Robust Adaptive Sliding Mode Control Based on Fuzzy Compensation for Robot

Yufeng Li1*, Kuiwu Li2, Yutian Pan1, Kelei Li1
1School of Computer and Control Engineering, North University of China, Taiyuan 030051, China
2Northwest Institute of Mechanical & Electrical Engineering, Xianyang 712099, China
*Corresponding author, e-mail: 11334671@qq.com

Abstract
Aimed at the problems of low control accuracy and weak robustness influenced by external disturbance, friction, load changes, modeling errors and other issues in ammunition auto-loading robot control system, a new robust adaptive fuzzy sliding mode controller based on fuzzy compensation is proposed. The control architecture employs fuzzy systems to compensate adaptively for plant uncertainties to distinguish different disturbance compensation terms and approximate each of them respectively. The stability of the robust adaptive fuzzy sliding mode control (SMC) and the convergence of the tracking errors are ensured by using the Lyapunov theory. By analyzing and comparing the simulation results, it is obviously shown that the control system can lighten the effect on the control system caused by different disturbance factors and eliminate the system chattering instead of traditional SMC. As a result, the control system has great dynamic features and robust stability and meets the requirement that the actual motion of ammunition auto-loading robot quickly tracks the scheduled trajectory.

Keywords: fuzzy compensation, sliding mode control, nonlinear system, uncertainty disturbance

1. Introduction
It is widely recognized that robotic robot have to face many uncertainties in their dynamics, in particular structured uncertainty, such as payload parameter, and unstructured one, such as friction and disturbance [1]. It is difficult to obtain the desired control performance when the control algorithm is only based on the robot dynamic model. To overcome this problem, an adaptive control of robot using fuzzy compensator was designed in [1], and the adaptive control schemes utilized an fuzzy logic system (FLS) as a compensator for any uncertainty. In [2] a fuzzy compensation based on computed torque controller is designed to handle inevitable uncertainties, and the fuzzy compensative controller is used for approximating lumped uncertainty. A practical and effective compensator based on adaptive fuzzy logic systems is employed to compensate the joint friction in [3-6]. The basic idea of the adaptive fuzzy logic control arises from the fact a wide class of nonlinear system can be approximated to arbitrary closeness by a fuzzy logic system. Adaptive fuzzy estimator provides a tool for making use of the fuzzy information in a systematic and efficient manner. Hsu Chun-Fei et al [7, 8] suggested the methods of robust wavelet-based adaptive neural controller design with a fuzzy compensator. But the structure of neural network and the adaptive laws have to be found by the trial-and-error method. To overcome these difficulties, in [9] a robust adaptive fuzzy control of robots based on fuzzy compensation was proposed.

In the recent decade, sliding mode control (SMC) which has good control performance for nonlinear systems, was used in robotic robot control in [10, 11], and the most significant property of SMC is its robustness. In fact, a pure SMC suffers from some disadvantages. First, there is the problem of chattering, which is the high-frequency oscillations of the controller output, brought about by the high speed switching necessary for the establishment of a sliding mode. Second, an SMC is extremely vulnerable to measure noise since the input depends on the sign of a measured variable that is very close to zero. Third, the SMC may employ unnecessarily large control signals to overcome the parametric uncertainties. To attenuate these difficulties, several methods were proposed in [12-14]. To overcome these difficulties, in this paper we propose the robust adaptive fuzzy sliding mode control schemes which utilize fuzzy logic systems as compensators for any uncertainty to alleviate the chattering and reduce...
the tracking errors, and restrain friction, disturbance, load variations and other nonlinear influencing factors.

This paper is organized as follows: Section 2 provides the dynamic model of ammunition auto-loading robot. In section 3 classical sliding mode controller for robot and its stability are given. Section 4 presents robust adaptive fuzzy sliding mode controllers with fuzzy compensation are given. Finally, experiment results and conclusions are given in section 5 and 6 respectively.

