Research on Warehouse Target Locating and Tracking Based on EKF and UKF

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Abstract
This paper presents a new locating and tracking method based on WSN, EKF and UKF. The principle of Location and tracking is applying maximum likelihood estimation algorithm of multilateral measurement method to calculating the coordinates of the unknown node. According to monitoring motion trajectory of the same unknown target node within a continuous period of time, the motion equation can be established. When the state equation of warehouse target tracking system is non-linear, EKF and UKF filtering algorithm are respectively applied to acquiring the state estimate of the warehouse target motion equation, so as to achieve the effective tracking of warehouse target.

Keywords: WSN, EKF, UKF, Locating, Tracking

1. Introduction
Recent advancements of micro sensors technology have allowed a wide range of Wireless Sensor Networks (WSN) implementations to be realized. WSN are used in monitoring and detecting information of detection objects in many types of industrial and military environments. The information is sent to the gateway node, in order to realize monitoring and tracking the target in a certain area. Warehouse target localization and tracking system consists of many wireless sensor nodes, those nodes include beacon nodes and unknown nodes. Beacon nodes can obtain their precise location by carrying BDS (GPS) positioning equipment. Beacon nodes are the reference points of unknown node location [1].

The unknown node can be personnel, vehicles, operation machinery and other mobile nodes. By communicating with nearby beacon nodes or the unknown nodes which have acquired their own position information, the unknown nodes can calculate their own position according to a certain location algorithm. The paper firstly established the warehouse target locating and tracking system model based on WSN, maximum likelihood estimation algorithm of multilateral measurement method is applied to calculating the coordinates of the unknown node. According to monitoring motion trajectory of the same unknown target node within a continuous period of time, the motion equation can be established. When the state equation of warehouse target tracking system is non-linear, EKF and UKF filtering algorithm are respectively applied to acquiring the state estimate of the warehouse target motion equation, so as to achieve the effective tracking of warehouse target.

2. Background
2.1. Problem Statement
Warehouse target locating and tracking system based on WSNs is composed of many wireless sensor nodes. These nodes contain beacon nodes and unknown nodes. The proportion of beacon nodes in the network is small. Beacon nodes can get their own precise location by some means such as carrying BDS (GPS) positioning equipments. Beacon nodes are the reference points of unknown nodes location. And beacon nodes are arranged inside and outside the warehouse evenly. The unknown nodes can be active nodes of personnel, vehicles and warehouse equipments. By communicating with nearby beacon nodes or the unknown
nodes which have acquired their own position information, the unknown nodes can calculate their own position according to a certain location algorithm. As is shown in Figure 1.

![Figure 1. Schematic Diagram of Warehouse Target Location and Tracking System](image)

2.2. Design Analysis of Warehouse Target Localization and Tracking System Based on WSN

The basic thought of wireless sensor networks localization can be represented as follows. Some special nodes which occupy a certain proportion are deployed in the wireless sensor networks. This kind of nodes which are called beacon nodes also have strong energy and can be equipped with BDS(GPS) positioning system, or can acquire their own coordinates by other ways. By measuring the distance and angle between unknown nodes and beacon nodes, or doing certain calculation according to the relative position relationship, their own coordinates can be worked out [2].

The positioning principle of warehouse target localization and tracking system is to calculate the coordinates of the unknown nodes with node position calculation method. The maximum likelihood estimation method of the multilateral measurement is used to calculate the coordinates of the unknown nodes [1].

The multilateral measurement method is often applied to calculating the coordinates of the unknown nodes. As shown in Fig.2, there are n reference nodes \( M_1(x_1,y_1), M_2(x_2,y_2), \ldots, M_n(x_n,y_n) \), and their distance to unknown node N is respectively \( r_1(t), r_2(t), \ldots, r_n(t) \). We set the coordinates of N as \( (x(t), y(t)) \). Then the coordinates satisfy Equation (1).

