An Exact Model for Rotor Field-Oriented Control of Single-Phase Induction Motors

M. Jannati*, A. Monadi, S. A. Anbaran, N. R. N. Idris, M. J. A. Aziz
Universiti Teknologi Malaysia, UTM-PROTON Future Drive Laboratory, Faculty of Electrical Engineering, Universiti Teknologi Malaysia, 81310 Skudai, Johor Bahru, MALAYSIA
*Corresponding author, e-mail: Jannatim94@yahoo.com

Abstract
This work presents a new Rotor Field-Oriented Control (RFOC) technique for single-phase Induction Motors (IMs). The proposed method uses two rotational transformations, which extract from the steady-state equivalent circuit of single-phase IM. It is proved by using proposed rotational transformations, the single-phase IM asymmetrical equations change into symmetrical equations. In the proposed technique, the assumption of \((M_q/M_d)^2 = L_{qs}/L_{ds} = a^2\) which is usually used in other FOC of single-phase IMs, is not considered. Performance of the proposed technique is assessed using MATLAB/SIMULINK. Extensive simulation results show the performance and correctness of the proposed method.

Keywords: new RFOC technique, rotational transformations, single-phase induction motor

1. Introduction
Single-phase Induction Motors (IMs) have been broadly employed in low power applications. In these applications the motor operate at low efficiency, fixed speed and consume about 10% of electrical energy. During the last decades, single-phase IM drives have been thoroughly discussed by researchers. Single-phase IMs with two main and auxiliary windings can be considered as two-phase IMs, since these stator windings are displaced 90° from each other. A Four-Switch Inverter (FSI) topology is used in this paper as illustrated in Figure 1, has been proposed as a low cost solution for single-phase IM drives [1, 2].

Variable speed drives can provide reliable dynamic systems and important savings in energy custom and costs of the electrical machines [1-11], [13-16]. Recently, some high performance single-phase IM drive systems were proposed. In these schemes, rotor and stator Field-Oriented Control (FOC) principles are adapted to the single-phase IM model [1-3], [6-10]. Winding asymmetry due to the different inductances and resistances of the main and auxiliary stator windings in single-phase IM causes extra coupling between windings. This asymmetry affects on the motor operation and produces torque and current pulsations [11]. By using a transformation matrix, the compensation of the single-phase IM asymmetry was presented in [2]. In the previous methods for vector control of single-phase or unbalanced two-phase IMs, the assumption of \((M_q/M_d)^2 = L_{qs}/L_{ds}\) is normally applied [2-3], [6-10]. The differences in the d and q stator with considering of \((M_q/M_d)^2 = L_{qs}/L_{ds}\) and without considering of \((M_q/M_d)^2 = L_{qs}/L_{ds}\), are reflected in the speed and hence the torque responses of the drive system. In this paper, a new RFOC strategy for single-phase IM drive is investigated. In the proposed scheme, the backward component in the stator voltages due to this assumption \((M_q/M_d)^2 = L_{qs}/L_{ds}\) as previously ignored in [2-3], [6-10] will be taken into account. The remainder of this work is organized as follows. The modeling of the single-phase IM is presented in section 2. In section 3, the main idea of proposed vector control for single-phase IM discussed and subsequently a new control strategy based on RFOC is presented. The effectiveness of the proposed method is verified using MATLAB/SIMULINK and presented in section 4. Finally, conclusion is presented in section 5.
2. Modeling of the Single-Phase IM

The basic drive system which is studied in this paper is shown in Figure 1.