2. Dynamic Model of Ammunition Auto-loading Robot

The ammunition auto-loading robot is shown in Fig.1. Its dynamic model may be expressed in the following Lagrange form:

\[ D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + F(q, \dot{q}, \ddot{q}) = \tau \]  

Where \( q = [q_1, \ldots, q_n]^T \) is an \( n \times 1 \) vector of joint position, \( \dot{q} = [\dot{q}_1, \ldots, \dot{q}_n]^T \) is an \( n \times 1 \) vector of joint velocity, \( \ddot{q} = [\ddot{q}_1, \ldots, \ddot{q}_n] \) is an \( n \times 1 \) vector of joint acceleration, \( D(q) \) is an \( n \times n \) inertia matrix, \( C(q, \dot{q}) \) is an \( n \times n \) matrix resulting from Coriolis and centrifugal forces, \( G(q) \) is an \( n \times 1 \) gravity vector, \( \tau \) is the control input. \( F(q, \dot{q}, \ddot{q}) \) is the uncertainty generated by friction \( F_r \), load changes and the disturbance \( \tau_d \) adding on the \( \tau \).

3. Classical Sliding Mode Controller for Robot

In the design of sliding mode controller for ammunition auto-loading robot, the control objective is to drive the joint position \( q \) to the desired position \( q_d \). So by defining the tracking error to be in the following form: \( e = q_d - q \). The sliding surface can be written as: \( s = \dot{e} + \lambda e \), where \( \lambda = \text{diag}[\lambda_1, \ldots, \lambda_n] \), in which \( \lambda_i \) is a positive constant. The control objective can now be achieved by choosing the control input so that the sliding surface satisfies the following sufficient condition: \( \frac{1}{2} \frac{ds}{dt} \leq -\eta_i |s_i| \), where \( \eta_i \) is a positive constant, which indicates that the energy of \( s \) should decay as long as \( s \) is not zero. To set up the control \( \tau \), define the reference states to be: \( \dot{q}_r = \dot{q} - s = \dot{q}_d - \lambda e \), and \( \ddot{q}_r = \ddot{q} - \ddot{s} = \ddot{q}_d - \lambda \ddot{e} \). Now the control input \( \tau \) can be chosen to be in the following form:

\[ \tau = \hat{\tau} - Ks \text{sgn} s, \hat{\tau} = \hat{D}\ddot{q}_r + \hat{C}\dot{q}_r + \hat{G} - As \]
Where $K = \text{diag}[k_{11}, \ldots, k_{nn}]$ is a diagonal positive definite matrix in which $k_{ii}$'s are positive constants and $A = \text{diag}[a_1, \ldots, a_n]$ is a diagonal positive definite matrix in which $a_i$'s are also positive constants. So we have:

$$Ds + (C + A)s = \Delta f - K \text{sgn}(s)$$  \hspace{1cm} (3)

Where $\Delta f = \Delta D \ddot{q} + \Delta C \dddot{q} + \Delta G, \Delta D = \hat{D} - D, \Delta C = \hat{C} - C, \Delta G = \hat{G} - G$. It can be proved that by choosing $K$ such that $k_{ii} \geq |\Delta f|_{\text{bound}}$, where $|\Delta f|_{\text{bound}}$ is the boundary of $|\Delta f|$, the overall system is asymptotically stable.

A Lyapunov function is a scalar function $L(x)$ defined on a region $D$ that is continuous, positive definite, ($L(x) > 0$ for all $x \neq 0$), and has continuous first-order partial derivatives at every point of $D$. The derivative of $L$ with respect to the system $x' = f(x)$, written as $\dot{L}(x)$ is defined as the dot product. The existence of a Lyapunov function for which $\dot{L}(x) \leq 0$ on some region $D$ containing the origin, guarantees the stability of the zero solution of $x' = f(x)$, while the existence of a Lyapunov function for which $\dot{L}(x)$ is negative definite on some region $D$ containing the origin guarantees the asymptotical stability of the zero solution of $x' = f(x)$.