![Figure 2. Schematic Diagram of Maximum Likelihood Estimation algorithm of Multilateral Measurement Method](image)

Equation (2) can be acquired by the maximum likelihood estimation method.

\[
\begin{align*}
(x(t) - x_1)^2 + (y(t) - y_1)^2 &= r_1^2(t) \\
&\ldots \\
(x(t) - x_n)^2 + (y(t) - y_n)^2 &= r_n^2(t)
\end{align*}
\]
Applying system of linear equations, it can be expressed as \( A \mathbf{X}(t) = b(t) \), where:

\[
A = \begin{bmatrix}
2(x_1 - x_n) & 2(y_1 - y_n) \\
\vdots & \vdots \\
2(x_{n-1} - x_n) & 2(y_{n-1} - y_n)
\end{bmatrix}
\]

\[
b(t) = \begin{bmatrix}
x_1^2 - x_n^2 + y_1^2 - y_n^2 + r_{11}^2(t) - r_{nn}^2(t) \\
\vdots \\
x_{n-1}^2 - x_n^2 + y_{n-1}^2 - y_n^2 + r_{n-1,n}^2(t) - r_{nn}^2(t)
\end{bmatrix}
\]

\[
\mathbf{X}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}
\]

Where \( x(t) \) represents coordinates of unknown node \( N \) on \( x \) direction at time \( t \);

\( y(t) \) represents coordinates of unknown node \( N \) on \( y \) direction at time \( t \);

\( r_{ij}(t) \) represents the distance from reference node \( M_i \) to unknown node \( N \) at time \( t \).

The coordinates of the node \( N \) can be acquired by standard minimum mean variance estimation method.

The coordinates is:

\[
\hat{\mathbf{X}}(t) = (A^T A)^{-1} A^T b(t)
\]  

Warehouse target tracking based on wireless sensor networks is applying wireless sensor network to monitor, identify and track moving warehouse target within the monitoring area. Warehouse target tracking system can monitor motion characteristics information of warehouse target and warehouse target attribute information in real time. From the acquiring process of position and velocity of the mobile targets, it must be that multiple sensor nodes work together to complete the target tracking.

Figure 3. Schematic Diagram of Warehouse Target Tracking Principle
Warehouse target tracking is essentially a estimate problem of hybrid system, in other words, it is applying discrete sensor measurements to estimate the target's continuous state, the principle is shown in Figure 3. Residual is the difference between the sensor measurement and the state prediction value. According to changes of the residual vector, the maneuver detection and maneuver identification are conducted. In accordance with certain criteria or logic, the motion characteristics of the targets can be identified in real time. By the filtering algorithm the value of state estimate and prediction of the target can be obtained. Therefore, the essential factors of the warehouse target tracking consist of the modeling of warehouse target motion model, target locating, target recognition and filtering algorithms.

3. The Warehouse Target Tracking Research Based on EKF and UKF

Currently, the most common nonlinear filtering algorithm is Extended Kalman filter (EKF) and Unscented Kalman filter (UKF). EKF has a few disadvantages, including that first-order linearization accuracy is low and it need to calculate the Jacobian matrix of nonlinear function, which easily cause that EKF numerical stability is poor and even diverging. UKF is based on UT transformation, which apply the framework of Kalman filter and UT transform to dealing with the nonlinear transmission of the mean and covariance for one step prediction equation, rather than apply approximating nonlinear function. It need not to calculate the Jacobian matrix by derivation.

3.1. Extended Kalman Filter (EKF) of Nonlinear Discrete System

Computing model of EKF is as follows:

(1) Prediction equation

\[ \hat{x}_{k|k-1} = \Phi \hat{x}_{k-1} + U_k = f_{k-1}(\hat{x}_{k-1}, u_{k-1}, q_{k-1}) \]  

(4)

\[ \hat{z}_{k|k-1} = H_k \hat{x}_{k|k-1} + y_k = h_k(\hat{x}_{k|k-1}, r_k) \]  

(5)

(2) State estimation

\[ \hat{x}_k = \hat{x}_{k|k-1} + K_k (z_k - z_{k|k-1}) \]  

(6)

(3) Filter gain matrix

\[ K_k = P_{k|k-1} H_k (H_k P_{k|k-1} H_k^{T} + A_k R_k A_k^{T})^{-1} \]  

(7)

(4) One-step prediction covariance equation

\[ P_{k|k-1} = \Phi \hat{x}_{k|k-1} P_{k-1} \Phi^{T} + \Gamma \hat{x}_{k|k-1} Q_{k-1} \Gamma^{T} \]  

(8)

(5) Filter covariance

\[ P_k = (I - K_k H_k) P_{k|k-1} \]  

(9)

According to (4) to (9), as long as the initial values of \( \hat{x}_0 \) and \( P_0 \) are given, according to the measurement value \( z_k \) at time k, the state estimation \( \hat{x}_k \) can be figured out by the recursive algorithm at time k.

