Weakest Buses Identification and Ranking in Large Power Transmission Network by Optimal Location of Reactive Power Supports

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Abstract
The detection of voltage collapse is essential to avoid possible voltage collapse for the preventive control actions and voltage security assessment. One effective way to know the locations where voltage collapses could be appear is to identify weakest buses in the systems. The weakest bus is the first point where voltage collapses appear in a severe contingency. This paper proposes a technique to evaluate the weakest bus in large scale power system based on the optimal position of reactive power supports. To solve the optimization problem, Differential Evolutionary (DE) technique is used. The fitness function consists of cost, power losses and Load voltage stability index ($L_{mn}$) which satisfying all operational constraints. $L_{mn}$ is used as the indicator for voltage stability margin and weakest bus identification. The method is applied on standard IEEE 30 bus, 57 bus and 118 bus test systems to show their comparative computing effectiveness.

Keywords: weakest bus, voltage collapse, reactive power support, voltage stability, differential evolution

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1. Introduction
A system experiences a state of voltage instability when there is a progressive or uncontrollable drop in voltage magnitude after a disturbance, increase in load demand or change in operating condition. The main factor, which causes these unacceptable voltage profiles, is the inability of the power system to meet the demand for reactive power. Under normal operating conditions, the bus voltage magnitude increases as reactive power injected at the same bus is increased. However when voltage magnitude of any one of the system’s buses decreases with the increase in reactive power for that same bus, the system is said to be unstable. Although the voltage instability is a localized problem, its impact on the system can be wide spread as it depends on the relationship between transmitted active power, injected reactive power and receiving end voltage [1]. The main challenge of this problem is to identify the locations where voltage instability could be initiated and to understand the origin of the problem. One effective way to known the voltage instability origin is to identify weakest buses in the systems. The weakest bus has been identified as the bus which lacks reactive power supports the most to defend against voltage collapse.

The identification of weakest buses is an important task for the analysis of power system stability [2, 3]. To identify the weak buses several methods has been proposed in the literature, the most of these methods are based on Voltage Stability Indices. In Ref. [4] the voltage collapse proximity indicator (VCPI) method for each load bus is applied to identify the weak buses of the system. Ref. [5] proposes the use of Continuation Power Flow (CPF) to identify the weak buses. The Ref. [6] uses line stability index designated as fast voltage stability index (FVSI) to determine the maximum reactive load-ability and the weakest buses. A New Voltage Stability Index (NVSI) is proposed in Ref. [7]. A fuzzy logic based fast decoupled load flow method is considered to estimate the value of NVSI. Ref. [8] presented the use of Line voltage stability index ($L_{mn}$) for Weak Bus Identification for FACTS location. Ref. [9] proposes the identification of the weakest buses over 24 hours in order to study and compensate the detrimental impacts of PEV charging stations on voltage profiles and voltage stability of smart grid.

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The identification of the weakest buses using above mentioned methods is based on graduation increasing of load at chosen load bus and calculating of the Voltage Stability Indices (VSI). The value of VSI close to 1.00 indicates that the particular bus is close to its instability point. In the final step the maximum power loading or maximum load-ability limit (MLL) is extracted for every load buses and the smallest MLL is ranked the highest implying the weakest bus in the system. The major weakness of these methods is that required a large calculation time particularly for large scale power systems. In recent years evolutionary/meta-heuristic computing techniques like Genetic Algorithm (GA), Particle Swarm Optimization (PSO), evolutionary programming and others have emerged as very powerful general purpose solution tools. Basically these tools are search techniques capable of finding the optimum solution of a problem. The most remarkable feature of these tools is that they do not impose any restriction to the nature of the search space and type of the variables [10]. In this paper a technique to evaluate the weakest bus in large scale power systems based on the optimal location of reactive power supports is proposed. Planning of reactive power supports would give benefits to the users of the transmission systems, in terms of loss reduction, among other technical benefits, such as improving steady state and dynamic stability; improve system voltage profiles [11]. The reactive power planning problem involves optimal allocation of reactive power sources (Var sources) to improve the system voltage stability and reduce fuel cost and power losses. In this paper the optimization problem is solved using Differential Evolutionary (DE) technique. The Load voltage stability index (Lmn) is used as the indicator for voltage stability margin and weakest bus identification and ranking. Simulations are performed on IEEE 30, 57 and 118 bus systems.

