Optimization Design of Cantilever Beam for Cantilever Crane Based on Improved GA

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Abstract

Based on in depth study of optimization design methods, according to analyzing the forces of cantilever beam for Cantilever Crane, with the feature that I-beams used in cantilever beam mostly, combining structural optimization technique with discrete variables with GA, an optimization design method of cantilever beam for Cantilever Crane based on improved GA was proposed for the problems of huge material redundancy and high production cost. Mathematical model and fitness function of the structural optimization with discrete variables were built, optimization design of cantilever beam structure was achieved, the efficiency of this optimization design method was validated and the consumption of steel for production was reduced. This method had a certain guiding significance for engineering application.

Keywords: index terms-optimization design, improved GA, discrete variables, fitness function, cantilever crane

1. Introduction

The geometry and loading conditions of Cantilever Crane are quite complex, so structural parameters are primarily determined by experience, no precise calculation of strength and safety factor is often a larger value in order to meet the reliability requirements in traditional design methods. Therefore, material waste and cost improving are produced.

At present, there are many algorithms and theories about optimum structural design. As a global optimization search algorithm, Genetic Algorithm (GA) is a general framework for solving complex optimization problems which has few limitations on searching space, no need of continuity of solutions and strong robustness. Existing research on genetic algorithm is mainly about optimization method for continuous variables. Due to cantilever beam with I-beams and discrete design variables, discrete results which obtained by adjusting the optimization methods of continuous variables and need to test the feasibility and reliability are usually infeasible solutions. Therefore, research on calculation method for optimum structural design with discrete variables has a certain practical value.

By study on optimization technique of discrete variables and improving standard genetic algorithm in this article, mathematical model and fitness function of the optimization with discrete variables have built and cantilever beam structure of Cantilever Crane has optimized by selection, crossover and mutation. Finally, structural optimization problem with discrete variables has solved.

2. Simple GA

GA is a kind of global optimization search algorithm which simulates the process of genetic and evolution in nature and proposed by Holland at University of Michigan in his paper for the first in 1975. The algorithm, firstly, encodes according to the solution of the problem, translates solution space into GA searching space and produces an initial group, then, simulates the phenomenon of breeding, crossover and genetic mutation in the process of natural selection and heredity, improves the adaptability of individual in the group through a variety of genetic manipulation operators, thereby obtains better individuals, use genetic operators (selection, crossover and mutation) to combine these individuals, produce a new
candidate group, having several generation of genetic evolution until after some convergence index is satisfied, that means get the optimal solution of the problem [1-3]. The process shows in Figure 1.

Simple genetic algorithms can be defined as a 7 - element array:

$$GA = (M, F, s, c, m, p_c, p_m)$$  \hspace{1cm} (1)

In this type, $M$ is population size, $F$ is individual fitness evaluation function, $S$ is selection operator, $C$ is crossover operator, $m$ is mutation operator, $P_C$ is crossover rate, $P_m$ is mutation rate.

![Figure 1. Simple GA Flowchart](image)

The key problems in the process of implementation as follows:

(a) Coding

Coding is the process that representing the first generation of individuals in the group with a fixed-length binary encoding string. Usually coding length is determined by the requirement of actual precision and the size of the group is generated by random method. In general, the larger the population size is, the better it is. But oversize scale will reduce computational efficiency and small size will cause algorithm convergence in advance, then optimal solution could not be obtained.

(b) Fitness function

Fitness function is a corresponding relationship between individuals and their fitness in a group. In genetic algorithm, the probability of individual heredity to the next generation of group is decided by the size of the individual fitness. The greater the individual fitness, the

![Figure 2. Schematic Diagram of Cantilever Beam Structure and Load](image)
bigger the probability that inherit to the next generation is. Otherwise, the less it is. In general, take objective function as the fitness evaluation function in the maximum problem.

(c) Genetic operator
The simple genetic algorithm usually includes three genetic operators:
1. Select operation: individuals with higher fitness selected from the current population, put into the matching set (buffer), prepared for later chromosome crossing-over, mutation and producing new individual. Whether each individual will be selected and copied to the next generation based on cumulative probability mainly. Basic operator is the choosing probability:

\[ s = \frac{F(x_i)}{\sum F(x_i)} \]

In this formula, \( x_i \) is the i-th chromosome in the population, \( F(x_i) \) is fitness function, \( \sum F(x_i) \) is the sum of all individual fitness in the population.

2. Crossover operation: Two individuals binary code were chosen from the current population and one or more bits were exchange from each other, then new individuals were produced. Take two individuals, \( A \) and \( B \), as example in the current population, two new individuals \( A' \) and \( B' \) were generated through single-point crossover.

\[
\begin{array}{c}
A & 1011011101 \\
B & 0001010111 \\
\end{array}
\xrightarrow{\text{crossover}}
\begin{array}{c}
A' & 1011011111 \\
B' & 0001010011 \\
\end{array}
\]

3. Mutation operation: take one bit of the current binary code individual as a variation point, invert gene values of it. For example, take the forth bit of individual \( C \) as mutation point, new individual \( C' \) was generated after mutation.

\[
\begin{array}{c}
C & 1001010011 \\
\end{array}
\xrightarrow{\text{mutation}}
\begin{array}{c}
C' & 1000100111 \\
\end{array}
\]

(d) Operation parameters
\( M \): population size, which is the number of individuals in the group, is 20-100 usually; \( T \): terminate evolitional generation of genetic algorithms, usually the value is 100-500; \( Pc \): crossover probability, usually the value is 0.4 -0.99; \( Pm \): mutation probability, usually the value is 0.0001-0.1.