3.2. Unscented Kalman Filter (UKF)

Differing from EKF, by the unscented transformation UKF can make the nonlinear system equations apply to standard Kalman filtering system under the linear hypothesis, rather than EKF achieve recursive filtering by linearizing nonlinear function. UKF is a kind of nonlinear
gaussian state estimator based on minimum variance estimate criterion, It can better approximate the nonlinear characteristics of the state equation than EKF, and has a higher estimation accuracy while the order of calculation amount is as same as EKF, thus it arouses the widespread attention.

Assuming that nonlinear gaussian system state equation and measurement equation are respectively as follows:

\[
\begin{align*}
\mathbf{x}_k &= f_k(x_{k-1}, u_{k-1}, \mathbf{w}_{k-1}) \\
\mathbf{z}_k &= h_k(x_k, v_k)
\end{align*}
\] (10)

Where the process noise \( \mathbf{w}_k \) and measurement noise \( \mathbf{v}_k \) are all uncorrelated zero mean white Gauss noise, their covariance are respectively \( \mathbf{Q}_k \) and \( \mathbf{R}_k \).

UKF filter algorithm steps are as follows:

(1) The statistical properties of the initial state are as follows:

\[
\begin{align*}
\hat{\mathbf{x}}_0 &= E(\mathbf{x}_0) \\
\mathbf{P}_0 &= \text{Cov}(\mathbf{x}_0, \mathbf{x}_0) = E[(\mathbf{x}_0 - \hat{\mathbf{x}}_0)(\mathbf{x}_0 - \hat{\mathbf{x}}_0)^T] \\
E[\mathbf{x}_0, \mathbf{w}_k] &= 0 \\
E[\mathbf{x}_0, \mathbf{v}_k] &= 0
\end{align*}
\] (11)

(2) The expansion state vector of the system is expressed as follows:

\[
\begin{align*}
\mathbf{x}_{\hat{x}}^+ &= [\hat{\mathbf{x}}_k^T \ \mathbf{w}_k^T \ \mathbf{v}_k^T]^T \\
\mathbf{P}_{\hat{x}}^+ &= E[(\mathbf{x}_{\hat{x}}^+ - \hat{\mathbf{x}}^+_k)(\mathbf{x}_{\hat{x}}^+ - \hat{\mathbf{x}}^+_k)^T] = 
\begin{bmatrix}
\mathbf{Q}_k \\
\mathbf{R}_k
\end{bmatrix}
\end{align*}
\] (12)

(3) Time’s updating

\[
W_{0}^{(m)} = \frac{\hat{\lambda}}{n_x + \hat{\lambda}}, \quad W_{0}^{(c)} = \frac{\hat{\lambda}}{n_x + \hat{\lambda}} + (1 - \alpha^2 + \beta), \quad \hat{\lambda} = \alpha^2 (n + k) - n
\] (13)

\[
W_{i}^{(m)} = W_{i}^{(c)} = \frac{1}{2(n_x + k)}, \quad i = 1, 2, \ldots, 2n_x
\] (14)

Without considering the input function, weight \( W_i \) can be calculated by Equation (13) and Equation (14), there are:

\[
\mathbf{x}_{4|k-1} = f(x_{4|k-1}, u_{k-1}, \mathbf{x}_{k-1}^\nu)
\] (15)

\[
\hat{\mathbf{x}}_{4|k-1} = \sum_{i=0}^{2n_x} W_i^{(m)} \mathbf{x}_{4|k-1}^i
\] (16)

\[
\mathbf{P}_{4|k-1} = \sum_{i=0}^{2n_x} W_i^{(c)} [\mathbf{x}_{4|k-1}^i - \hat{\mathbf{x}}_{4|k-1}] [\mathbf{x}_{4|k-1}^i - \hat{\mathbf{x}}_{4|k-1}]^T
\] (17)

\[
\mathbf{Z}_{4|k-1} = h(\mathbf{x}_{4|k-1}^i, \mathbf{x}_{k-1}^\nu)
\] (18)
\[ \hat{z}_{k|k-1} = \sum_{i=0}^{2n} W_i^{(m)} z_{i,k|k-1} \]  
(19)