2. Formulation of Voltage Stability Index

Voltage stability is currently one of the most important research areas in the field of electrical power system. Voltage instability problem is associated with the increased loading of system (heavily loaded), and insufficient local reactive supply. The main challenge of this problem is to identify the locations where voltage instability could be initiated and to understand the origin of the problem. One effective way to known the voltage instability origin is to identify weakest buses in the systems. The weakest bus has been identified as the bus which lacks reactive power supports the most to defend against voltage collapse.

Identifying weak buses can give correct information for the optimal reactive power planning involved that would decide which buses are the most severe and need to have new reactive power sources installed and distributed generator to enhance load-ability of the system [4, 12]. There are many methods currently in use to help in the voltage stability analysis and weak area identification. Some of them are PV and QV analysis [13], Modal Analysis [14], Maximum Loading Margin Index (MLM) [15], load proximity index [16, 17], impedance index [18], Fast Voltage Stability Index (FVSI) [6], Line stability index [19]. In this paper the line voltage stability index (Lmn) is used for weakest buses identification. The line voltage stability Index symbolized (Lmn) proposed by Moghavvemi [20] is formulated based on a power transmission line. This index is basically used to determine the maximum load-ability in a power system. The voltage stability index referred to a line was formulated from the 2-bus representation of power system. The value of line index that is closed to the unity indicates that the respective line is closed to its stability limit. The representation of a 2-bus model is illustrated in Figure 1.

![Figure 1. Model of Simple Branch for Voltage Stability Research](image-url)
The voltage stability index for a line is defined as follows:

\[ L_{mn} = \frac{4XQ_s}{[V_s \sin(\theta - \delta)]^2} \leq 1.0 \]  

(1)

Where:

X: Line reactance;
Q_s: Reactive power at the receiving end;
V_s: Sending end voltage;
\theta: Line impedance angle;
\delta: Angle difference between the supply voltage and the receiving voltage.

The value of \( L_{mn} \) ranges from 0 (no load) to 1 (voltage collapse), and it must be less than 1 for stable systems. The \( L_{mn} \) is used to find the stability index for each line connected between two buses in an interconnected network, line with the highest value of \( L_{mn} \) index is considered to be weak compared to a line with the lower value of \( L_{mn} \) index.

3. Formulation of the Optimization Problem

This section presents a methodology to find the optimal positions of Var sources on an existing power network, these positions or nodes is considered as the weakest nodes in the system. The objective of optimal positions of Var sources is to optimize a certain objective function such as cost, loss, and voltage stability index while satisfying all operational constraints. The optimization of voltage stability index is included in the objective function to improve system voltage stability. In this context the general optimization problem can be written in the following form:

\[ \text{Min} \left[ f \right] = \sum_{i=1}^{NG} f_i + P_{LOSS} + \sum_{i=1}^{NB} L_{mn} \]  

(2)

Where, \( f_i \) is the fuel cost of the \( i^{th} \) generator.

The fuel cost curve is modeled by quadratic function as:

\[ f_i = a_i + b_i P_{gi} + c_i P_{gi}^2 \]  

(3)

In this equation, \( P_{gi} \) is the actual power produced in the generator \( i \). \( a_i \), \( b_i \) and \( c_i \) are the invariant factors and \( NG \) is the number of generators in the system.

The active power losses are expressed from the equation of active power balance:

\[ P_{LOSS} = \sum_{i=NG} P_{gi} - \sum_{j=NL} P_{gj} \]  

(4)

\( L_{mn} \) is the line voltage stability index and \( NB \) is the number of branches in the power system.

The equality and inequality constraints to be satisfied while searching for the optimal solution can be written as:

\[
\begin{align*}
U_i & = \sum_{j=1}^{NB} \left| U_j \right| \left( G_{gj} \cos \delta_j + B_{gj} \sin \delta_j \right) \\
Q_{gi} & = \sum_{j=1}^{NB} \left| U_j \right| \left( G_{gi} \sin \delta_j + B_{gi} \cos \delta_j \right)
\end{align*}
\]  

(5)
The system inequality operation constraints include:

\[ P_{gi}^{\min} \leq P_{gi} \leq P_{gi}^{\max} \]  
(6)

\[ Q_{gi}^{\min} \leq Q_{gi} \leq Q_{gi}^{\max} \]  
(7)

\[ P_{DGi}^{\min} \leq P_{DGi} \leq P_{DGi}^{\max} \]  
(8)

\[ V_{i}^{\min} \leq V_{i} \leq V_{i}^{\max} \]  
(9)

Where, \( NB \) is the number of buses; \( P_{gi} \) and \( Q_{gi} \) are the active and reactive power generations at \( i^{th} \) bus; \( P_{li} \) and \( Q_{li} \) are the active and reactive power demands at \( i^{th} \) bus; \( P_{i} \) and \( Q_{i} \) are the active and reactive power injections at \( i^{th} \) bus; \( \delta_{ij} \) is the deference between voltage angles at bus \( i \) and \( j \).

