Adaptive Fuzzy Sliding Mode Control for Hydraulic Servo System of Parallel Robot

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Abstract

The hydraulic servo control system, which is the important component of parallel robot, is a high order, nonlinear, dynamic asymmetry, parameter uncertain system, which seriously affect dynamic performance of robot, so it is very difficult to gain good performance with traditional control. Based on sliding mode variable structure control method combined with fuzzy control and adaptive control, an adaptive fuzzy sliding mode control algorithm was presented by taking the Stewart platform servo control system of parallel robot as the research object, and the uncertain term of the sliding mode controller was approximated with adaptive fuzzy control method. The simulation results showed that compared with the conventional PID control applied to the same hydraulic control system, the new controller has a strong robustness, good traceability with regard to model uncertainties, unknown external disturbances and changes in the operation conditions, as well as much better adaptive characteristics. The simulation results also showed that the adaptive fuzzy sliding mode control system can solve the dynamic asymmetrical performance and poor stability of parallel robot, and can greatly improve the robot control precision.

Keywords: parallel robot, Stewart platform, hydraulic servo system, adaptive fuzzy sliding mode control

1. Introduction

Serious problems like environmental pollution, energy shortage, are increasing serious, therefore, energy conservation and emission reduction technology naturally draw more and more attention. Robots can not only reduce the waste of resources, but also effectively reduce the energy loss, and gradually have attracted the attention of the government and research institutes. Since Hunt, a renowned Australian institutional professor puts forward the theory of using 6 degrees of freedom Stewart platform [1] (as shown in Figure 1) as a robot mechanism in 1978, the parallel robot, as a new kind of robot, has been attracting much attention of many scholars. The Stewart platform is a parallel mechanism composed of six same hydraulic servo systems [2]. Compared with traditional serial mechanism, it is qualified with better stiffness, carrying capacity, location accuracy and smaller working space. Therefore, it can reduce the static errors (owing to its high rigidity) and the dynamic error (owing to its low moment of inertia) when applied to the robot. Because of the viscosity of the oil, the friction between the cylinder and piston, nonlinear servo valve flow and time-varying system parameters, Stewart platform often uses the symmetrical valve controlling asymmetrical cylinder [3-4]. Stewart platform hydraulic control system tends to be high order system with highly nonlinear, parameter uncertainties, which seriously affect the dynamic performance of the Stewart platform hydraulic control system. And the nonlinear and parameter uncertainties make the system dynamic characteristics complex, as it is difficult to establish accurate mathematical model and the traditional control algorithm is difficult to perform satisfactory controlling effect [5-6].

It is hard to implement the trajectory tracking control of the parallel robot steadily, precisely and quickly. Literature [7] stated that the trajectory tracking performance of sliding mode control was better than that of the fuzzy control. Sliding mode variable structure control for the system parameter perturbation and external disturbances has strong robustness, so it provides a good solution to complicated nonlinear system controlling problem [8]. Fuzzy control is set up based on fuzzy reasoning, and it is unnecessary to establish a precise mathematical model. It is a good way to solve uncertain systems, which has been presented and applied [9]. The research object of adaptive control theory can also be aimed at uncertain systems, which is
a kind of nonlinear control method. So we can design an adaptive fuzzy sliding mode control system with more superior performance through using the reasoning abilities of fuzzy control, the learning ability of adaptive control and the rapidity of sliding mode control [10-11].

![Stewart Platform](image1)

In this paper, the adaptive fuzzy sliding mode control algorithm on the basis of adaptive control algorithm combining fuzzy control and sliding mode control was proposed for the hydraulic servo system, and the uncertainties of the sliding mode controller was fuzzy approximated by using the adaptive fuzzy control method [12-14]. We carried on a simulation research about the single channel valve control unsymmetrical hydraulic cylinder of the hydraulic position servo system of the Stewart platform of parallel robot. Compared with the conventional PID control, adaptive fuzzy sliding mode control method not only has better robustness and tracking ability, but also has a good adaptive ability about the variability of parameter and external disturbance. It meets the requirements of dynamic and steady state index, and has good trajectory tracking precision and the ability to suppress crosslink coupling load.