Consider a Lyapunov function candidate $L = \frac{1}{2} s^T D s$, since $D$ is symmetric and positive definite, then for $s \neq 0, L > 0$. It can be proved that:

$$\dot{L} = s^T D \dot{s} + \frac{1}{2} s^T \dot{D} s = s^T (- (C + A)s + \Delta f - K \text{sgn}(s) + Cs)$$

$$= \sum_{i=1}^{n} s_i (\Delta f_i - K_s \text{sgn}(s_i)) - s^T As \leq -s^T As \leq 0$$  \hspace{1cm} (4)

Thus, Equation (4) guarantees the decay of the energy of $s$ as long as $s \neq 0$. The sufficient condition is thus satisfied.

4. Sliding Mode Controller Based on Fuzzy Compensation

Select the Lyapunov function as: $L = \frac{1}{2} (s^T D s + \sum_{i=1}^{n} \Theta_i^T \Gamma_i \Theta_i)$, where $\Theta_i = \Theta_i^* - \Theta_i$. $\Theta_i^*$ is the desired parameter, $\Gamma_i > 0$. We have:

$$Ds = D\ddot{q} - D\dot{\dot{q}} = \tau - C\dot{q} - G - F - D\ddot{q}$$  \hspace{1cm} (5)

$$L = s^T Ds + \frac{1}{2} s^T \dot{D} s + \sum_{i=1}^{n} \Theta_i^T \Gamma_i \dot{\Theta_i} = -s^T (D\ddot{q} + C\dot{q} + G + F - \tau) + \sum_{i=1}^{n} \Theta_i^T \Gamma_i \dot{\Theta_i}$$  \hspace{1cm} (6)

Where $F(q, \dot{q}, \ddot{q})$ is unknown nonlinear function. $\hat{F}(q, \dot{q}, \ddot{q} | \Theta)$ based on the multi input-multi output (MIMO) fuzzy system is adopted to approximate to $F(q, \dot{q}, \ddot{q})$. In order to eliminate the influence of approximation error and keep stabilization, robust adaptive fuzzy sliding mode controller is designed as:

$$\tau = D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + \hat{F}(q, \dot{q}, \ddot{q} | \Theta) - K_s \dot{s} - W \text{sgn}(s)$$  \hspace{1cm} (7)
Where $K_D = \text{diag}(K_i)$, $K_i > 0, i = 1, 2, \ldots, n$, $W = \text{diag}\{w_{M_1}, \ldots, w_{M_n}\}$, $w_{M_i} \geq |a|, i = 1, 2, \ldots, n$, and:

$$
\hat{F}(q, \dot{q}, \ddot{q} | \Theta) = \begin{bmatrix} \hat{F}_1(q, \dot{q}, \ddot{q} | \Theta_1) \\ \hat{F}_2(q, \dot{q}, \ddot{q} | \Theta_2) \\ \vdots \\ \hat{F}_n(q, \dot{q}, \ddot{q} | \Theta_n) \end{bmatrix} = \begin{bmatrix} \Theta_1^T \xi(q, \dot{q}, \ddot{q}) \\ \Theta_2^T \xi(q, \dot{q}, \ddot{q}) \\ \vdots \\ \Theta_n^T \xi(q, \dot{q}, \ddot{q}) \end{bmatrix}
$$

Fuzzy approximating error is $\omega = F(q, \dot{q}, \ddot{q}) - \hat{F}(q, \dot{q}, \ddot{q} | \Theta^*)$, and we have:

$$
\dot{L} = -s^T (F(q, \dot{q}, \ddot{q}) - \hat{F}(q, \dot{q}, \ddot{q} | \Theta) + K_D s + W \text{sgn}(s)) + \sum_{i=1}^{n} \hat{\Theta}_i^T \Gamma_i \hat{\Theta}_i
$$

$$
= -s^T (\omega + K_D s) - W \|s\|^2 + \sum_{i=1}^{n} (\hat{\Theta}_i^T \Gamma_i \hat{\Theta}_i - s \hat{\Theta}_i^T \xi(q, \dot{q}, \ddot{q}))
$$

