Automatic Selection for Optimal Calibration Model of Camera

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Abstract
When calibration model express camera status well, calibration result will be accurate. How to select optimal calibration model for different camera by program is significant to calibrate automatically. A method of selecting optimal calibration model was proposed in this paper. First, the models including enough possible status were selected from physical models and Chebyshev models. These models would be taken as the input of our method. Second, candidate calibration models were obtained by variance detection. Third, optimal calibration model was extracted by utilizing detection of minimum description length. Experimental results show that calibration residuals is lowest via optimal model, computation error of principle distance is no more than 0.2 pixels and variance lies in the bounds of 0.5 pixels.

Keywords: camera calibration, calibration model, variance detection, minimum description length

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1. Introduction
When camera was fixed on the meter, the data of the meter can be directly read [1]. A lot of image processing technologies have been used in these meters such as grey transforming, edge detecting [2], profile extracting, segmentation [3] and template matching. However, precision of reading data does not satisfy the requirements of users. In order to read data of the meter accurately and automatically, camera calibration was needed. Camera calibration is the key step in machine vision. The result of image processing will be influenced greatly by calibration result [4]. There are many good methods for camera calibration such as traditional calibration, self-calibration and calibration based on active vision [5].

In traditional calibration, all parameters are computed by using relation between points in calibration body and homologous points in image [6]. In calibration based on active vision, camera parameters are obtained by controlling camera do some special movements[7,8]. In self-calibration, plenty of restriction information are used in computing camera parameters[9]. Because the camera on meter is limited by operating space and fixing condition, these calibration methods are hard to be applied to direct-reading meter.

Another problem is that optimal calibration model should not be exclusive to different camera. When condition have been changed inside and outside, optimal calibration model should be different even if the same camera. Once meter was used in locale, camera fixed on meter was difficult to be adjusted again. So, accurate and automatic calibration method should be developed. In this paper, a method of selecting optimal calibration model was proposed.

2. Automatic Selection of Optimal Calibration Model
2.1. Camera Projection Model
Camera projection can be represented by a collinearity equation shown as formula (1).

\[
P_{\text{CCD}} = R(P_{\text{object}} - P_O)
\]

\[
P_{\text{image}} = \frac{c}{z_c} \begin{pmatrix} x_c \\ y_c \\ z_c \end{pmatrix}
\]

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Where, $P_{\text{object}}$ is the coordinate of object point in the world. $P_{\text{CCD}}$ is the coordinate of object point in the camera system. $R$ is the rotation matrix. $P_{\text{image}}$ is the coordinate of object point in image. $c$ is the principle distance of the camera. Plane $z_c = c$ denotes image plane. $P_{\text{CCD}}$ is described by the 3D coordinates $(x_c, y_c, z_c)$ describes. $P_{\text{image}}$ is described by the 3D coordinates $(x, y)$ describes.

During actual projection process, ideal instance in formula (1) is disturbed by many factors. These disturbances are described by image distortion. Considering this condition, formula (1) turn into formula (2).

$$P_{\text{image}} - \delta P_{\text{image}} = \frac{c}{z_c} \left( \begin{array}{c} x_c \\ y_c \end{array} \right)$$

(2)

Where, $\delta P_{\text{image}}$ describes distortion by $(\delta x, \delta y)$.

How to describe image distortion in calibration model accurately is the key during calibration process. Radial distortion, tangential distortion and tilt distortion are included in many methods.

### 2.2. Physical Model

All kinds of distortion are considered in physical model. This model is shown as formula (3):

$$\begin{align*}
\delta x &= \sum_{i=1}^{3} A_i (r^{2i} - r_0^{2i}) x + B_1 (r^2 + 2x^2) + 2B_2 xy + C_1 x + C_2 y \\
&+ (D_1 (x^2 - y^2) + 2D_2 x^2 y^2 + D_3 (x^4 - y^4)) x / c + x_H \\
\delta y &= \sum_{i=1}^{3} A_i (r^{2i} - r_0^{2i}) y + B_1 (r^2 + 2y^2) + 2B_2 xy \\
&+ (D_1 (x^2 - y^2) + 2D_2 x^2 y^2 + D_3 (x^4 - y^4)) y / c + y_H
\end{align*}$$

(3)

Where, $A_i$ and $A_j$ describe radial distortion. $B_1$ and $B_2$ describe radial-asymmetric and tangential distortion. $C_1$ and $C_2$ describe tilt distortion. $D_1$, $D_2$ and $D_3$ describe global deformation of image. $x_H$ and $y_H$ are the coordinates of the principle point. And $r^2 = x^2 + y^2$.

When items of physical model was accepted or rejected according to different instance, physical model in formula (3) would turn into many calibration models.

### 3. Chebyshev Model

Solution to physical model is very complex. Compared to physical model, solution to Chebyshev model is simple.

Chebyshev calibration model can be expressed by the normalized orthogonal polynomial. Coefficients of polynomial reflect correction of image distortion. High correlations between the polynomial coefficients can be avoided by estimation in Chebyshev calibration. Chebyshev calibration model is shown as formula (4).

$$\begin{align*}
\delta x &= \sum_{m=0}^{M} \sum_{n=0}^{N} \alpha_{mn} K_m (k_x x) K_n (k_y y) \\
\delta y &= \sum_{m=0}^{M} \sum_{n=0}^{N} \beta_{mn} K_m (k_x x) K_n (k_y y)
\end{align*}$$

(4)

Automatic Selection for Optimal Calibration Model of Camera (Xuecong Li)
Where, $\alpha_{mn}$ and $\beta_{mn}$ are constant coefficients. $K_m(x)$ is defined like formula (5).

$$K_m(x) = \cos(m \arccos(x)) \quad (-1 \leq x \leq 1)$$

(5)

In formula (5), $k_x$ and $k_y$ can map coordinate of image to range $[-1,1]$.