![Single-phase IM Drive System](image)

Figure 1. Single-phase IM Drive System

Neglecting the core saturation, the electrical and mechanical equations of the single-phase IM in the stator reference frame (superscript "s") are given as follows (All of the parameters in Equation (1) have been defined in [2]):

\[
\begin{align*}
     v_s^i &= r_s^i i_s^i + \frac{d \lambda_s^i}{dt}, \quad v_q^i &= r_q^i i_q^i + \frac{d \lambda_q^i}{dt}, \\
     0 &= r_s^i i_s^i + \frac{d \lambda_s^i}{dt} + \alpha \lambda_q^i, \quad 0 = r_q^i i_q^i + \frac{d \lambda_q^i}{dt} - \alpha \lambda_s^i, \\
     \lambda_s^i &= L_s^i i_s^i + M_s^a i_a^i, \quad \lambda_q^i = L_q^i i_q^i + M_q^a i_a^i, \\
     L_s^i &= M_s^a + I_s^i i_s^i, \quad L_q^i = M_q^a + I_q^i i_q^i, \\
     r_s &= \frac{\text{Pole}}{2} \left( M_s^a i_a^i - M_s^i i_s^i \right), \\
     \frac{\text{Pole}}{2} \left( r_q - r_s \right) &= J \frac{d \omega_s}{dt} + F \omega_s
\end{align*}
\]

(1)

3. Equations of Proposed RFOC for Single-Phase IM

Steady-state equivalent circuit of the single-phase IM can be shown as Figure 2 [12]. In this Figure, \( V_m, V_a, I_m \) and \( I_a \) are the main and auxiliary voltages and currents, "\( a \)" is the turn ratio (\( a = N_a/N_m \)) and "\( j \)" is the square root of "-1". \( E_{fm}, E_{fa}, E_{lm} \) and \( E_{la} \) are the forward and backward voltage of magnetizing branch of the main and auxiliary windings. \( R_f, R_b, X_f \) and \( X_b \) are the forward and backward stator resistance and inductance of the main and auxiliary windings. \( R_{lm}, R_{la}, X_{lm} \) and \( X_{la} \) are the leakage resistance and inductance of the main and auxiliary winding. With the following change of variables:

\[
\begin{align*}
     v_f &= \frac{1}{2} \left( v_m - j v_a \right), \quad i_f = \frac{1}{2} \left( i_m - j i_a \right), \\
     v_a &= \frac{1}{2} \left( v_m + j v_a \right), \quad i_a = \frac{1}{2} \left( i_m + j i_a \right)
\end{align*}
\]

(2)
Figure 2. Steady-state Equivalent Circuit of the Single-phase IM

Figure 3 can be simplified as two balanced forward and backward circuit as follows [13, 14]:

Where:

\[
\begin{align*}
Z_1' &= Z_{im} + 2Z_f + Z_d - Z_f Z_d \\
Z_1'' &= Z_{im} + 2Z_s + Z_d - Z_f Z_d \\
Z_f &= \frac{Z_d}{Z_{im} + 2Z_f + Z_d} \\
I_d &= \frac{1}{2} \left( \frac{Z_d}{Z_f} - \frac{1}{Z_{im}} \right) \\
I_s &= \frac{Z_d}{Z_{im} + 2Z_f + Z_d}, \quad I_1 = \frac{j}{2} \left( \frac{Z_d}{Z_f} - \frac{1}{Z_{im}} \right)
\end{align*}
\]

As you can see, the equivalent circuit of single-phase IM splits to two circuits, each of them indicates a balanced motor which rotates in the forward and backward direction. By considering Figure 3 and Equations (3) we have:

\[
\begin{bmatrix}
V_f \\
I_f
\end{bmatrix}
\rightarrow
\begin{bmatrix}
jV_f \\
jI_f
\end{bmatrix}
= \begin{bmatrix}
\frac{N_a}{N_s} & \frac{N_a}{N_s} \\
\frac{N_a}{N_s} & \frac{N_a}{N_s}
\end{bmatrix}
\begin{bmatrix}
V_s \\
I_s
\end{bmatrix}
\]

\[
\begin{bmatrix}
V_f \\
I_f
\end{bmatrix}
\rightarrow
\begin{bmatrix}
jV_f \\
jI_f
\end{bmatrix}
= \begin{bmatrix}
\frac{N_a}{N_s} & \frac{N_a}{N_s} \\
\frac{N_a}{N_s} & \frac{N_a}{N_s}
\end{bmatrix}
\begin{bmatrix}
jV_s \\
jI_s
\end{bmatrix}
\]