Where $\hat{\Theta} = \Theta^* - \Theta, \xi(q, \dot{q}, \ddot{q})$ is the fuzzy system. The adaptive rule is:

$$
\tilde{\Theta} = -\Gamma_i^s \xi(q, \dot{q}, \ddot{q}), i = 1, 2, \ldots, n
$$

Therefore, we have:

$$
\dot{L} = -s^T \omega - s^T K_D s - W \|s\|^2 \leq -s^T K_D s \leq 0
$$

Because the uncertainty of ammunition auto-loading robot, including friction, external disturbance, and load change exist simultaneously, we can consider the case of robust adaptive fuzzy compensation with respect to friction, external disturbance and load change. Because the load change is relative to velocity, the fuzzy system which approximate external disturbance can be written as $\hat{F}_1(q, \dot{q}, \ddot{q} | \Theta)$. In order to decrease the number of fuzzy rules, the uncertainties are decomposed and the method based on the traditional fuzzy compensation is adopted to design the controller. Therefore, the dynamic equation for ammunition auto-loading could be described a:

$$
D(q) \ddot{q} + C(q, \dot{q}) \dot{q} + G(q) + e(q, \dot{q}, \ddot{q}, t) + F_r(q) + \tau_d = \tau
$$

Where, $C(q, \dot{q}) = C(m_n, \dot{q})$; $G(q) = G(m_n, q)$; $e(q, \dot{q}, \ddot{q}, t) = e_d[D(q) \ddot{q}] + e_c[C(q, \dot{q}) \dot{q}] + e_g[G(q)]$; $e_d = D(m_n, q) \ddot{q} - D(m_n, q) \ddot{q}$; $e_c = C(m_n, \dot{q}) \dot{q} - C(m_n, \dot{q}) \dot{q}$; $e_g = G(m_n, q) - G(m_n, q)$; and $m_n$ is known nominal value and $m_n$ is actual value. The uncertain parts can be expressed as:

$$
F(q, \dot{q}, \ddot{q}) = e(q, \dot{q}, \ddot{q}, t) + F_r(q) + \tau_d
$$

The above formula can be divided into: $F(q, \dot{q}, \ddot{q}) = F^1(q, \dot{q}) + F^2(q, \ddot{q})$. Where, $F^1(q, \dot{q}) = e_c[C(q, \dot{q}) \dot{q}] + e_g[G(q)] + F_r(q) + \tau_d$, $F^2(q, \ddot{q}) = e_d[D(q) \ddot{q}]$. The robust adaptive fuzzy adaptive sliding mode controller is designed as:

$$
\tau = D(q) \ddot{q}_r + C(q, \dot{q}) \dot{q}_r + G(q) + \hat{F}^1(q, \dot{q} | \Theta^1) + \hat{F}^2(q, \ddot{q} | \Theta^2) - K_D s - W \text{sgn}(s)
$$
Where $W = \text{diag}[w_{m_1}, \cdots, w_{m_n}], w_{m_i} \geq |\omega_i^1| + |\omega_i^2|, i = 1, 2, \cdots, n$. The adaptive rule is:

$$
\dot{\Theta}^1_i = -\Gamma^{-1}_{i1} s\xi^1(q, \dot{q}), \dot{\Theta}^2_i = -\Gamma^{-1}_{i2} s\xi^2(q, \ddot{q}), i = 1, 2, \cdots n
$$

(15)

Select the Lyapunov function as:

$$
L = \frac{1}{2}(s^T Ds + \sum_{i=1}^{n} \tilde{\Theta}^T_i \Gamma_i \tilde{\Theta}^1_i) + \sum_{i=1}^{n} \tilde{\Theta}^T_i \Gamma_i \tilde{\Theta}^2_i
$$

(16)

Therefore:

$$
\dot{L} = -s^T (D\dot{q} + C\dot{q} + G + F - \tau) + \sum_{i=1}^{n} \tilde{\Theta}^T_i \Gamma_i \dot{\Theta}^1_i + \sum_{i=1}^{n} \tilde{\Theta}^T_i \Gamma_i \dot{\Theta}^2_i
$$

(17)

