RBFNN Variable Structure Controller for MIMO System and Application to Ship Rudder/Fin Joint Control

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Abstract

Aiming at a class of multiple-input multiple-output (MIMO) system with uncertainty, a sliding mode control algorithm based on neural network disturbance observer is designed and applied to ship yaw and roll joint stabilization. The nonlinear disturbance observer is finished by radial basis function neural network and with that a terminal sliding mode control algorithm is proposed. The asymptotic stability of closed-loop system is proved based on Lyapunov theorem. The proposed control law is applied to anti-roll control under simulative wave disturbances. Simulation results verified robustness and effectiveness of the suggested algorithm. A good anti-rolling effect is achieved and yaw angle is also reduced greatly with less mechanical loss.

Keywords: sliding mode, radial basis function neural network, disturbance observer, roll/yaw, ship anti-roll

1. Introduction

Sliding mode variable structure control has merits such as invariance to matching uncertainties. It is effective means for nonlinear control problem and is widely used [1, 2]. The development of sliding mode theory for single-input single-output (SISO) is going to be accomplished, but sliding mode methods for SISO couldn’t generalized and applied to MIMO system simply and what’s more, many actual industrial objects are MIMO nonlinear. Therefore, the control problem on MIMO nonlinear uncertainty system becomes a research hotspot [3]. Paper [4] discussed a class of high order MIMO system terminal sliding mode control, but odd problem was not considered and boundary layer method was employed to reduce chattering such that the robustness was influenced. A higher sliding mode controller was designed for MIMO nonlinear system in [5]. Approaching precision was reserved and chattering is reduced significantly, but the decoupling matrix was gotten approximatively. Algebraic strong observability and system smoothness was put into use to realize finite time stability in [6]. The algorithm was also based on higher order sliding mode, but the unknown input observer was hard to design. Although some achievements which were about MIMO nonlinear control problem and only based on sliding mode theory were got, chattering, unknown upper bound of uncertainty and algorithm complexity are hard to handle. It is difficult to solve complex problem only by one control theory. Good result can be achieved if combining sliding mode with other control algorithms. Such as paper [7] proposed an adaptive fuzzy sliding mode control law and realized finite time stability based on final attractor. An adaptive sliding mode controller for perturbed nonlinear time varying systems was designed in [8].

Because radial basis function neural network (RBFNN) can approach any nonlinear function under certain condition and its self-learning, self-adaption and fault-tolerant abilities are good, many control schemes based on RBFNN are proposed [9]. RBFNN can be used as equivalent control part, to learn unknown upper bound, to adjust switching gain etc, in a word, there are many successful applications based on RBFNN sliding mode control [10-12] which are mainly adopted by SISO system. This paper combines RBFNN with sliding mode theory. A RBFNN disturbance observer is designed to approximate compound disturbance online and terminal sliding mode method is employed to cut down response time to complete the control for MIMO nonlinear system. System robustness is strengthened and chattering is lowered because of disturbance observer.
In shipping business, demand for sailing performances such as comfortability, safety and economy become higher. Rolling control can improve these performances exactly. There are many kinds of ship stabilizer [13]: fin stabilizer, tank stabilizer, rudder stabilizer and rudder/fin joint stabilizer. Rudder/fin joint stabilization is considered based on the fact that the ship motion is nonlinear and strong coupling in essence. In 1981 Kallstrom proposed rudder/fin joint control based on multivariable quadric form theory which improved rolling and yaw simultaneously [15]. Rudder/fin joint control also has been studied based on sliding mode [16, 17], but the common drawbacks are unsatisfactory chattering and long response time.

Rudder/fin joint system is MIMO nonlinear typically, such that the proposed neural sliding mode control algorithm is suitable for it. Firstly, rudder/fin joint nonlinear state equation is deduced according to the known linear transfer function. Then the proposed algorithm is used to simulate under wave disturbance. The results indicate good anti-rolling effect is got. The rolling angle is within ±1.8°. The control chattering of fin stabilizer and rudder is greatly weakened comparing with the sliding mode control without disturbance observer.

The paper’s structure arrangement is as follow. The problem description is in section 2. The terminal sliding mode controller based on RBFNN observer is designed in section 3. Section 4 studies the application to yaw/roll control and the conclusions are made at last.