2. Mathematical Model

The researched hydraulic servo system was consisted of servo valve, hydraulic motor and load and so forth. The schematic diagram of valve-controlled hydraulic motor was showed in Figure 2, which has composite load with the mass, damping and the spring, and the system’s external leakage was ignored [15].

![Schematic Diagram of Valve-controlled Hydraulic Motor](image2)

In order to establish mathematical model of the hydraulic system, the following assumptions were made [2]:

1) The four throttling windows of servo valve were matching and symmetrical, the supply oil pressure was constant; the return oil pressure was zero.
2) Connecting pipeline was short and thick, friction loss in pipeline, the impact of fluid mass, and pipeline’s dynamics were ignored.
3) The flowing state of the internal and external leakage was laminar flow.
4) The pressure of liquid chamber in the hydraulic motor was equal. Bulk modulus and oil temperature were constant.
5) When the hydraulic cylinder piston rod was protruding, the servo valve core was shifting right, namely of $x > 0$.

Under the above assumptions, we can get the dynamic equation of the hydraulic system.

$$
\begin{align*}
Q_L &= K_v x_v - K_v P_L \\
Q_i &= D_m \frac{d\theta_m}{dt} + C\omega P_L + \frac{V_m}{4\beta_r} \frac{dP_i}{dt} \\
T_i &= D_m P_L + J \frac{d^2\theta_m}{dt^2} + B_m \frac{d\theta_m}{dt} + G\theta_m + T_l
\end{align*}
$$

Where $Q_L$ is load flow of the valve, $x_v$ is spool displacement, $K_v$ is flow-pressure coefficient of valve, $P_L$ is load pressure, $K_v$ is flow enhancement of valve on steady working-point, $D_m$ is theoretical displacement of hydraulic motor, $\theta_m$ is angular displacement of hydraulic motor, $C\omega$ is total leakage coefficient of hydraulic motor, $V_m$ is total volume of connected pipes, Hydraulic motor and valve chamber, $\beta_r$ is effective bulk modulus of working oil, $J$ is the total inertia (the inertia between spool of Hydraulic motor and load (excluding oil) transfer into the motor shaft), $B_m$ is viscous damping coefficient of load and motor, $G$ is torsion spring stiffness of load, $T_i$ is external load torque loaded on the motor shaft.

After Laplace transformation for (1), we can obtain:

$$
\begin{align*}
Q_L(s) &= K_v x_v (s) - K_v P_L (s) \\
Q_i(s) &= D_m s \theta_m (s) + C\omega P_L (s) + \frac{V_m}{4\beta_r} P_L (s) \\
T_i &= D_m P_L (s) = J s^2 \theta_m (s) + B_m s \theta_m (s) + G\theta_m (s) + T_l (s)
\end{align*}
$$

From (2), we can obtain the transfer function:

$$
\frac{\theta_m}{x_v} = \frac{\omega_n^2 K_v D_m}{s^2 + 2\omega_n \xi_s s + \omega_n^2 s}
$$

Where $\omega_n$ is natural frequency of hydraulic system, $\xi_s$ is hydraulic damping ratio.

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Figure 3. Control Structure of Hydraulic Servo System
The structure of hydraulic servo system is shown in Figure 3. From the figure we know that the system is consisted of electro-hydraulic servo valve, servo amplifier, hydraulic motor, sensors and so forth.

In order to simplify the servo system, supposed that the system is only inertia loads, while the elastic load, viscous load were 0, and the dynamic characteristics of servo valve, amplifiers and other electrical components are ignored. The transfer function of servo system is simplified as:

\[ \frac{\omega_m}{x_v} = \frac{\omega_h^2 K_u D_m}{s^2 + 2\omega_h \zeta s + \omega_h^2} \]  

By (4), the status equation can be written as:

\[
\begin{align*}
\dot{x} &= Ax + B\omega_m \\
y &= Cx 
\end{align*}
\]  

Where:

\[ x = [x_1 \ x_2]^T, \dot{x} = [\dot{x}_1 \ \dot{x}_2]^T, A = \begin{bmatrix} 0 & 1 \\ -\omega_h^2 & -2 \zeta \omega_h \end{bmatrix}, B = \begin{bmatrix} 0 & k_x \omega_h^2 / D_m \end{bmatrix}^T, C = [1 \ 0] \]

3. Design of Controller

Consider the following SISO uncertainty system:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f(x, t) + g(x, t)u(t) + d(x, t) \\
y &= x 
\end{align*}
\]  

Where \( \dot{x} = [x_1 \ x_2] = [x_1 \ \dot{x}_1] \) is the system’s state vector, \( u(t) \) is the control input of the system, \( y \) is the output, \( d(x, t) \) is a bounded disturbance, \( f(x, t) \) and \( g(x, t) \) are the unknown nonlinear function, and \( g(x, t) > 0, |d(x, t)| \leq D \), \( D \) is the upper bound function.