(4) Measurement’s updating

\[ \begin{align*}
\mathbf{P}_{z[k|k-1]} &= \sum_{i=0}^{2n} W_i^{(c)} [z_{i,k|k-1} - \hat{z}_{i,k|k-1}] [z_{i,k|k-1} - \hat{z}_{i,k|k-1}]^T \\
\mathbf{P}_{x[k|k-1]} &= \sum_{i=0}^{2n} W_i^{(c)} \left[ \frac{3 \cos x_{2,k}}{x_{1,k} + e^{x_{1,k}} + 8} \right] + \mathbf{K}_k (z_k - \hat{z}_{k|k-1}) \\
\hat{x}_k &= \hat{x}_{k|k-1} + \mathbf{K}_k (z_k - \hat{z}_{k|k-1}) \\
\hat{p}_k &= \mathbf{K}_k \mathbf{P}_{x[k|k-1]} \mathbf{K}_k^T + \mathbf{P}_{x[k|k-1]} - \mathbf{K}_k \mathbf{P}_{z[k|k-1]} \mathbf{P}_{x[k|k-1]}^{-1} \mathbf{K}_k^T
\end{align*} \]  
(20)  
(21)  
(22)  
(23)  
(24)

At this point, the filtering state and the variance of UKF at time \( k \) are obtained. As the function values by Unscented Transformation are not be linearized, not ignoring its higher-order terms, avoiding the calculation of jacobian matrix (linear), thus the estimate of mean and covariance acquired by UKF are more accurate than EKF method.

4. Simulation Analysis and Comparison of EKF and UKF

4.1. Precision Comparison of EKF and UKF in the Case of Nonlinearity

We assume the motion characteristics of warehouse targets are described as the non-linear system model, which are shown in the following:

State equation:

\[ \mathbf{x}_{k+1} = \begin{bmatrix} x_{1,k+1} \\ x_{2,k+1} \\ x_{3,k+1} \end{bmatrix} = \begin{bmatrix} 3 \cos x_{2,k} \\ x_{1,k} + e^{x_{1,k}} + 8 \\ x_{1,k} (x_{2,k} + x_{3,k}) \\ 6 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} w_k \]  
(25)

Measurement equation:

\[ \mathbf{z}_k = x_{1,k} \cdot x_{2,k} + 2x_{3,k} + v_k \]  
(26)

Where \( w_k \) and \( v_k \) are Gaussian white noise, and their constant statistical characteristics are as follows:

\[ q = 0.3, \quad Q = 0.7 \quad r = 0.5 \quad R = 1.0 \]  
(27)

The theoretical initial value of linear system formula (25) and (26) is assumed as follows:

\[ \mathbf{x}_0 = \begin{bmatrix} -0.6 \\ 1 \end{bmatrix} \]  
(28)
Meanwhile the initial value of the state estimation is set as follows:

\[
\hat{x}_0 = 0 \quad P_0 = I
\]  

(29)

And \( \hat{x}_0 \), \( w_k \) and \( v_k \) is not relevant.

UT transform symmetric sampling strategy is applied to the simulation, proportionality coefficient \( k = 0.6 \). EKF and UKF are respectively applied to estimating the system state, Figure 4 and Figure 5 are the estimation curves of nonlinear system state in the case of two kinds of algorithm, Figure 6 and Figure 7 describe the simulation diagram of estimation error and mean square error in the case of UKF and EKF respectively.