2. Controller Design
2.1. Problem Description

Considering nonlinear system with uncertainty:

\[
\begin{align*}
\dot{x}(t) &= f(x) + (G(x) + \Delta G(x))u + \Delta f(x) \\
y(t) &= h(x(t))
\end{align*}
\]  

Where \( x \in R^n \) is state vector, \( u \in R^m \) is control inputs, \( y \in R^n \) represents outputs. \( \Delta f(x) \in R^n \) is unknown modeling error, \( \Delta G(x) \in R^{m \times n} \) stands for system uncertainties, \( f(x), G(x) \) are smooth function with suitable dimensions. Without loss of generality, assume \( G(x) \) is non-singular.

In order to design terminal sliding mode control, suppose \( \Delta f(x) = 0 \) and \( \Delta G(x) = 0 \). The tracking errors are defined as:

\[
e = y - y_d
\]  

Where \( y_d \) are expected values.

Represent sliding mode surfaces as follow:

\[
\sigma = Ce
\]  

Where \( C = \text{diag}[c_1, c_2, \ldots, c_m] \).

For the convenience of description, define the following variables:

\[
\sigma^T = [\sigma_1, \sigma_2, \ldots, \sigma_m]
\]  

\[
|\sigma| = [|\sigma_1|, |\sigma_2|, \ldots, |\sigma_m|]
\]  

\[
\Phi_{si \text{ gn}(\sigma)} = \begin{bmatrix}
\Phi_{s1 \text{ gn}(\sigma_1)} \\
\Phi_{s2 \text{ gn}(\sigma_2)} \\
\vdots \\
\Phi_{sm \text{ gn}(\sigma_m)}
\end{bmatrix}
\]
Where $0 < \gamma = a / b < 1$, $a, b$ are positive odd numbers and the meaning of $\hat{\Phi}$ will be given when designing controller.

For nominal part of system (1), derivative of sliding mode surfaces satisfy:

$$\dot{\sigma} = -\Lambda_{1}\sigma - \Lambda_{2}\sigma'$$  \hfill (7)

Where $\Lambda_{1} = \text{diag} \{\Lambda_{11}, \Lambda_{12}, ..., \Lambda_{1n}\} > 0$, $\Lambda_{2} = \text{diag} \{\Lambda_{21}, \Lambda_{22}, ..., \Lambda_{2n}\} > 0$.

From (1), (3) and (7)

$$C (f(x) + G(x)u - \dot{y}_{d}) = -\Lambda_{1}\sigma - \Lambda_{2}\sigma'$$ \hfill (8)

Then the control law of nominal model of system (1) is:

$$u_{a} = (CG(x))^{-1}u_{a}$$ \hfill (9)

Where $u_{a} = Cy_{d} - Cf(x) - \Lambda_{1}\sigma - \Lambda_{2}\sigma'$

2.2. Design of Terminal Sliding Mode Controller Based on RBFNN Observer

$\Delta f(x)$ and $\Delta G(x)$ must be considered when designing terminal sliding mode controller for robustness of system (1). The compound disturbance is defined as:

$$d = \Delta G(x)u + \Delta f(x)$$ \hfill (10)

For any given $x \in M$, ($M$ is compact set), optimal weight $W^{*}$ may be defined as:

$$W^{*} = \text{arg min}_{W \in \Omega, W \in \mathbb{R}^{n}} [\sup_{x \in M} |W - \hat{W}|]$$ \hfill (11)

$$\Omega = \{W : ||W|| \leq M \}$$ \hfill (12)

Where $\Omega$ is parameter feasible region, $M$ is design parameter, $W$ represents neural network weights, $\hat{W}$ stands for adjustable neural network weights. $\partial_{i}(x | \hat{W})$ denotes the $i^{th}$ element of $\partial x | \hat{W}$.

$$\partial_{i}(x | \hat{W}) = \hat{W}_{i}^{T} \xi_{i}(x)$$ \hfill (13)

Where $\hat{W}_{i} = \text{diag}[\hat{W}_{i1}, \hat{W}_{i2}, ..., \hat{W}_{im}]^{T}$, $\xi_{i}(x) = [\xi_{i1}(x), \xi_{i2}(x), ..., \xi_{im}(x)]^{T}$ is neural network basis function. $\xi_{i}(x) = \exp(-||x - c_{i}||^{2} / \delta_{i}^{2})$, $c_{i}, \delta_{i}$ are the center and width values of RBFNN. The Approximation value of RBFNN is:

$$\partial(x | \hat{W}) = \hat{W}^{T} \xi(x)$$ \hfill (14)

$$d = W^{T} \xi(x) + e_{i}, |e_{i}| \leq \Phi_{i}^{*}$$ \hfill (15)