Assuming \( f(x, t) = \hat{f}(x, t) + \Delta f(x, t) \), \( \hat{f}(x, t) \) is the estimated value of \( f(x, t) \), \( \Delta f(x, t) \) is the uncertainty of model.

The tracking error vector is defined:

\[ e^T = [e_1, e_2]^T = [x_1 - x_d, \dot{x}_1 - \dot{x}_d]^T = [\dot{e}, e]^T \]  

The nonlinear equation is obtained:

\[
\begin{align*}
\dot{\dot{e}}_1 &= e_2 \\
\dot{\dot{e}}_2 &= f(e + x_d, t) + g(e + x_d, t)u(t) + d(e + x_d, t) - \ddot{x}_d(t) 
\end{align*}
\]  

The sliding mode switching function is selected as:

\[ s(t) = \lambda e \]
Where \( \lambda = [\lambda_1, 1] \) (\( \lambda_1 > 0 \)), the correlation coefficient satisfies the Hurwitz polynomial, namely \( \dot{s}(t) = s(t) = 0 \). So we obtain the sliding mode surface equation as:

\[
s(t) = \lambda_1 e_1 + e_2 = 0
\] (10)

Namely,

\[
\dot{s}(t) = \lambda_1 \dot{e}_1 + f(e + x_d, t) + g(e + x_d, t)u(t) + d(e + x_d, t) - \ddot{x}_d(t) = 0
\] (11)

So the sliding control law is designed as:

\[
u(t) = \frac{-1}{g(e + x_d, t)}[\dot{\lambda}_1 \dot{e}_1 + f(e + x_d, t) + d(e + x_d, t) - \ddot{x}_d(t) - u_{sw}]
= \frac{-1}{g(e + x_d, t)}[\dot{\lambda}_1 \dot{e}_1 + f(e + x_d, t) + d(e + x_d, t) - \ddot{x}_d(t) - u_{sw}]
\] (12)

Where \( u_{sw} = k \text{sgn}(s) \), \( k > 0 \), \( \text{sgn}(s) = \begin{cases} 1, & s > 0 \\ 0, & s = 0 \\ -1, & s < 0 \end{cases} \)

Then we can obtain:

\[
\dot{s}(t) = -u_{sw} = -k \text{sgn}(s)
\] (13)

Then,

\[
\dot{s}(t) \cdot s(t) = d(t)s - k|s| \leq 0
\] (14)

But in the actual servo system, \( f(x, t), g(x, t), d(t) \) are unknown, the control law \( u(t) \) cannot be implemented in practical applications. Especially when the disturbance item \( d \) is larger, the switch gain \( k \) of controller will be also increased, which will cause chattering. In order to reduce the chattering, using the fuzzy system to approximate the control law \( u(t) \) is proposed in this paper, namely \( \hat{f}, \hat{g}, \hat{h} \) approximates \( f, g, u_{sw} \). In the fuzzy system of approximating equivalent control, the \( \dot{s}(t) \) and \( s(t) \) are selected as input variables, then the output \( u(t) \) of fuzzy control is controlled by fuzzy rules of fuzzy controller. The fuzzy rules are given in the following form [5]:

\[
R^{(i)}: IF \dot{s}(t) is A^j_1 and s(t) is A^j_2 THEN u(t) is B^j
\]

Based on the adopting of the single value fuzzification, the product inference engine and the center average defuzzifier, the output of fuzzy system is given as:

\[
y(x) = \frac{\sum_{i=1}^{m} y^i \left( \prod_{j=1}^{n} \mu A^i_j (x_j) \right)}{\sum_{i=1}^{m} \left( \prod_{j=1}^{n} \mu A^i_j (x_j) \right)}
\] (15)
Where $\mu A_i^j(x_i)$ is the membership function of $x_i$, $\xi(x)$ is introduced vector, namely 
$y(x)=\theta^T \xi(x)$, where $\theta=[y_1 \cdots y_m]^T$, $\xi(x)=[\xi^1(x) \cdots \xi^m(x)]$.