(4)
Equation (4) is the transformation matrixes for changing the variables from unbalanced form to balanced form (e.g., Figure 2 to Figure 3). With substitution of Equations (5) in equations (4), we can obtain Equation (6) and (7).

\[
\begin{align*}
    \begin{bmatrix}
        v_{d}^e \\
        v_{q}^e
    \end{bmatrix} &= \begin{bmatrix}
        M_s \cos \theta_e & M_d \\
        -M_s \sin \theta_e & M_d
    \end{bmatrix} \begin{bmatrix}
        v_{d}^s \\
        v_{q}^s
    \end{bmatrix} \\
    \begin{bmatrix}
        i_{d}^e \\
        i_{q}^e
    \end{bmatrix} &= \begin{bmatrix}
        M_s \cos \theta_e & M_d \\
        -M_s \sin \theta_e & M_d
    \end{bmatrix} \begin{bmatrix}
        i_{d}^s \\
        i_{q}^s
    \end{bmatrix}
\end{align*}
\]  

(6)

Rotational transformation for voltage variables:

(7)

In this equation, \( \theta_e \) is the angle between the stationary reference frame and rotating reference frame. It is expected by using (6) and (7), unbalanced single-phase IM equations change into balanced equations form. By applying of these rotational transformations to the equations of single-phase IM and without the assumption of \((M_q/M_d)^2 = L_{qs}/L_{ds}\), we have (in the previous presented methods for FOC of single-phase IM the assumption of \((M_q/M_d)^2 = L_{qs}/L_{ds}\) is considered [2-3, 6-10]):

Rotor voltage equations:

(8)

After simplifying the equations of rotor voltages can be written as:

Rotor voltage equation:

(9)

Rotor flux equations:

(10)
After simplifying the equations of rotor flux can be written as follows:

Rotor flux equation:

\[
\lambda_{dq} = M_q i_d + L_d i_q
\]

\[
\lambda_{q} = M_q i_q + L_q i_d
\]

(11)

Electromagnetic torque equation:

\[
\tau_e = \frac{Pole}{2} \left( M_{q}\dot{i}_q - M_{d}\dot{i}_d \right)
\]

\[
= \frac{Pole}{2} \begin{bmatrix}
\dot{i}_d \\
\dot{i}_q
\end{bmatrix} \begin{bmatrix}
0 & M_q \\
M_d & 0
\end{bmatrix} \begin{bmatrix}
i_{sh} \\
i_{sq}
\end{bmatrix}
\]

\[
- \left( \frac{Pole}{2} \begin{bmatrix}
\dot{i}_d \\
\dot{i}_q
\end{bmatrix} \begin{bmatrix}
T_p & \left( T_p \right)^{-1}
\end{bmatrix} \begin{bmatrix}
i_{sh} \\
i_{sq}
\end{bmatrix} \right)
\]

\[
\begin{bmatrix}
0 & M_q \\
-M_d & 0
\end{bmatrix} \begin{bmatrix}
T_p^{-1} \left( T_p \right)^{-1}
\end{bmatrix} \begin{bmatrix}
i_{sh} \\
i_{sq}
\end{bmatrix}
\]

Therefore, we have:

\[
\tau_e = \frac{Pole}{2} M_q (i_q^e - i_q^b)
\]

(13)