3. Improved Genetic Algorithm of Cantilever Beam for Cantilever Crane
3.1. Cantilever Beam Structure and Stress Analysis
Taking the column cantilever crane as instance with Lifting weight \( Q=3t \), Weight rotating radius \( R=4000mm \) and Lifting height \( H=3000mm \). At present, I iron with the particular type 36 c, height \( h = 360mm \), width \( b = 140mm \) and weight is about 297kg served as cantilever beam in finalized product [4]. Cantilever beam structure and load were shown in the following figure.

In Figure 2, \( Q \) as maximum lifting load, \( G1 \) as trolley deadweight, \( q \) as cantilever beam deadweight load set, \( L \) as cantilever length, \( L1 \) as cantilever length of cantilever beam, \( N1 \) as the level reaction of point \( C \), \( N2 \) as the level reaction of point \( A \), \( P2 \) as vertical reaction of point \( A \), \( P1 \) as vertical reaction of point \( C \).

3.2. Cantilever Beam Structure Optimized Model
Due to adoption of I steel in cantilever beam, optimized possible values were discrete variables. The characteristics of discrete variable optimization problems were that the variable values were discreteness, feasible solution set was distributed mainly as scattering spots and object functions and constraint functions in the mathematical model were no longer had the continuity and differentiability, so many effective parsing algorithm in the original continuous variable optimization could not be applied [5-7].

Structure optimization mathematical model of discrete variable could be described as follows:
In this type, \( F(X) \) is the objective function, \( X \) is the design variables, \( S_i \) is discrete variables values set of \( x_i \), \( S \) is set for \( S_i(i = 1, 2, \ldots, n) \), \( g_j(X) \) is the inequality constraint function, \( h_k(X) \) is the equality constraint function.

(1) Objective Function
The cantilever beam of cantilever crane is a weldment which consists of I steel and bracket. I steel was standardized and bracket was welded with steel plate after cutting. Values of section parameters and support plate thickness parameters were the set of discrete variables and optimization design must correspond with the specifications and standards. Therefore, objective function could be rewritten as:

\[
\min F(X) = \rho_1 L x_1 + \rho_2 A x_2 \\
\text{s.t. } g_j(X) \geq 0, \quad j = 1, 2, \ldots, m \\
\quad h_k(X) = 0, \quad k = 1, 2, \ldots, p
\]

In this type, objective function \( F(X) \) is quality of cantilever beam, \( \rho_1 \) is density of I steel, \( L \) is length of I steel, \( x_1 \) is plate area of I steel, \( \rho_2 \) is density of bracket plate, \( A \) is area of steel plate, \( x_2 \) is thickness of steel plate.

(2) Constraints
Strength constraints:

\[
\sigma_i \leq [\sigma_i], \quad \sigma_2 \leq [\sigma_2]
\]

Stiffness constraint:

\[
f_1 \leq [f_1], \quad f_2 \leq [f_2]
\]

Among them: \( \sigma_1 \) is maximum stress of the cantilever beam, \( \sigma_2 \) is maximum stress of cantilever bracket, \( f_1 \) is maximum displacement of I steel, \( f_2 \) is maximum displacement of bracket.

3.3. Improved Genetic Algorithm of Discrete Variable

(1) Initial conditions and constraints
Genetic algorithm took design variables coding as operation objects and four design variables \( L, x_1, x_2, A \) were contained in the objective function according to type (6). The cantilever crane had a fixed rotating radius in the current research (i.e., \( L \) is a fixed value) and \( A \) is cross-sectional area of the bracket. Compared to cantilever beam and uprights, length and width of cantilever bracket were small, so they were seen as constant values. Because of little effect on the overall quality, \( x_2 \) was not optimized. \( x_1 \) was cross-sectional area of I-beam which can be searched in the manual \( x_1 = \{21.516, 26.131, 30.765, 35.578, 39.578, 42.128, 46.528, 48.541, 53.541, 55.404, 61.004, 67.156, 73.556, 79.956, 76.480, 83.680\} \). It can be seen that \( x_1 \) is discrete variables optimization design.

Available by the stiffness and strength of theoretical knowledge, selection of I steel should satisfy the following constraints.

\[
\frac{(G_1 + q)(L - E/2) + q(L - E/2)/2}{W} \cdot [\sigma] \leq 0
\]
Available by cantilever crane design specifications, \( f = 156 \text{MPa} \) and \( q = 36 \text{mm} \). The values of W, \( I \) were discrete numerical and depend on iteration values of cross-sectional area of I-beam. Because the support plate located in constraints side, its stress and deformation were small, so constraint can be omitted.