Where $e_{i}$ is the $i^{th}$ component of $e$, $\Phi_{i}^{*}$ is upper bound of RBFNN error $e_{i}$, weight value error vector $\hat{W} = W^{*} - \hat{W}$. Then:

$$d - d(x | \hat{W}) = W^{T} \xi(x) + e$$ \hfill (16)
In order to design disturbance observer, considering the following form system

\[ \dot{\Gamma} = -\Xi \Lambda \Gamma + h(x | \dot{W}) \]  

(17)

Where \( \Gamma \) denote states of auxiliary system. Design parameter \( \Xi = \text{diag}\{\Xi_1, \Xi_2, \ldots, \Xi_n\} > 0 \), \( \Lambda = \text{diag}\{\Lambda_{11}, \Lambda_{22}, \ldots, \Lambda_{nn}\} > 0 \), \( h(x | \dot{W}) = \Xi \Lambda \dot{x} + \Xi (x - \Gamma)^T + f(x) + G(x)u + \hat{d}(x | \dot{W}) + \hat{d}\text{sign}(\zeta), \) \( \zeta \) is observer error of disturbance as \( \zeta = x - \Gamma \).

Considering (1) and (17), the dynamic equation of disturbance error.

\[ \dot{\zeta} = -\Xi (\Lambda \dot{\zeta} + \zeta^T) + \dot{W}^T \zeta(x) + \varepsilon - \hat{d}\text{sign}(\zeta) \]  

(18)

As is indicated in (18), if \( \zeta \rightarrow 0 \) then \( \dot{\zeta} \rightarrow 0 \) which means the RBFNN observer can approach compound disturbance effectively. Weights \( \dot{W} \) of network and adaptive law \( \dot{\Phi} \) are designed respectively.

\[ \dot{W} = \eta_2 \xi \left( KC \sigma + \Lambda_1 \dot{\zeta} + \zeta^T \xi \right)^T \]  

(19)

\[ \dot{\Phi} = \eta_2 (KC \sigma + \Lambda_1 \dot{\zeta} + \zeta^T \xi) \]  

(20)

Where \( \dot{W} = \text{diag}\{\dot{W}_1, \dot{W}_2, \ldots, \dot{W}_n\} \), \( \xi(x) = \begin{bmatrix} \xi_1(x), \xi_2(x), \ldots, \xi_n(x) \end{bmatrix} \), \( \eta_1 > 0 \), \( \eta_2 > 0 \), \( K = K^T > 0 \).

The robust control law is deduced as:

\[ u = (CG(x))^{-1} (u_0 - C\hat{d}(x | \dot{W}) - C\hat{d}\text{sign}(\xi)) \]  

(21)

Then the terminal sliding mode control algorithm based on RBFNN observer can be concluded as Theorem 1.

**Theorem 1.** For MIMO nonlinear system (1), RBFNN disturbance is designed based on (17), parameters adjustment formulas are as (19) and (20), the control law is designed as (21), then tracking errors of closed-loop system and disturbance observation errors are asymptotic convergence.

**Proof:**

Considering (1), (3) and (21):

\[ \sigma = -\Lambda \sigma - \Lambda \dot{\zeta}^T + C(\dot{W}^T \dot{\zeta} + \zeta^T \xi) - C\hat{d}\text{sign}(\sigma) \]  

(22)

Choose Lyapunov function:

\[ V = \frac{1}{2} \sigma^T K \sigma + \frac{1}{2} \zeta^T A_1 \zeta + \frac{a}{a + b} \zeta^T \zeta + \frac{1}{\eta_1} \text{tr}(\dot{W}^T \dot{W}) + \frac{1}{\eta_2} \dot{\Phi}^T \dot{\Phi} \]  

(23)

The derivative of (23) is:

\[ \dot{V} = \sigma^T K \dot{\sigma} + \left(\Lambda_1 \dot{\zeta}^T + \zeta^T \xi \right) \dot{\zeta} + \frac{1}{\eta_1} \text{tr}(\dot{W}^T \dot{W}) + \frac{1}{\eta_2} \dot{\Phi}^T \dot{\Phi} \]  

(24)
Take (18) and (21) to (24):
\[
\dot{V} = -\sigma^{T} K (\Lambda_{x} + \Lambda_{y} \zeta) - (\Lambda_{x} + \Lambda_{y} \zeta) (\Xi (\Lambda_{x} + \zeta)) + (KC\sigma + \Lambda_{x} + \Lambda_{y} \zeta) (\ddot{\theta} + \zeta) \\
- (\Lambda_{x} + \Lambda_{y} \zeta) (\ddot{\theta} + \zeta) \Phi \text{ sign}(\zeta) - \sigma^{T} K C \Phi \text{ sign}(\sigma) + \frac{1}{\eta_{1}} \text{tr}(\ddot{\theta} \ddot{\theta}) + \frac{1}{\eta_{2}} \dddot{\theta}
\]
(25)

Because of \(0 < \sigma = \alpha b \sigma \) and \(a, b \) are positive odd numbers, then \( \text{sign}(\zeta) = \text{sign}(\zeta') = \text{sign}(\Lambda_{x} + \zeta) \).

