion of Table 5, ELAN neural network should be the better way to realize passenger flow forecast, its error is 5.487%. However, in this paper, the result was enhanced to 4.0% after data correction through Kalman filter.

4. Conclusion

It can be drawn that when the rail transit passenger flow was forecasted by only neural network algorithm, the maximum absolute error reached 4.8%. After correction using Kalman filter, the maximum absolute error was reduced to 4.0%. It was because that Kalman filter took historical error into account during calculation, thereby improving the forecasting accuracy.

In the next study, the following aspects should be considered: firstly, in order to improve the forecasting accuracy of neural network, training samples should contain more conditions, i.e. neural network need larger training sample. Secondly, the passenger flow in every station are different, hence, it should be forecasted separately, so as to realize the management of a certain station.

References