In formula (4), Chebyshev model would also turn into many calibration models when different values are given to $m$ and $n$.

4. Iterative Solution of Calibration Model

When $\delta_{\text{image}}(\delta x, \delta y)$ is taken into formula (2), intact projection model is built. By computing unknown parameter in formula (2), camera calibration can be accomplished.

Formula (2) can be looked on as a Markoff model. So its solution can be realized by least squares method. Markoff model is shown as formula (6).

$$u + v = X\eta$$

$$D(u) = \sigma^2_0 T^{-1}$$

(6)

Where, $X$ is a matrix of $p \times q$ rank. $\eta$ is an unknown vector of $q \times 1$ dimension. $u$ is an observation vector of $p \times 1$ dimension. $D(u)$ is a covariance matrix of $p \times p$ rank. $T$ is a weight matrix.

When relation among all observation vectors is unknown, $D(u)$ in formula (6) can be predigested into $\sigma^2_0 I$. $I$ is an unit matrix. When covariance is obtained, formula (6) can be solved.

All observation vectors are correlative in camera calibration. This correlation is described by $T$.

The solution of $\eta$ is like formula $\hat{\eta} = (X'X)^{-1}X'Tu$. When formula was changed a little, correction item of observation vector can be shown by formula $\hat{\nu} = X\hat{\eta} - u$. So estimation of difference is shown as formula (7).

$$\hat{\sigma}^2_0 = \frac{\hat{\nu}'T\hat{\nu}}{p-q}$$

(7)

Then, covariance of unknown parameter $\eta$ can be estimated by formula (8).

$$\hat{D}(\hat{\eta}) = \hat{\sigma}^2_0 (X'TX)^{-1}$$

(8)

Take each item of calibration model in formula (2) in formula (6) and carry out iterative calculation like from formula (6) to formula (8). Then, each parameter of calibration model can be solved.

5. Automatic Selection of Optimal Calibration Model

To apply camera to direct-reading meter, camera calibration should be optimal and automatic. So, this work should hand to program. In our program, this work can divided into two steps.

(a) Creation of candidate calibration model

Select a group of models from physical models and Chebyshev models enough to include possible conditions in calibration process, and take these models as input. Then, solve each of these models by the method in 2.4 section.
After every model was solved, calibration parameters were taken into formula (2). Then, world coordinate of calibration point would be computed according image coordinate of calibration point.

After these work, difference $\sigma_0^2$ would be computed again. If the result was different with anterior difference, this calibration model would be abandoned. Contrarily, this model was taken as candidate calibration model.

(b) Selection of optimal calibration model
If there were two or plural models in final concourse of candidate calibration models, minimum description length rule would be used in selecting optimal model.

Minimum description length can be described in formula (9) in camera calibration

$$B_L = B_M + B_D$$

Where, $B_M$ are bits of model parameter. $B_D$ are bits of model description.

When observation vector defers uniform distribution, formula (9) turns to formula (10).

$$B_L = \frac{q}{2} \log_2(p) + \hat{\sigma}(T_v)$$

3. Experimental Results and Analysis
3.1. Design of Experiments
In order to validate the method in this paper, two cameras (camera 1 and camera 2) had been calibrated. In calibration, ten calibration models were taken as input of selecting optimal model from physical models and Chebyshev models. Parameters configuration of these model are listed in Table 1.

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameters configuration</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physical model</td>
<td>$x_H, y_H, A_1, C_1$</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>$x_H, y_H, A_1, A_2, B_1, B_2, C_1, C_2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x_H, y_H, A_1, A_2, A_3, B_1, B_2, C_1, C_2, D_1, D_2, D_3$</td>
<td></td>
</tr>
<tr>
<td>Chebyshev model</td>
<td>$\alpha_m, \beta_m, k_i (i = M = N = 0, \cdots, 6)$</td>
<td>7</td>
</tr>
</tbody>
</table>

3.2. Automatic Selection Experiment of Optimal Calibration Model
Optimal calibration model for two camera were selected from 10 models in table I. The results are shown as Figure 1 and Figure 2.

![Figure 1. Selection Result of First Camera](image1)

![Figure 2. Selection Result of Second Camera](image2)
In Figure 1 and Figure 2, abscissa of X axis denotes 3 physical models and 7 Chebyshev models for first camera in turn. Forked model is abandoned because they do not pass difference checking. Integer of Y axis describes minimum description length $B_L$. Arrowhead model is the optimal model because it has passed difference checking and its $B_L$ is least.

4. Performance Experiment for Optimal Model

The optimal calibration model is obtained by the method in this paper. By comparing residuals of 10 calibration model in Table 1, we can find that residual of the optimal model is least. Residuals of third model and optimal model for first camera are shown as in Figure 3.

![Figure 3. Residuals in Different Models of First Camera](image)

In Figure 3(a) is residual of third model and (b) is residual of optimal model. It is obvious that performance of optimal model is better. Some calibration results of optimal model are listed in Table 2 for camera 1 and camera 2.

From Table 2, we can see that computation error of principle distance is no more than 0.2 pixels and variance lies in the bounds of 0.5 pixels. Root mean square error (RMS) is a key parameter between observations and model in image plane. RMS in Table 2 also shows that selected model is optimal.

4. Conclusion

In order to calibrate camera fixed on the direct-reading meter automatically, a method of selecting optimal model was founded. Some models were selected from physical models and taken as input. Then, optimal model was obtained by difference checking and detection of minimum description length. By analyzing qualitative and quantitative experimental results, we can find that optimal model had been selected and calibration method in this paper had a high precision.

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References