4. Differential Evolution Based Optimal Location of Var Sources

Differential Evolution (DE) is a population based algorithm proposed by Strom and Price (1995) [21] whose main strategy is to generate a position for an individual with the help of vector difference among other randomly selected members of the population.

The advantage of DE can be summarized as follows [22, 23]:
- DE is an effective, fast, simple, robust, inherently parallel, and has few control parameters need little tuning. It can be used to minimize non-continuous, non-linear, non-differentiable space functions, also it can work with noisy, flat, multi-dimensional, and time dependent objective functions and constraint optimization in conjunction with penalty functions.
- The optimization process in DE is carried out using the following steps:

**Step 1:** Initialization of power flow data and DE control parameters such as the size of population (NP), the maximum number of iteration, the mutation factor (F); the crossover factor (CR) and the number of variables to be optimized (D).

**Step 2:** Initialization of population: The initial population is generated randomly using the following equation:

\[ x_{ij}(0) = x_{ij}^L + rand(0,1)(x_{ij}^U - x_{ij}^L) \]

Where \( x_{ij} \) is the variable that should be optimized (the exact location where it will be installed the Var source (Load Buses)), and \( x_{ij}^L, x_{ij}^U \) are the lower and the upper bound (the buses of power network except where the generators are installed). The random number \( rand(0,1) \) is uniformly distributed in interval \((0, 1)\).

**Step 3:** evaluate the fitness for each individual in the population according to the objective function.

**Step 4:** Create a new population by:
- a) **Mutation:** For each target vector a mutant vector is generated according to following equation:

\[ v_{ij}(g+1) = x_{ij}(g) + F \cdot (x_{ij}^*(g) - x_{ij}(g)), \]

Where \( r1, r2, r3 \in \{1, 2, ..., NP\} \) integer, mutually different and \( F > 0 \), the randomly chosen integers \( r1, r2 \) and \( r3 \) are also chosen to be different from the running index \( i \). \( F \) is a real constant factor usually within range of \([0.4 \ 1.0]\).

- b) **Crossover:** In order to increase the diversity of the perturbed parameter vectors, crossover is introduced. To this end, the trial vector is formed, where:

\[ u_{ij}(g+1) = \begin{cases} v_{ij}(g+1) & \text{if } rand(0,1) \leq CR \text{ or } j = j_{rad} \\ x_{ij}(g) & \text{otherwise} \end{cases} \]
Where $j_{\text{rand}}$ is a randomly chosen index to ensure that the trail vector $u_{ij}$ does not duplicate $x_{ij}$. $CR$ is the crossover constant which has to be determined by the user in the range of $[0, 1]$.

c) **Selection:** the trial vector is compared to the target vector and the better one is selected into the next generation as follows:

$$x'_{i}(g + 1) = \begin{cases} u_{i}(g + 1), & \text{if } f(u_{i}(g + 1)) \leq f(x_{i}(g)) \\ x_{i}(g), & \text{otherwise} \end{cases}$$

Where $x'$ is the offspring of $x_i$ for the next generation.

**Step 5:** end of the process and save the best individual (optimal location of Var source) if the stopping criterion is satisfied, else go back to step 4.

The DE control parameters are set as follows: the number of population is 20; the mutation factor $F = 0.8$; the crossover factor $CR = 0.8$ and the iteration number is 150.

5. **Simulation Results and Discussion**

The solutions results for optimal location of reactive power sources to minimize the fitness function mentioned above in objective to find the weakest buses for IEEE 30, 57 and 118 power systems are obtained and discussed below. The important parameter values of IEEE 30, 57 and 118 test systems are given in Table 1 and the detailed parameters are listed in [24].