Stator voltage equations:

\[
\begin{bmatrix}
\dot{v}_{a} \\
\dot{v}_{q}
\end{bmatrix} = \begin{bmatrix}
\dot{r}_{ab} + L_{ab} \frac{d}{dt} \\
\dot{r}_{q} + L_{q} \frac{d}{dt}
\end{bmatrix} \begin{bmatrix}
0 \\
0
\end{bmatrix} \begin{bmatrix}
T_{a} \\
T_{q}
\end{bmatrix}^{-1} \begin{bmatrix}
\dot{i}_{a} \\
\dot{i}_{q}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\dot{r}_{ab} \\
\dot{r}_{q}
\end{bmatrix} = \begin{bmatrix}
M_{d} d \frac{d}{dt} \\
M_{q} d \frac{d}{dt}
\end{bmatrix} \begin{bmatrix}
T_{a} \\
T_{q}
\end{bmatrix}^{-1} \begin{bmatrix}
\dot{i}_{a} \\
\dot{i}_{q}
\end{bmatrix}
\]

(14)

Equations of stator voltage can be simplified as following equation:

\[
\begin{bmatrix}
\dot{v}_{a} \\
\dot{v}_{q}
\end{bmatrix} = \begin{bmatrix}
\dot{r}_{ab} + L_{ab} \frac{d}{dt} - \omega_L i_{qs} \\
\omega_L J_{qs} \dot{i}_{qs} + L_{q} \frac{d}{dt} i_{qs}
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
M_{q} d \frac{d}{dt} - \omega_L M_{q} \\
\omega_L M_{q} d \frac{d}{dt}
\end{bmatrix} \begin{bmatrix}
i_{a} \\
i_{q}
\end{bmatrix}
\]

\[
\begin{bmatrix}
M_{a} \left( \frac{M_{q}^2}{M_{a}^2} I_{p} - I_{q} \right) + I_{a} \\
M_{q} \left( \frac{M_{q}^2}{M_{a}^2} I_{p} - I_{q} \right) \frac{d}{dt}
\end{bmatrix}
\]

\[
\begin{bmatrix}
M_{a} \left( \frac{M_{q}^2}{M_{a}^2} I_{p} - I_{q} \right) + I_{a} \\
M_{q} \left( \frac{M_{q}^2}{M_{a}^2} I_{p} - I_{q} \right) \frac{d}{dt}
\end{bmatrix}
\]

(15)

Generally, Equation (15) are included two terms; forward terms (superscript "^e") and backward terms (superscript "^-e"). It is because of unequal main and auxiliary resistances and
inductances in the single-phase IM equations. Equation between forward and backward term is as follows:

\[
\begin{align*}
\ell_{ds}^e &= -\cos^2 \theta \ell_{ds}^{pe} - \sin \theta \ell_{ds}^{qf} + \ell_{ds}^{h}\sin \theta \ell_{ds}^{qf} \\
\ell_{qs}^e &= -\sin \theta \ell_{qs}^{pe} - \cos \theta \ell_{qs}^{qf} + \ell_{qs}^{h}\sin \theta \ell_{qs}^{qf}
\end{align*}
\]  

As expected, using proposed transformation matrixes (Equations (6) and (7)) equations of the rotor voltage, flux, torque and stator voltage are obtained like balanced motor. In RFOC method, the rotor flux vector is aligned with d-axis; \((\lambda_{dr} = \lambda_{d}, \lambda_{qr} = 0)\), based on this assumption, Equation (15) can be classified as follows:

\[
\begin{align*}
\ell_{ds}^e &= \ell_{ds}^{d-f} + \ell_{ds}^{ref-f} + \ell_{ds}^{d-b} + \ell_{ds}^{ref-b} \\
\ell_{qs}^e &= \ell_{qs}^{d-f} + \ell_{qs}^{ref-f} + \ell_{qs}^{d-b} + \ell_{qs}^{ref-b}
\end{align*}
\]