Considering (16) and (25), then:
\[
\dot{V} \leq -\sigma^{T} K (\Lambda_{x} + \Lambda_{y} \zeta) + (\Lambda_{x} + \Lambda_{y} \zeta) (\Xi (\Lambda_{x} + \zeta)) + (KC\sigma + \Lambda_{x} + \Lambda_{y} \zeta) \\
+ \zeta^{T} \dddot{\theta} (\ddot{\theta} - \dddot{\theta}) + (KC\sigma + \Lambda_{x} + \Lambda_{y} \zeta) \dddot{\theta} \xi + \frac{1}{\eta_{1}} \text{tr}(\ddot{\theta} \dddot{\theta}) + \frac{1}{\eta_{2}} \ddot{\theta}
\]
(26)

Take into account equations: \( \Phi = \phi - \ddot{\theta} = \dddot{\theta} - \Phi \), \( | \Lambda_{x} + \zeta^{T} |^{T} = \Phi^{T} | \Lambda_{x} + \zeta |^{T} \), \( \text{tr}(\ddot{\theta} \xi (\Lambda_{x} + \zeta^{T})) = (\Lambda_{x} + \zeta^{T}) \dddot{\theta} \xi \). Then from formula (26):
\[
\dot{V} \leq -\sigma^{T} K \lambda_{x} \sigma - \sigma^{T} K \Lambda_{x} \sigma - (\Lambda_{x} + \Lambda_{y} \zeta) (\Xi (\Lambda_{x} + \zeta)) \\
+ \text{tr}(\ddot{\theta} \xi (KC\sigma + \Lambda_{x} + \Lambda_{y} \zeta) + \ddot{\theta} \xi) + \Phi^{T} (| KC\sigma | + \dddot{\theta}) \\
+ \Lambda_{x} + \Lambda_{y} \zeta + \zeta^{T} \dddot{\theta} + \frac{1}{\eta_{2}} \dddot{\theta}
\]
(27)

Because of \( \dddot{\theta} = -\dddot{\theta} \) and \( \dddot{\theta} = -\dddot{\theta} \), formula (27) can be written as:
\[
\dot{V} \leq -\sigma^{T} K \lambda_{x} \sigma - \sigma^{T} K \Lambda_{x} \sigma - (\Lambda_{x} + \Lambda_{y} \zeta) (\Xi (\Lambda_{x} + \zeta)) \\
\]
(28)

Considering \( K > 0, \Xi > 0, \Lambda_{x} > 0, \Lambda_{y} > 0 \) and \( \Lambda_{x} > 0 \), then:
\[
\dot{V} < 0
\]
(29)

Then closed-loop system is asymptotical stable.

3. Application to Ship Yaw/Roll Joint System
3.1. Rudder/Fin Joint Nonlinear System Mathematic Model

Ship motion is rather complex. Course angle keeping and rolling angle reduction are the main control objective when studying rudder/fin joint control. Figure 1 is functional block diagram of linear rudder/fin joint control system.

![Functional Block Diagram of Linear Rudder/Fin Joint Control System](image-url)
Transfer function of a ship rudder/fin joint system is given in literature [18]. The reduced order transfer function is:

\[
\begin{bmatrix}
    r \\
    \psi
\end{bmatrix} =
\begin{bmatrix}
    G_{r_1}(s) & G_{r_2}(s) \\
    G_{\psi_1}(s) & G_{\psi_2}(s)
\end{bmatrix}
\begin{bmatrix}
    \zeta \\
    \delta
\end{bmatrix}
= \begin{bmatrix}
    -0.2 & 0.1 \\
    (5s^2+1)(2s^2+1) & (5s^2+1)(2s^2+1) \\
    -0.0096 & 0.05 \\
    s(52.35s+1) & s(52.35s+1)
\end{bmatrix}
\begin{bmatrix}
    \zeta \\
    \delta
\end{bmatrix}
\tag{29}
\]

Where \(r, \psi, \zeta, \delta\) represent rolling angle, yaw angle, fin angle and rudder angle.