<table>
<thead>
<tr>
<th>IEEE test systems</th>
<th>Number of generators</th>
<th>Number of lines</th>
<th>Number of loads</th>
<th>Total $P_D$ (MW)</th>
<th>Total $P_Q$ (MVAr)</th>
<th>Number of Transformer tapings</th>
<th>Number of shunt capacitances</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 Bus</td>
<td>6</td>
<td>41</td>
<td>24</td>
<td>283.4</td>
<td>126.2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>57 Bus</td>
<td>7</td>
<td>80</td>
<td>50</td>
<td>1251.8</td>
<td>3364</td>
<td>15</td>
<td>3</td>
</tr>
<tr>
<td>118 Bus</td>
<td>54</td>
<td>186</td>
<td>64</td>
<td>3678</td>
<td>1438</td>
<td>9</td>
<td>15</td>
</tr>
</tbody>
</table>

5.1. **Determination of Weak Buses in 30-bus Test System**

The IEEE 30-bus test system consists of six generators at buses 1, 2, 5, 8, 11 and 13. The system has 41 transmission lines and 24 loads. The total system load is 283.4 MW. The cost coefficients for 30-bus system are taken from [25].

Figure 2 depicts the IEEE 30-bus test system load curve (hourly load curve), where is divided into two different periods, i.e. peak period and off-peak period. Figure 3 and Figure 4 show respectively the power generation and system voltage magnitudes for peak and off-peak periods. It is observed that more then allowed level of load increasing, power generation increased and voltage at all buses dropped. Figure 5 shows the $L_{mn}$ index in the normal and heavy load conditions (peak period). It can be seen that the $L_{mn}$ increased when the system operate in the heavy load conditions, for this reason it is more practical to find the weakest buses at heavy load conditions.

![Figure 2. IEEE 30-bus System Load Curve](image-url)
In this subsection DE optimization technique is used to define the best location to provide desired reactive power support under heavy load conditions. The find buses are considered as the weakest buses in the system from the point of view of voltage stability. The result of the weakest bus ranking under heavy load conditions obtained is presented in Table 2. The buses are ranked starting with the most critical bus which is the bus 30. The results obtained from the method proposed in Ref. [26-28] are used for making a comparison with those results obtained from the proposed method. From Table 2, we conclude that the proposed method can efficiently identify the weakest buses. Figure 6 show the weakest area in the system, from this figure we can observe that this area has not included any generators and it is remote from the generator buses. In otherwise the buses 30, 26 and 29 are all at the end of the radial network, they are requiring the reactive power compensation.

Table 2. Weakest Buses Ranking under Heavy Load Conditions in IEEE 30-bus System

<table>
<thead>
<tr>
<th>Reference</th>
<th>Rankings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref [26]</td>
<td>30, 26, 29, 25, 27</td>
</tr>
<tr>
<td>Ref [27]</td>
<td>30, 26, 29, 14, 23</td>
</tr>
<tr>
<td>Ref [28]</td>
<td>30, 26, 29, 19, 20</td>
</tr>
<tr>
<td>Proposed method</td>
<td>30, 26, 29, 21, 24</td>
</tr>
</tbody>
</table>
5.2. Determination of Weak Buses in 57-bus and 118-bus Test Systems

In this case, the proposed method is applied on the IEEE 57-bus and IEEE 118-bus systems [25]. Based on DE optimization technique the first five weakest buses are presented in Table 3. For IEEE 57-bus system the weak area is recovered the following buses: 37, 15, 31, 52 and 13. The second test system can be regarded as a realistic transmission level power network in terms of number of nodes and branches. It consists of 118 nodes and 186 branches but using the proposed method the identification and ranking of weakest buses can only take a few of minutes. For this test system the following buses 95, 63, 22, 94 and 101 are defined as the weakest buses.

Table 3. Weakest Buses Ranking under Heavy Load Conditions in IEEE 57-bus and IEEE 118-bus Test Systems

<table>
<thead>
<tr>
<th>Test system</th>
<th>Weakest buses</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE 57-bus</td>
<td>37, 15, 31, 52, 13</td>
</tr>
<tr>
<td>IEEE 118-bus</td>
<td>95, 63, 22, 94, 101</td>
</tr>
</tbody>
</table>

6. Conclusion

Differential Evolution Based Optimal Location of reactive power supports (Var sources) is proposed to identify the weakest buses for different IEEE standard test systems. The weakest buses identification problem is modeled as optimization problem considering the voltage stability of power system. The scheme optimizes the cost, the power losses and the load voltage stability index to find the buses were the Var sources to be installed, and these buses are considered as the weakest buses in the system. Simulations were performed on IEEE 30, 57 and 118 bus systems. The simulation results authenticate the effectiveness of the proposed method.

References