Fuzzy approximating error is $\omega^1 = F^1(q, \dot{q}) - \hat{F}^1(q, \dot{q} | \Theta^1)$, $\omega^2 = F^2(q, \dot{q}) - \hat{F}^2(q, \dot{q} | \Theta^2)$

Therefore,

$$
L = -s^T K_{p1}s - s^T (\omega^1 + w^2) + \sum_{i=1}^{n} \tilde{\Theta}^T_i \Gamma_i \dot{\Theta}^1_i - s\tilde{\Theta}^T_i \xi^1(q, \dot{q})) + \sum_{i=1}^{n} \tilde{\Theta}^T_i \Gamma_i \dot{\Theta}^2_i - s\tilde{\Theta}^T_i \xi^2(q, \ddot{q}))
$$

$$
= -s^T K_{p1}s - s^T (\omega^1 + w^2)
$$

(18)

The FLS is composed of four main components: a fuzzifier, a fuzzy rule base, a fuzzy inference engine and a defuzzfier as shown in Figure 2.

![Figure 2. Block Diagram of Fuzzy Logic Systems](image)

And the fuzzy system is:

$$
\hat{F}(q, \dot{q}, \ddot{q} | \Theta) = \left[ \begin{array}{c}
\hat{F}_1(q, \dot{q} | \Theta^1) + \hat{F}_x^2(q, \dot{q} | \Theta^2) \\
\hat{F}_2(q, \dot{q} | \Theta^1) + \hat{F}_x^2(q, \dot{q} | \Theta^2) \\
\vdots \\
\hat{F}_n(q, \dot{q} | \Theta^1) + \hat{F}_x^2(q, \dot{q} | \Theta^2)
\end{array} \right]
$$

(19)

5. Experiments for Ammunition Auto-loading Robot

The kinetics equation of ammunition auto-loading robot is:

$$
\begin{bmatrix}
D_1(q_2) & D_2(q_2) \\
D_3(q_2) & D_4(q_2)
\end{bmatrix}
\begin{bmatrix}
\dot{q}_1 \\
\dot{q}_2
\end{bmatrix}
+ \begin{bmatrix}
-C_1(q_2)q_2 & -C_4(q_2)q_2 - \dot{q}_2 \\
C_3(q_2)\dot{q}_1 & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{q}_1(q_2 + q_2) \\
\ddot{q}_2(q_2 + q_2)
\end{bmatrix}
+ F(q, \dot{q}, \ddot{q}) = \begin{bmatrix}
\tau_1 \\
\tau_2
\end{bmatrix}
$$

(20)
\[ D_{12}(q_2) = (m_1 + m_2) r_1^2 + m_2 r_2^2 + 2m_2 r_1 r_2 \cos(q_2), \quad D_{11}(q_2) = D_{21}(q_2) = m_2 r_2^2 + m_1 r_1 r_2 \cos(q_2), \]
\[ D_{22}(q_2) = m_2 r_2^2, \quad C_{12}(q_2) = m_2 r_1 r_2 \sin(q_2). \]
Where \( m_1 \) and \( m_2 \) are the mass of link1 and link2, and \( r_1 \) and \( r_2 \) are the lengths of link1 and link2. Let \( y = [q_1, q_2]^T, \quad \tau = [\tau_1, \tau_2]^T, \quad x = [q_1, \dot{q}_1, q_2, \dot{q}_2]^T. \)

The parameters: \( r_1 = 1m, r_2 = 0.8m, m_1 = 1kg, m_2 = 1.5kg \). The control objects to make the outputs \( q_1, q_2 \) track the desired trajectories \( y_{d1} = 0.3 \sin t \) and \( y_{d2} = 0.3 \sin t \) respectively. The membership function as shown in Figure 3 is defined as:

\[
\mu A_i'(x_i) = \exp\left(-\frac{(x_i - \bar{x}_i)^2}{\pi / 24}\right) \tag{21}
\]

Where \( \bar{x}_i \) are \(-\pi / 6, -\pi / 12, 0, \pi / 12 \) and \( \pi / 6 \), respectively, \( i = 1, 2, 3, 4, 5 \), \( A_i' \) is the fuzzy set including NB,NS,ZO,PS,PB belong to the fuzzy rule.