(2) Encode When design variables are discrete numerical
In the discrete variable structural optimization of genetic algorithm, the length of binary coded depended on the number of discrete variable number. Each individual adopt hybrid coded method which included binary coded, decimal integer coded (serial number of discrete variable values) and real-coded (discrete variable values). Binary coded was used in crossover and mutation operations. Coded relationship was shown in Table 1.

<table>
<thead>
<tr>
<th>Table 1. Hybrid Coded Comparison Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>binary coded</td>
</tr>
<tr>
<td>0000</td>
</tr>
<tr>
<td>0001</td>
</tr>
<tr>
<td>0010</td>
</tr>
<tr>
<td>...</td>
</tr>
<tr>
<td>1111</td>
</tr>
</tbody>
</table>

(3) Fitness function
Simple genetic algorithm usually used for solving the problem of unconstrained maximum principle, generally took objective function as fitness function and fitness value was required to be greater than zero. But cantilever beam optimize in the article belonged to solving the problem of minimum principle with constrains. So some methods were adopted to transform general fitness function to another kind of fitness function which included constrained. Refusal strategies were used here.

\[
F(X) = \begin{cases} 
C_{\text{max}} - (p_1X_1 + p_2X_2) & \text{if } (\sigma_1 \leq \sigma_1^r \text{ and } \sigma_2 \leq \sigma_2^r \text{ and } f_1 < f_1^r \text{ and } f_2 < f_2^r) \\
0 & \text{if } (\sigma_1 > \sigma_1^r \text{ or } \sigma_2 > \sigma_2^r \text{ or } f_1 > f_1^r \text{ or } f_2 > f_2^r)
\end{cases}
\]

In this formula, \( F(X) \) is fitness function, \( C_{\text{max}} \) is a larger given constant.

(4) Genetic operators
Selection operator: Fitness value proportion select method was adopted and accomplished by roulette wheel method in selection.
In this way, firstly, individual fitness was calculated, and then the proportion of individual fitness in the total population’s fitness was counted which represented the chosen probability in the process of selection. Whether the individual was selected or not was determined by a random number, so as to ensure the good genes transmitted to the next generation of individuals.

Crossover operator: Due to the fewer optimal design variables and the smaller design variables’ string length, the crossover probability was 100 \% (It means that all the chromosomes were involved in the crossover operation) in this optimal design, the intersection was generated randomly.

Mutation operator: Variation was an important method to prevent precocious, the number of variation digits were determined by the mutation probability, the length of encoding string and the population size, and then do mutation operation.

(5) Operation parameters
The population size \( M = 50 \), the genetic algebra is 50, the crossover rate \( P_c = 100 \% \), the mutation rate \( P_m = 0.001 \), \( C_{\text{max}} = 100 \). The known parameters of cantilever:
- the lifting capacity \( Q = 3t \), \( R = 4000 \text{mm} \), \( H = 3000 \text{mm} \), the type of I-beams is 36c, \( h = 360 \text{mm} \), \( b = 140 \text{mm} \), \( x_1 = 90.88 \text{cm}^2 \), \( G_T \) is the trolley’s self-weight, the cantilever’s self-weight load concentration \( q = 31.069 \text{kg/m} \), the cantilever’s total length \( L = 4300 \text{mm} \), the cantilever length \( L_1 = 3000 \text{mm} \), the scaffold material is Q235, the cross-sectional area \( A = 1462.5 \text{cm}^2 \), the thickness of steel plate \( x_2 = 26 \text{mm} \), \( [\sigma_1] = 156 \text{MPa} \), \( [f_1] = 1 \text{mm} \).
3.4. The Optimization Decisions

This paper has took the cantilever’s weight as objective function with constraints that the maximal displacement was not more that allowable values, the maximum stress was less than 156Mpa and the minimum safety factor was less than 1.5. Then, the cantilever had optimized. The convergence result was shown in Figure 3. The objective function began to converge to the optimal solution after 61th generation; the target value was 236.9Kg.

The stress and displacement before and after optimization were shown in Table 2.

<table>
<thead>
<tr>
<th>design variables</th>
<th>weight</th>
<th>maximum displacement</th>
<th>maximum stress</th>
<th>Minimum safety factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>before</td>
<td>90.880</td>
<td>26 287.3 Kg</td>
<td>13.8 mm</td>
<td>92.3 MPa</td>
</tr>
<tr>
<td>after</td>
<td>73.556</td>
<td>24 236.9Kg</td>
<td>15.3mm</td>
<td>106.5MPa</td>
</tr>
</tbody>
</table>

4. Conclusion

Through the global optimization ability of genetic algorithm, the traditional algorithm in the discrete space optimization problems was solved. Taking cantilever crane cantilever beam structure as an example, the cross-sectional area was taken as the design variables, the minimum weight was taken as the objective function, and the discrete variable optimization design based on genetic algorithm was carried out. According to the data which were shown in table 2, the optimized quality is about 20% lighter than before, the utilization rate of the material has improved, the size of the structure parts has reasonably matched and the research provided a theoretical basis for the lightweight optimization design of the cantilever crane.

References