Applying Extenics Theory to Motion Performs Evaluation of Numerical Machines

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Abstract
The motion performs evaluation indexes of numerical control machine tools (NC) are made of multiple elements. In order to evaluate motion performs of numerical control machine tools objectively, we have applied the extenics theory to establish the matter-element model for the motion performs evaluation of the NC. The relational degree was calculated by introducing the relational function in the extension set theory, and the evaluation indexes weight coefficient of NC were defined by using the 1-9 scale law in AHP (the analytic hierarchy process). A numerical control machine tool as a study object, the motion performs was studied by using the extenics theory. The result indicates that the extenics theory is appropriate for the motion performs evaluation. The motion performs of the evaluated NC was good, the evaluation method possesses a certain practicability.

Keywords: extenics theory, motion performs evaluation, AHP (analytic hierarchy process), numerical control machine tool

1. Introduction
A numerical control machine tool (NC) has some characteristics, such as high processing precision, the stabilizing quality, high production efficiency, better adaptability to the changed design of products. A numerical control machine tool can process the parts that the common machine tool can’t do. It is beneficial to the development of the comprehensive and automatic manufacture technology [1]. Along with the extensive use of numerical machine tools, the users demand higher performances. Because numerical machine tools have a great variety and the performance indexes are made of multiple elements, it is not good to evaluate the motion performs qualitatively or quantitatively with a single method.

The motion performs of NC is a significant part of the overall functions. It shows the technicality of NC. So, the study of motion performs is followed with interest, it is necessary to evaluate and analyse the motion performs of NC. The motion performs indexes involve feed velocity, the rotation rate of chief axis and the adjustment range of it, the power of chief axis, the torque of feed axis, the coordinate strokes, the range of swing angle, the capacity of tool magazine, the tool changing time, etc [2]. From the above analysis and due to more indexes of the motion performs, extenics theory leded-in, the matter-element model of motion performs evaluation of NC is founded abusing the matter-element theory and extension set theory, the motion performs level of NC is determined by solving its matter-element model [3]. The paper selects six indexes to evaluate the NC, which form the motion performs evaluating system, they are feed velocity (m/min), the rotation rate of chief axis(r/min), the x/y/z coordinate stroke(mm), the range of swing angle (°), the capacity of tool magazine and the tool changing time(s).

2. Founding the Model
2.1. Founding the Matter-element Model
2.1.1. Identifying Classical Matter-element
The quantity of motion performs evaluation index is n, shown c_{1}, c_{2}, c_{3}, …, c_{n}, the quantity of motion performs level of NC is m [4].
\[ R_j = (N_j, C_i, X_{ji}) = \begin{bmatrix}
(N_j) & c_1, x_{ji} \\
c_2, x_{j2} \\
\vdots \\
c_n, x_{jn} \\
\end{bmatrix} \begin{bmatrix}
(N_j) & c_1, \langle a_{j1}, b_{j1} \rangle \\
c_2, \langle a_{j2}, b_{j2} \rangle \\
\vdots \\
c_n, \langle a_{jn}, b_{jn} \rangle \\
\end{bmatrix} \] 

\( i=1,2,...,n; \ j=1,2,...,m \)

Where equation (1) is matter-element model, \( N_j \) is shown the motion performs level \( j \), \( C_i \) is the motion performs evaluation index of \( N_j \), \( x_j \) is the magnitude limits of \( C_i \) in \( N_j \) [4].

2.1.2. Identifying Extensional Matter-element

Extensional matter-element is the magnitude limits of every evaluation index in all the motion performs level. Where Equation (2) is shown, \( p \) is all the evaluation index, \( X_{pi} \) is the magnitude limits of \( c_i \), \( X_{pi} = \langle a_{pi}, b_{pi} \rangle \).

\[ R_p = (P, C, X_p) = \begin{bmatrix}
p & c_1, x_{p1} \\
c_2, x_{p2} \\
\vdots \\
c_n, x_{pn} \\
\end{bmatrix} \begin{bmatrix}
p & c_1, \langle a_{p1}, b_{p1} \rangle \\
c_2, \langle a_{p2}, b_{p2} \rangle \\
\vdots \\
c_n, \langle a_{pn}, b_{pn} \rangle \\
\end{bmatrix} \] 

2.1.3. Matter-element Awaiting Evaluation

The machining center awaiting evaluation is shown PM, the concrete value of every evaluation index is shown by matter-element. Where Equation (3) is shown. \( x_i \) is the value of motion performs index of \( c_i \).