From Figure 4 to Figure 7, we can see easily, the value EKF estimates system state is better in some cases, state estimation error is small, but sometimes the state estimation error is large, and the state estimation mean variance accumulates rapidly with time, because the EKF’s posterior mean and variance of nonlinear system state can only reach first order Taylor approximation. In the course of filtering the nonlinear system formula (25) and (26), this first order approximation precision is high at some point, can meet the practical system
requirements, and at some point it is really poor which make EKF state estimation error large, and even diverging, while UKF can maintain effectively tracking true value of the target state, estimation accuracy is better than EKF, because UKF can approach posterior mean and covariance of nonlinear Gaussian system state at the accuracy of second-order Taylor approximation, this is the primary reason that UKF filtering accuracy is better than EKF.

4.2. Precision Comparison of EKF and UKF in the Case of the Discontinuous Situation of Nonlinear System State Equation

We assume the motion characteristics of warehouse targets are described as the nonlinear system model, which are shown in the following:

State equation:

\[
x_{k+1} = 
\begin{bmatrix}
x_{1,k+1} \\
x_{2,k+1} \\
x_{3,k+1}
\end{bmatrix} = 
\begin{bmatrix}
3\cos x_{2,k} \\
x_{1,k} + e^{8x_{3,k}} + 8 \\
x_{1,k}(x_{2,k} + x_{3,k})
\end{bmatrix} + 
\begin{bmatrix}
1 \\
1 \\
6 \\
3
\end{bmatrix} w_k
\]  

(30)

Measurement equation:

\[
z_k = x_{1,k} + x_{2,k}x_{3,k} + v_k
\]  

(31)

The \( w_k \) and \( v_k \) are gaussian white noise, and their constant statistical properties are as follows:

\[
g = 0.3, \quad Q = 0.7 \quad r = 0.5 \quad R = 1.0
\]  

(32)

The theoretical initial value of linear system formula (30) and (31) is assumed as follows:

\[
x_0 = [-0.6 \quad 1]^T
\]  

(33)

Meanwhile the initial value of the state estimation is set as follows:

\[
\hat{x}_0 = 0 \quad P_0 = I
\]  

(34)

And \( \hat{x}_0 \), \( w_k \) and \( v_k \) is not relevant.

UT transform symmetric sampling strategy is applied to the simulation, proportionality coefficient \( \bar{k} = 0.6 \).

EKF and UKF are respectively applied to estimating the system state. Figure 8 shows the estimation curve of the system state under UKF filtering algorithm, Figure 9 shows the estimation curve of the system status under EKF filtering algorithm.

We can see from Figure 8, when the state equation shown in formula (30) is not differentiable at some point, UKF remains effective tracking for the state changes, this is because UKF need not to calculate the Jacobian matrix, so it is suitable for nonlinear systems filtering while the state function is discontinuous or non-differentiable, while EKF requires nonlinear function should be continuously and differentiable when it calculate Jacobian matrix. Therefore, EKF failed to filter the nonlinear system state such as formula (30) and (31). As is shown in Figure 9, when the sample number is more than 250, EKF has been unable to proceed, probably because at some point it can't solve the Jacobian matrix of formula (30).
5. Conclusions and recommendations

The paper firstly established the warehouse target locating and tracking system model based on wireless sensor networks. When the state equation of warehouse target tracking system is non-linear, EKF and UKF filtering algorithm are respectively applied to acquiring the state estimate of the system and simulation analysis.

Simulation results show that, the value EKF estimated target tracking state is better in some cases, state estimation error is small, but sometimes the state estimation error is large, and the state estimation mean variance accumulates rapidly with time, because the EKF’s posterior mean and variance of nonlinear system state can only reach first order Taylor approximation, this first order approximation precision is high at some point, can meet the practical system requirements, and at some point it is really poor which makes EKF state estimation error large, and even diverging, while UKF can maintain effectively tracking true value of the target state, estimation accuracy is better than EKF, because UKF can approach posterior mean and covariance of nonlinear Gaussian system state at the accuracy of second-order Taylor approximation, this is the fundamental reason that UKF filtering accuracy is better than EKF.

When the warehouse target tracking system state equation is nonlinear and discontinuous, EKF and UKF filtering algorithm are respectively applied to acquiring the state estimate of the system. Simulation results show that, when the state equation of the system is not differentiable at some point, UKF remains effective tracking of the state changes, while EKF filtering algorithm has failed and even unable to filter.

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