PTS Method with Combined Partitioning Schemes for Improved PAPR Reduction in OFDM System

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Abstract
Although orthogonal frequency division multiplexing (OFDM) is an efficient wireless transmission system, it suffers from a crucial drawback namely high peak-to-average power ratio (PAPR) that limits transmitter power efficiency. Thus, different PAPR reduction algorithms have been introduced. Partial transmit sequence (PTS) is the most attractive solution which can provide good PAPR reduction performance without distortion. In any PTS system, partitioning of the OFDM frame into disjoint sub-blocks is a significant step. Out of the existing partitioning techniques, adjacent partitioning (AP) is a fairly simple partitioning scheme achieving efficient PAPR reduction performance. This paper presents an enhanced PTS approach that combines two PTS partitioning schemes, adjacent and interleaved partitioning, in order to effectively reduce the PAPR of OFDM systems. With an aim of determining the effects of length variability of adjacent partitions, we performed an investigation into the performances of a variable length adjacent partitioning (VL-AP) and fixed length adjacent partitioning in comparison with the enhanced PTS scheme. From the various computer simulation results with different types of modulation, we confirmed that the enhanced PTS method offers better PAPR reduction performance compared to adjacent partitioning for fixed and variable length which itself is based on PTS scheme considered efficient in PAPR reduction.

Keywords: orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAPR), partial transmit sequences (PTS), adjacent partitioning PTS (AP-PTS), interleaved partitioning PTS (IP-PTS).

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1. Introduction
Orthogonal frequency division multiplexing (OFDM), is well-known that has been adopted in different standard of wireless communication [1]. OFDM provides many advantages such as narrow band interference and robustness against frequency selective fading [2]. OFDM has been adopted for broadband wireless communications and have been suggested in many standards. OFDM received a lot of attention, especially in the field of wireless communications because of its efficient use of frequency bandwidth and robustness to multi-path fading, immunity to the inter-symbol interference (ISI), and ability for high data rates [3]. From these advantages, the OFDM has already been proposed as the digital terrestrial broadcasting systems and the standard transmission technique in the wireless LAN systems [4]. Also, the OFDM technique is regarded as one of the candidate transmission techniques for the next generation of mobile communications systems [5]. However, one major drawback of OFDM is a large peak to average power ratio (PAPR). It is caused nonlinear distortions after amplified by a power amplifier. Over the past decade, many PAPR reduction techniques have been proposed in the literatures [6], such as clipping [7], coding [8], selective mapping (SLM) [9], companding methods [10], tone reservation (TR) [11], tone injection (TI) [12], active constellation extension (ACE) [13], partial transmit sequences (PTS) [14, 15]. Each of these techniques has various costs of the reduced PAPR and bit error rate (BER). Among these techniques, partial transmit sequence (PTS) technique is one type of PAPR reduction methods of the probabilistic schemes.
The idea of the PTS is based on combining of signal sub-blocks, which are multiplied by weighting factors. Multiple sequences with the lowest PAPR are transmitted [16-18]. In this paper, we describe a PTS technique that combines two schemes interleaved and adjacent to reduce the PAPR, as well as we study and analyzed adjacent and interleaved sub-block partitioning scheme, and focusing on analyses the effect of PAPR performance for different length of disjoint sub-blocks PTS scheme when compared with the enhanced PTS method. The paper is organized as follows: In section II, we briefly review the PAPR of an OFDM signal, and PTS scheme is presented. Section III presents the sub-block partitioning schemes. Section IV presents the enhanced PTS method. Section V shows the performance of the enhanced method with simulation results. Finally, we end the paper with brief conclusions in Section VI.

2. System Model and PTS Scheme

2.1. PAPR of the OFDM Signal

In OFDM systems, the discrete-time domain transmitted signals with $N$ subcarriers are generated by applying the IFFT operation, can be expressed as:

$$
x(n) = \text{IFFT}[X] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{(\frac{j2\pi kn}{N})}, \quad 0 \leq n \leq N - 1
$$

(1)

Where $X_k, k = 0, 1, \ldots, N - 1$, are input symbols modulated into phase shift keying (PSK) or quadrature amplitude modulation (QAM).

In general, the PAPR of OFDM signal in one symbol period in Eq. (1) is defined as the ratio between the maximum instantaneous power and its average power, can be given by [19]:

$$
PAPR = \frac{\max[|x(t)|^2]}{E[|x(t)|^2]}
$$

(2)

Where $E[.]$ denotes the expected value.

The OFDM system uses the complementary cumulative distributed function (CCDF) to evaluate the performance of any PAPR reduction techniques. The CCDF of PAPR denotes the probability that the PAPR is below the threshold $PAPR_0$. CCDF is defined as [20]:

$$
\text{CCDF}(PAPR_0) = P_r (PAPR > PAPR_0)
$$

(3)

2.2. PTS Scheme

The basic principles of PTS, as shown in Figure 1, the input data symbol is partitioned into $M$ sub-vectors $X_m$, which is non-overlapping with each other, so each sub-vector’s length becomes to $N/M$ with all subcarriers positions are occupied by the other sub-block are set to zero, where $m = 1, 2, \ldots, M$. Therefore, is given as [21]:

$$
X = \sum_{m=1}^{M} X_m
$$

(4)