\[ R_M = (P_M, C, X) = \begin{bmatrix}
p_M & c_1, x_1 \\
c_2, x_2 \\
\vdots \\
c_n, x_n \\
\end{bmatrix} \] 

2.2. Relational Degree of Every Evaluation Level in NC Awaiting Evaluation

The interval distance of \( P_M \) and \( N_j \) with regard to the motion performs evaluation index \( C_i \) is shown \( \rho(x_i, x_j) \), which of \( P_M \) and \( N_p \) is shown \( \rho(x_i, x_p) \). The relational degree of evaluation index \( C_i \) with regard to the evaluation grade \( j \) is shown in Equation (4).

\[ K_j(x_i) = \frac{\rho(x_i, x_j)}{\rho(x_i, x_p) - \rho(x_i, x_j)} \] 

The weight of \( C_i \) is \( w_i \), so the relational degree of NC awaiting evaluation \( P_M \) with regard to the evaluation grade \( j \) is shown in Equation (5).

\[ K_j(p_M) = \sum_{i=1}^{n} w_i K_j(x_i) \]
2.3. Defining the Weight of Every Evaluation Index by AHP

The gist of AHP is as follows, the influence of different factors on a target is compared, the people’s subjective evaluations is shown and handled by quantum shape, thereby its proportion is founded in the target, finally the conclusion of estimate is defined. [3] The moves and substances of AHP roughly consist of the following aspects: establish the hierarchical structure; build the judgment matrix of every layer factors; test the consistency of the judgment matrix and revise it; ascertain the sequence preference of every layer factors; ascertain the overall sequence preference; inspect the totality consistency.[2] AHP is used to obtain the proportions of evaluation indexes in the motion performs.

3. An Example of Synthetically Analysis of a Motion Performs Evaluation of NC

As the NC machine tool is developing and its structure characteristics is improved, the function and the quality are raising substantially. The motion performs evaluation indexes of NC are feed velocity (m/min), the rotation rate of chief axis(r/min), the x/y/z coordinate strokes(mm), the range of swing angle (°), the capacity of tool magazine and the tool changing time(s).

The feed velocity affects the processing quality, produce efficiency and the tool life. The chief axis of NC is drived by the direct-current or alternating-current motor to have wide speed regulation range and high rotation accuracy. High rotation rate of chief axis is beneficial to raising the processing quality of parts. The x/y/z coordinate strokes are the size of coordinate strokes, it determines the processing range, namely the size of the processed part. The paper select x coordinate stroke as the evaluation index. The range of swing angle is the controlled range of swing angle coordinates, it affects the processing ability to the space position of parts. The capacity of tool magazine and the tool changing time straightly influence the productivity of NC. The capacity of tool magazine is the quantity of tools that can be checked in it. The tool changing time is the time that the tool used in chief axis is changed by the tool used in next production processes in the tool magazine.

In evaluation model, n equals 4. The motion performs grades of NC are A, B, C and D.

3.1. Founding the Matter-element Model

Identifying classical matter-element is shown in Equation (6)

\[
R_a = \begin{bmatrix}
(N_a) & c_1(20,30) \\
& c_2(8000,10000) \\
& c_3(1000,1200) \\
& c_4(240,260) \\
& c_5(80,100) \\
& c_6(1,5)
\end{bmatrix}
\]

\[
R_b = \begin{bmatrix}
(N_b) & c_1(15,20) \\
& c_2(6000,8000) \\
& c_3(800,1000) \\
& c_4(220,240) \\
& c_5(60,80) \\
& c_6(5,12)
\end{bmatrix}
\]

\[
R_c = \begin{bmatrix}
(N_c) & c_1(10,15) \\
& c_2(5000,6000) \\
& c_3(600,800) \\
& c_4(200,220) \\
& c_5(30,60) \\
& c_6(12,18)
\end{bmatrix}
\]

\[
R_d = \begin{bmatrix}
(N_d) & c_1(8,10) \\
& c_2(3000,5000) \\
& c_3(400,600) \\
& c_4(180,200) \\
& c_5(16,30) \\
& c_6(18,25)
\end{bmatrix}
\]