Where:

\[
\begin{align*}
\ell_{ds}^{d-f} &= \alpha_k(-I_{qs} + \frac{M_q^2}{L_r} \ell_{qs}^e + (\frac{M_q^2}{L_r}) \ell_{ds}^{h} - \frac{\lambda_r}{T_r} \ell_{qs}^e) \\
\ell_{ds}^{ref-f} &= M_q^2 (\frac{L_r}{M_d}) \frac{d\ell_{ds}^e}{dt} \\
\ell_{ds}^{d-b} &= -\alpha_k(\frac{M_q^2}{L_r} \ell_{ds}^{h} - L_{qs} - L_{qs}^e) \ell_{qs}^e \\
\ell_{ds}^{ref-b} &= \frac{M_q^2}{M_d} (\frac{L_{ds} - L_{qs}}{M_{ds}}) \frac{d\ell_{ds}^e}{dt} \\
\ell_{qs}^{d-f} &= -\omega_k(-I_{qs} + \frac{M_q^2}{L_r} \ell_{ds}^e + \frac{\alpha_k}{L_r} M_q^2) + \frac{\omega_k}{L_r} \frac{d\lambda_r}{dt} \\
\ell_{qs}^{ref-f} &= r_{qs}^e (\frac{L_{qs} - M_q^2}{L_{qs}}) \frac{d\ell_{qs}^e}{dt} \\
\ell_{qs}^{d-b} &= \omega_k(\frac{M_q^2}{M_d} L_{ds} - L_{qs}) \ell_{ds}^e \\
\ell_{qs}^{ref-b} &= \frac{M_q^2}{M_d} (\frac{L_{ds} - L_{qs}}{M_{ds}}) \frac{d\ell_{qs}^e}{dt}
\end{align*}
\]

In Equation (18), \(\ell_{qs}^{d-f}, \ell_{ds}^{d-b}, \ell_{qs}^{d-f}\) and \(\ell_{qs}^{d-b}\) can be generated by Decoupling Circuit and \(\ell_{qs}^{ref-f}, \ell_{ds}^{ref-b}, \ell_{qs}^{ref-f}\) and \(\ell_{qs}^{ref-b}\) can be generated by current PI controller as it is shown in Figure 4.

According to (9), (11) and (13), the equations of RFOC can be formulated and shown as Equation (19) and Figure 5 respectively (In Equation (19), \(T_r\) is rotor time constant).

\[
\begin{align*}
|\lambda_r| &= \frac{M_q^2}{1 + \frac{L_{ds}}{T_r} \frac{d\ell_{ds}^e}{dt}} \\
\omega_k &= \omega_k + \frac{M_q^2}{L_r} \frac{d\ell_{qs}^e}{dt} \\
r_s &= \frac{P_{oe} \ell_{qs}^e}{2} (\frac{M_q^2}{L_r} |\lambda_r|)^2
\end{align*}
\]
4. Results and Comparisons
4.1. Performance Evaluation

The proposed controller based on Figure 4 and Figure 5 is applied to a commercial 0.25 hp single-phase IM with the nominal values and parameters as in Table 1. The developed scheme performance is then simulated with different values of rotor speed. Extensive simulation results are presented to evaluate the proposed single-phase IM drive performance.

Figure 6 illustrates the reference and the real rotor speed signals towards two different steady-state rotor speed values (the reference real speed varies from zero to the rated and...
rated to zero value. A load torque equal to 1 N.m is introduced at $t = 9$ s and removed at $t = 11$ s. It is seen that the real rotor speed signals are so accurate that hardly can be distinguished from the corresponding reference speed signals even after applying load torque (the oscillation of speed is about 0.2 rpm in steady-state and after applying load torque). The motor electromagnetic torque is also shown in Figure 6(b). It can be seen that the electromagnetic torque has a quick response with no pulsations.

Figure 6. Simulation Results of RFOC at Zero and Nominal Command Speed; (a) Speed, (b) Torque

Figure 7. Simulation Results of RFOC for a Trapezoidal Command Speed; (a) Stator currents, (b) Speed, (c) Torque
Figure 7 shows simulation results of the command and actual rotor speed according to proposed method for a trapezoidal command speed from 500rpm to -500rpm. It is evident from Figure 7(b) that the real speed follows the command speed. The auxiliary and main stator currents and electromagnetic torque for trapezoidal command speed are shown in Figure 7(a) and Figure 7(c) respectively. In this case, as can be seen in Figure 7(b), by using proposed controller, the speed oscillation at steady-state is ~ 0.07rpm at rotor speed of 500rpm.