\(G_{r_2}(s)\) is 2 order Nomoto model and can be changed to nonlinear response dynamic.

\[
\dot{\psi} + \left( \frac{K_o}{T_o} \right) H(\dot{\psi}) = \left( \frac{K_o}{T_o} \right) \delta
\tag{30}
\]

Where \(H(\dot{\psi}) = \lambda \dot{\psi} + \beta \dot{\psi}^3, \lambda = 20.01, \beta = 29415.13\).

Then the linear model is changed to nonlinear mathematic model by using (30).

The influence of \(G_{r_2}(s)\) to the whole system is rather small, such that it can be neglected.

Considering mechanical characteristics of fin and rudder.

\[
\begin{bmatrix}
    T_F \dot{\zeta} + \zeta = \zeta_c \\
    T_E \dot{\delta} + \delta = \delta_c
\end{bmatrix}
\tag{31}
\]

Where \(\zeta_c\) is fin control angle, \(\delta_c\) is rudder angle, \(T_F = 0.5s, T_E = 2.5s\) the control constraints are \(\dot{\delta}_{\max} = 4.4^\circ s^{-1}, \dot{\zeta}_{\max} = 20^\circ, \zeta_{\max} = 8^\circ s^{-1}, \zeta_{\max} = 20^\circ\).

Take \(x_1 = r, x_2 = \psi, x_3 = \psi, x_4 = \phi, x_5 = \delta, u_1 = \zeta, u_2 = \delta, y_1 = r, y_2 = \psi\). The nonlinear mathematic model of the rudder/fin joint system

\[
\begin{bmatrix}
    \dot{x}_1 \\
    \dot{x}_2 \\
    \dot{x}_3 \\
    \dot{x}_4 \\
    \dot{x}_5 \\
    \dot{x}_6 \\
    \dot{y}_1 \\
    \dot{y}_2
\end{bmatrix} =
\begin{bmatrix}
    x_2 \\
    -0.1x_1 - 0.7x_2 - 0.02x_5 + 0.01x_6 \\
    x_4 \cos x_4 \\
    -0.0182x_4 - 28.0366x_4^3 + 0.00096x_6 \\
    -2x_5 + 2u_1 \\
    -0.4x_6 + 0.4u_2 \\
    x_1 \\
    x_3
\end{bmatrix}
\tag{32}
\]

\subsection*{3.2. Wave Disturbance Model}

In fact, ship stabilization problem is to restrain wave’s influence. The wave disturbance could not be neglected. This study adopts a simple method to simulate wave disturbance, which is band-limited white noise to drive two-order oscillation element. The wave disturbance simulation schematic diagram is shown in Figure 2, Figure 3 and Figure 4 show equivalent rolling angle disturbance and yaw angle disturbance which will be brought to nominal model of rudder/fin joint system in matlab simulation.
3.3. Ship Stabilization Simulation

Each parameter should be chosen properly according to (21). The given rolling angle and yaw angle are both zero. The simulation is performed under wave disturbance recommended in section 3.2 and the simulation step time is set 0.01s. Figure 5 shows the rolling angle under three different conditions. It is indicated that rolling angle is reduced and the anti-rolling effect is similar under both SMC and RBFNN SMC. Yaw angle curves shown as Figure 6 indicate that yaw angle is smaller under RBFNN SMC than under SMC and both algorithms are effective.

The control variables are rudder angle and fin angle as is shown in Figure 7 and Figure 8. Both values of rudder angle and fin angle are smaller under RBFNN SMC than that under
SMC. From the partial enlarged drawing in Figure 7 and Figure 8, the chattering is greatly reduced under RBFNN SMC.

4. Conclusion

Nonlinear disturbance observer based on RBFNN approximation capability is proposed in this paper and on this basis terminal sliding mode controller for MIMO nonlinear uncertainty system is designed. The RBFNN SMC is applied to ship rudder/fin control. Wave disturbance is simulated by combining band-limited white noise with two-order oscillation element. Simulation results show the designed observer can approach wave disturbance. The system robustness is realized and chattering of fin and rudder is rather smaller than that without disturbance observer. The rudder/fin model adopted in this paper is deduced from linear model and so it is rather easy and rough. Next we will study control method of rudder/fin system with four degrees of freedom. Furthermore, the idea combining RBFNN observer with dynamic sliding mode is promising.

Acknowledgements

The research work was supported by A Project of Shandong Province Higher Educational Science and Technology Program under Grant No.J12LN29 and Shandong Provincial Natural Science Foundation under Grant No. ZR2013EE014, No. ZR2013ZEM006 and Shandong Province Transportation Innovation Program (No. 2012-33).

References