Identifying extensional matter-element is shown in Equation (7)

\[
\begin{align*}
(N_a) & \begin{bmatrix} c_1(20,30) \\ c_2(8000,10000) \\ c_3(1000,1200) \\ c_4(240,260) \\ c_5(80,100) \\ c_6(1,5) \end{bmatrix} \\
(N_b) & \begin{bmatrix} c_1(15,20) \\ c_2(6000,8000) \\ c_3(800,1000) \\ c_4(220,240) \\ c_5(60,80) \\ c_6(5,12) \end{bmatrix} \\
(N_c) & \begin{bmatrix} c_1(10,15) \\ c_2(5000,6000) \\ c_3(600,800) \\ c_4(200,220) \\ c_5(30,60) \\ c_6(12,18) \end{bmatrix} \\
(N_d) & \begin{bmatrix} c_1(8,10) \\ c_2(3000,5000) \\ c_3(400,600) \\ c_4(180,200) \\ c_5(16,30) \\ c_6(18,25) \end{bmatrix}
\end{align*}
\]
Identifying the matter-element awaiting evaluation is shown in Equation (8).

\[ R_p = \begin{bmatrix} c_1(8,30) \\ c_2(3000,10000) \\ c_3(400,1200) \\ c_4(180,260) \\ c_5(16,100) \\ c_6(1,25) \end{bmatrix} \]  

(7)

3.2. Calculating the Relational Degree

The relational degree of the NC is shown in Equation (9).

\[ K_u = \begin{bmatrix} -0.5455 & 0.036 & -0.583 & -0.5 & -0.875 & 1.75 \\ -0.25 & -0.033 & -0.5 & -0.25 & -0.818 & 0 \\ -0.231 & -0.42 & -0.25 & -0.5 & -0.429 & 0 \\ -0.47 & -0.167 & -0.25 & 3 & -0.353 & \end{bmatrix} \]  

(8)

3.3. Defining the Weight of every Evaluation Index by AHP

According to six evaluation indexes defined, quantifying the decision analysis, 1 to 9 and their reciprocal used, the matrix S about relative importance two each other indexes is founded. As shown in Equation (10)

\[ S = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 2 \\ 1/2 & 1 & 1 & 2 & 3 & 1 \\ 1/3 & 1 & 1 & 1 & 4 & 1 \\ 1/4 & 1/2 & 1 & 1 & 2 & 1 \\ 1/5 & 1/3 & 1/4 & 1/2 & 1/2 & 1 \\ 1/2 & 1 & 1 & 1 & 2 & 1 \end{bmatrix} \]  

(10)

Approximate method of solving is used, feature vectors \( w \) and maximal characteristic \( \lambda_{\text{max}} \) are gotten [5], and shown in Equation (11).

\[ w = [0.351, 0.169, 0.152, 0.128, 0.058, 0.142]^T \]

\[ \lambda_{\text{max}} = 6.132 \]  

(11)

Because of objective things’ complexity and the understanding variety of people, the conclusions probably are not same when a great quantity factors are compared with each other. So examining the consistency of matrix S is necessary. The consistency target is shown in Equation (12) [6].

\[ CI = \frac{\lambda_{\text{max}} - n}{n-1} = \frac{6.132 - 6}{5} = 0.0264 \]  

(12)
The average randomness target is shown \( RI = 1.26 \) through table look-up, so the randomness consistency ratio is shown in Equation (13).

\[
CR = \frac{CI}{RI} = \frac{0.0264}{1.26} = 0.021(0.1) \quad (13)
\]

The matrix S meets the needs of consistency. The feature vectors \( \mathbf{w} \) shows the weights of evaluation indexes.

### 3.4. Motion Performs Evaluation Grade

The feature vectors \( \mathbf{w} \) and equation (9) are substituted in Equation (5), the synthetical relational degree of motion performs evaluation grade of the NC is shown in Equation (14).

\[
K = \begin{bmatrix} -0.140 & -0.073 & -0.279 & 1.391 \end{bmatrix}^T \quad (14)
\]

Based on the largest degree of membership, the motion performs grade of this NC is B.

### 4. Conclusion

(1) In this paper, the reasonable motion performs evaluation indexes are selected, and the motion performs evaluation system of NC is founded. Relational degree is calculated through relational function, the weighs are calculated by AHP. Therefore, the motion performs evaluation is more reasonable.

(2) The motion performs evaluation of NC based on extenics theory accords with the reality of it, the evaluation conclusion is dependable.

(3) The paper considered the practice usage of the motion performs evaluation of NC, the evaluation conclusion is feasible. It supplied the machinery choice with an overall and reliable method.

### References