The control based on friction, external disturbance and load changes compensation is used for the case with friction, external disturbance and load changes, and the controller parameters are: \( \lambda_i = \lambda_2 = 10, K_D = 10I, \Gamma_1 = \Gamma_2 = 0.0001. \)

The initial states are: \( q_1(0) = q_2(0) = \dot{q}_1(0) = \dot{q}_2(0) = 0. \) The friction is

\[
F(\dot{q}) = \begin{bmatrix} 15\dot{q}_1 + 6 \text{sgn} (\dot{q}_1) \\ 15\dot{q}_2 + 6 \text{sgn} (\dot{q}_2) \end{bmatrix}, \quad \tau_d = \begin{bmatrix} 0.05 \sin(20t) \\ 0.1 \sin(20t) \end{bmatrix}, \quad W = \text{diag}[2, 2]. \]

The fuzzy sliding mode controller is given in Equation (14), and the adaptive rule is given in Equation (15). The simulation results by matlab are shown in Figure 4-Figure 8, and the sliding mode control (SMC) without fuzzy compensation and with adaptive fuzzy compensation are applied respectively for tracking control of ammunition auto-loading robot in the case with friction, external disturbance and load changes.

It is seen in Figure 4 and 5 that tracking accuracy with adaptive fuzzy compensation is more higher than that in SMC without fuzzy compensation, and tracking trajectory and desired trajectory almost coincide completely in the former.
In Figure 6 the speed trajectory without fuzzy compensation has serious chattering and great error, but there is a slight error at the beginning of the speed tracking in SMC with adaptive fuzzy compensation, then tracking effect is better. Referring to Figure 7, the control torque inputs chattering is effectively eliminated by using the adaptive fuzzy compensation which can well reduce the influence of friction, external disturbance and load change on the
system. Friction, external disturbance and load change fuzzy compensation $F_{p1}$ and $F_{p2}$ are shown in Fig. 8, and the adaptive fuzzy compensation can compensate the effect of friction, external disturbance and load changes in ammunition auto-loading robot.

![Figure 7. Control Torque Inputs of links 1 and 2](image)

(a) SMC without compensation and (b) SMC with adaptive fuzzy compensation

![Figure 8. Friction, External Disturbance and Load Change Fuzzy Compensation $F_{p1}$ and $F_{p2}$](image)

Table 1 summarizes a numerical comparison of the two control schemes, with maximum of absolute value and root-mean-square (rms) value of the tracking errors. From these comparative simulation results, it is found that the proposed control scheme is superior to the traditional SMC without compensation. Consequently, we have found that the SMC with adaptive fuzzy compensation scheme for ammunition auto-loading robot is feasible and robust to the frictions, external disturbances and load changes through the simulations.

<table>
<thead>
<tr>
<th>Type of Controller</th>
<th>Tracking Error (max)</th>
<th>Tracking Error (rms)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Joint 1</td>
<td>Joint 2</td>
</tr>
<tr>
<td>SMC without Fuzzy Compensation</td>
<td>0.0521</td>
<td>0.0252</td>
</tr>
<tr>
<td>SMC with Adaptive Fuzzy Compensation</td>
<td>0.0375</td>
<td>0.0181</td>
</tr>
</tbody>
</table>
6. Conclusion

In this paper, a sliding mode control with robust adaptive fuzzy compensation is presented to compensate the effect of friction, external disturbance and load changes on ammunition auto-loading robot. The control architecture employs adaptive fuzzy systems to compensate adaptively for plant uncertainties to distinguish different disturbance compensation terms and approximate each of them respectively. Results are compared with SMC without fuzzy compensation, which shows that fuzzy compensation is essential for obtaining low trajectory tracking errors. It is shown that the control system can lighten the effect on the control system caused by different disturbance factors and eliminate the system chattering that the traditional SMC without fuzzy compensation can not accomplish.

Acknowledgements

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References