Figure 8 and Figure 9 shows the good performance of the proposed drive system for controlling single-phase IM in the difference values of speed (±500rpm, ±1000rpm and ±1700rpm) and at very low speed operation respectively. It can be seen from Figure 6-9 that the dynamic performance of the proposed drive system for vector control of single-phase IM is extremely acceptable.

![Figure 7](image1)

**Figure 7.** Shows simulation results of the command and actual rotor speed according to proposed method for a trapezoidal command speed from 500rpm to -500rpm.

![Figure 8](image2)

**Figure 8.** Simulation Results of RFOC in the Difference Values of Command Speed; (a) Speed, (b) Torque

![Figure 9](image3)

**Figure 9.** Simulation Results of RFOC at Low Speed Operation; (a) Speed, (b) Speed Error

4.1. Comparisons

Based on Equation (15), the difference in the d and q stator voltages between the conditions in which the supposition of \((M_q/M_d)^2 = L_{qs}/L_{ds}\) is considered (e.g., [2-3], [6-10], [14-16]) and otherwise is as following equation:

\[
\begin{align*}
\Delta v_{d} &= \begin{bmatrix} \frac{M_q^2}{M_d^2} (I_d - I_{dq}) \frac{d}{dt} - \alpha \frac{M_q^2}{M_d^2} (I_d - I_{dq}) \frac{d}{dt} \\ \alpha \left( \frac{M_q^2}{M_d^2} (I_d - I_{dq}) \frac{d}{dt} - \frac{M_q^2}{M_d^2} (I_d - I_{dq}) \frac{d}{dt} \right) + \frac{M_q^2}{M_d^2} (I_d - I_{dq}) \frac{d}{dt} \end{bmatrix} 
\end{align*}
\]  

An evaluation between the steady-state rotor speed response of the RFOC with considering of \((M_q/M_d)^2 = L_{qs}/L_{ds}\) and with no considering of \((M_q/M_d)^2 = L_{qs}/L_{ds}\) is demonstrated in Figure 10. Small magnitude of oscillations at the rated reference rotor speed can be observed in the speed responses when the supposition \((M_q/M_d)^2 = L_{qs}/L_{ds}\) is employed. As can be seen in Figure 10, by using conventional controller (supposition \((M_q/M_d)^2 = L_{qs}/L_{ds}\)), the speed oscillation at steady-state is ~ 0.2rpm at rotor speed of 1800rpm but by using proposed controller the speed oscillation reduced ~ 0.08rpm at rotor speed of 1800rpm.

![Figure 10](image4)
An Exact Model for Rotor Field-Oriented Control of Single-Phase Induction Motors (M. Jannati)

It is concluded in comparison with the previous proposed schemes for FOC of single-phase or two-phase IMs (e.g., [2-3], [6-10], [14-16]), the proposed controller in this research produces fewer ripples in the torque and speed.

![Figure 10. Simulation Results of Comparison between Speed Response in Single-phase IM](image)

(a) not assuming \((M_q/M_d)^2 = L_{qs}/L_{ds}\) (b) assuming \((M_q/M_d)^2 = L_{qs}/L_{ds}\)

5. Conclusion

An accurate technique for speed control of single-phase IMs based on RFOC has been presented. The proposed method employs rotational transformations that transform the unbalanced single-phase IM equations into equations of RFOC that have the same structure as the balanced motor. Unlike other RFOC method implemented for the single-phase IMs, the proposed technique does not utilize the supposition \((M_q/M_d)^2 = L_{qs}/L_{ds}\). Simulation results proved the technique validity.

References