\[
\xi(x) = \frac{\prod_{j=1}^{n} \mu A_i^j(x_i)}{\sum_{j=1}^{m} \prod_{i=1}^{n} \mu A_i^j(x_i)} \tag{16}
\]

Assuming $\hat{f}(x | \theta_f), \hat{g}(x | \theta_g), \hat{h}(s | \theta_h)$ are selected as above fuzzy logic system, then:

\[
\hat{f}(x | \theta_f) = \theta_f^T \xi(x), \hat{g}(x | \theta_g) = \theta_g^T \xi(x), \hat{h}(s | \theta_h) = \theta_h^T \phi(s) \tag{17}
\]

The $\hat{f}, \hat{g}, \hat{h}$ are replaced by $f, g, h$, so the control law turns into the fuzzy vector of $\xi(x)$ and $\phi(s)$. The $\theta_f^T, \theta_g^T, \theta_h^T$ are changed based on the change of adaptive law. Where $\hat{h}(s | \theta_h^*) = k_s \text{sgn}(s)$, $k_s = D + k, k > 0$.

The adaptive law is designed as:

\[
\begin{align*}
\dot{\theta}_f &= r_s \xi(x) \\
\dot{\theta}_g &= r_s \xi(x)u(t) \\
\dot{\theta}_h &= r_s \phi(s)
\end{align*} \tag{18}
\]

The optimal parameter is defined as:

\[
\begin{align*}
\theta_f^* &= \arg \min \{ \sup_{x \in \mathbb{R}^n} | \hat{f}(x | \theta_f) - f(x, t) | \} \\
\theta_g^* &= \arg \min \{ \sup_{x \in \mathbb{R}^n} | \hat{g}(x | \theta_g) - g(x,t) | \} \\
\theta_h^* &= \arg \min \{ \sup_{x \in \mathbb{R}^n} | \hat{h}(s | \theta_h) - u_{sw} | \}
\end{align*} \tag{19}
\]

Where $\Omega_f, \Omega_g, \Omega_h$ are the set of $\theta_f, \theta_g, \theta_h$.

The Lyapunov function is selected as:

\[
V = \frac{1}{2} (s^2 + \frac{1}{r_1} \phi_f^T \phi_f + \frac{1}{r_2} \phi_g^T \phi_g + \frac{1}{r_3} \phi_h^T \phi_h) \tag{20}
\]

Where $r_1, r_2, r_3$ are the positive constant, $\phi_f = \theta_f^* - \theta_f$, $\phi_g = \theta_g^* - \theta_g$, $\phi_h = \theta_h^* - \theta_h$. Then:

\[
\begin{align*}
\dot{V} &= ss + \frac{1}{r_1} \phi_f^T \phi_f + \frac{1}{r_2} \phi_g^T \phi_g + \frac{1}{r_3} \phi_h^T \phi_h \\
&= s \phi_f^T \xi(x) + \frac{1}{r_1} \phi_f^T \phi_f + s \phi_g^T \xi(x)u(t) + \frac{1}{r_2} \phi_g^T \phi_g \\
&+ s \phi_h^T \phi(s) + \frac{1}{r_3} \phi_h^T \phi_h + s(d(t) - \hat{h}(s | \theta_h^*)) + sw
\end{align*} \tag{21}
\]
Where \( w = f(x, t) - \hat{f}(x | \Theta^*_v) + (g(x, t) - \hat{g}(x | \Theta^*_g))u \) is the approximate error. Because of \( \hat{h}(s | \Theta^*_v) = \eta_h \text{sgn}(s) \), we can obtain as:

\[
\dot{V} = \frac{1}{r_1} \varphi_{f_i}^T[r_i s \xi(x) + \dot{\varphi}_{f_i}] + \frac{1}{r_2} \varphi_{g_i}^T[r_i s \xi(x)u(t) + \dot{\varphi}_{g_i}]
\]
\[
+ \frac{1}{r_3} \varphi_{h_i}^T[r_i s \phi(s) + \dot{\varphi}_{h_i} + sd(t) + sw - (D + \eta)|s|]
\]
\[
\leq \frac{1}{r_1} \varphi_{f_i}^T[r_i s \xi(x) - \dot{\varphi}_{f_i}] + \frac{1}{r_2} \varphi_{g_i}^T[r_i s \xi(x)u(t) - \dot{\varphi}_{g_i}]
\]
\[
+ \frac{1}{r_3} \varphi_{h_i}^T[r_i s \phi(s) - \dot{\varphi}_{h_i}] + sw - \eta|s|
\]

Based on the adaptive law, we can also obtain \( \dot{V} \leq sw - \eta|s| \). According to the principle of universal fuzzy approximation, the \( w \) may be very infinitesimal, so we can obtain \( \dot{V} \leq 0 \). Namely it is proved that the control system is stable in the sense of Lyapunov.

4. Simulation and discussion

In order to verify the effectiveness of the controller and get the accurate simulation result, MATLAB is used to make simulation for the hydraulic servo system. According to looking up the relevant parameter of hydraulic servo system, the transfer function of hydraulic servo system is given as [5, 16]:

\[
G(s) = \frac{891572.069544}{s^2 + 81.978s + 18667.7569}
\]

The status equation is given as:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
-18667.7569 & -81.978
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix} 0 \\
1
\end{bmatrix} \omega_{in}
\]

(24)

Where the initial state value of system is \( [0 \ 0] \), the adaptive parameter is selected as \( r_1 = 0.5, r_2 = 0.1, r_3 = 1 \), the output signal is given as \( x_d = 0.2 \cos(\pi t) \), the switching function is given as \( s(t) = \lambda_1 e_1 + e_2, \lambda_3 = 5 \).

The sliding mode control with the following five kinds of membership function approximates the \( u(t) \), namely:

\[
\begin{aligned}
\mu_{PM}(s) &= \exp[-((s - \pi / 6) / (\pi / 12))^2] \\
\mu_{PS}(s) &= \exp[-((s - \pi / 12) / (\pi / 12))^2] \\
\mu_{ZD}(s) &= \exp[-(s / (\pi / 12))^2] \\
\mu_{NS}(s) &= \exp[-((s + \pi / 12) / (\pi / 12))^2] \\
\mu_{NM}(s) &= \exp[-((s + \pi / 6) / (\pi / 12))^2]
\end{aligned}
\]

(25)

The membership function of switching function is defined as:
\[ \mu_{PM}(s) = \frac{1}{1 + \exp(5(s + 3))} \]
\[ \mu_{ZO}(s) = \exp(-s^2) \]
\[ \mu_{NM}(s) = \frac{1}{1 + \exp(5(s - 3))} \]

(26)

With the same variation of parameter and external load, the simulations were made by using the adaptive fuzzy sliding mode control and traditional PID control. The results were showed from Figure 4 to Figure 9. Compared with the conventional PID control algorithm, the
system with adaptive fuzzy sliding mode control fast completed the convergence in 1.5 seconds, thus ensured the better tracking effect, which fully incarnates its higher response speed and better control precision. We can also find that the adaptive fuzzy sliding mode controller effectively weaken the chattering phenomenon, which made the external disturbance and parameter perturbation of the servo system predicted and compensated effectively. Moreover, the traditional PID control method existed obvious oscillation phenomena in the tracking phase, which may result instability and oscillation phenomena even causing resonance and damage the robot during working time. Thus the adaptive fuzzy sliding mode controller was more suitable to control the parallel robot.

5. Conclusion

Aiming at the highly nonlinear, load sensitivity, parameter uncertainty and time-varying load coupling interference of channels in the Stewart platform of hydraulic control system, an adaptive control algorithm combining fuzzy control with sliding mode control is proposed for the hydraulic servo system. Compared with the conventional PID control algorithm, the above proposed method can guarantee the better robust tracking performance and parameter perturbation of the hydraulic servo system without needing an accurate model. Moreover, the external disturbance is compensated effectively. The simulation results also show that the problem of chattering is inhibited obviously, and the ability of anti-interference, anti-parameters perturbation, stability and control quality of the system are improved effectively. At the same time, the scheme is simple and suitable for engineering application.

References