The sub-blocks partitioning $X_m$ are transformed from the frequency domain into time domain partial transfer sequence $x_m$ by used the Inverse Discrete Fourier Transform (IDFT), which can express as:

$$
x_m = \sum_{m=1}^{M} \text{IDFT} \{ X_m \}
$$

(5)

Then the phase weighting factors to all subcarriers for each sub-block $x_m$ are applied and combined together to generate a set of candidates. The candidate to minimize the PAPR is select for transmitting. Thus, the time domain signal after combination can then be represented as:

$$
x = \sum_{m=1}^{M} b_m x_m
$$

(6)

As mentioned, there are three well known partitioning schemes for PTS technique: adjacent, interleaved, and pseudorandom [22]. Among them, pseudorandom partitioning PTS
(PRP-PTS) scheme can be obtained the better PAPR performance but the computational complexity is higher than the other partitioning. In comparison, we used adjacent partitioning due to it simple to implement as well as it presented PAPR reduction performance very close to pseudorandom partitioning with less computational complexity [23]. Therefore, our analysis was restricted in comparison between the performance of the enhanced method with those of interleaved and adjacent partition schemes.

![Figure 1. Block Diagram of the PTS Method](image)

3. Sub-Block Partitioning on PTS Schemes for PAPR Reduction

3.1. Adjacent Partitioning PTS (AP-PTS)

In this section, we made simulation studies of the PTS method with an adjacent scheme for fixed and variable length of sub-blocks partitioning. Mathematical frameworks of these two different adjacent methods can describe by:

**a) Adjacent Partitioning with Variable Length (VL-AP)**

In this method, all subcarriers of OFDM frame were first partitioning into M disjoint sub-blocks with equal size. Then IFFT was computed for each sub-block. The output of IFFT in each sub-block was a phase rotated by the rotation factor and then the blocks are summed together to produce a transmitted signal. The mathematical process is represented by Equation (7-11).

As mentioned, first variable length partitions are generated as follows:

\[
X = [P_0 \ P_1 \ P_2 \ldots \ P_{M-1}]  
\]  

(7)

\[
P_0 = [\hat{P}_0 \ 00000 \ldots \ 0]  
\]  

(8a)

\[
P_1 = [00000 \ldots \ 0, \ \hat{P}_1, \ 00000 \ldots \ 0]  
\]  

(8b)

\[
P_{M-1} = [00000 \ldots \ 0, \ \hat{P}_{M-1}]  
\]  

(8c)

Where, \(P'_i\) are the variable length disjoint subsets of OFDM frame and \(P_i\) are the corresponding disjoint partitions, i.e. partitions with disjoint variable length supports.

Then time domain signal is obtained by taking IFFT of these partitions as shown by:

\[
x_0(n) = \text{IFFT}(P_0)  
\]  

(9a)

\[
x_1(n) = \text{IFFT}(P_1)  
\]  

(9b)

\[
x_{M-1}(n) = \text{IFFT}(P_{M-1})  
\]  

(9c)

Phase rotated \(x'^i(n)\) of the time domain signals are obtained simply through multiplication by phase factors, \(\varphi(r_i) = e^{j\theta_i}\), where \(\theta_i\) are the rotation angles. The rotating phase factors, as given by:

\[
x'^0(n) = \varphi(r_0)x_0(n)  
\]  

(10a)
The transmitted signal in this scheme can be represented by,

\[ \hat{x}(n) = \sum_{i=0}^{M-1} x_i^{(r_i)}(n) \]  

(11)

**b) Adjacent Partitioning with Fixed Length (AP)**

We repeated the process in step (a) with the fixed length size of sub-blocks partitioning for the same \( N \) complex symbols in OFDM frame. The mathematical formulas are similar to the one in Equation (7-11) with the difference that in variable length AP, the supports for the disjoint partitions have variable lengths, while for fixed length AP, all the partitions have supported with the same length.

### 3.2. Interleaved Partitioning PTS (IP-PTS)

In interleaved partitioning partial transmit sequence (IP-PTS) scheme, the \( N \) subcarriers is first divided into \( M \) groups with each group having \( L = \frac{N}{M} \) contiguous subcarriers and every subcarrier signal spaced apart is allocated at the same sub-block. Then the i-th interleaved partition is formed by assigning i-th subcarrier of each group to the i-th interleaved partition. The partitions can be represented by the following equations,

\[
P_0 = \begin{bmatrix} p_0^{(1)} & 0 & \cdots & 0 & p_0^{(M)} & 0 & \cdots & 0 \end{bmatrix}
\]  

(12a)

\[
P_1 = \begin{bmatrix} 0 & p_1^{(1)} & 0 & \cdots & 0 & p_1^{(M)} & 0 & \cdots & 0 \end{bmatrix}
\]  

(12b)

\[
P_L = \begin{bmatrix} 0 & \cdots & 0 & p_L^{(1)} & 0 & \cdots & 0 & p_L^{(M)} \end{bmatrix}
\]  

(12c)

Where, \( p_i^{(j)} \) is the j-th element of the i-th interleaved partition. Remaining steps to generate transmitted signal are similar to those of adjacent partitioning. IP-PTS has lower computational complexity when compared with AP-PTS and PRP-PTS [24]. However, PAPR reduction performance of (IP-PTS) is the worse than other schemes when the number of generated candidates is the same [25].

In the above this scheme, like the others sub-block partitioning schemes (AP-PTS) and (PRP-PTS), after partitioned an input symbol sequence of \( N \) sub-carriers into \( M \) disjoint sub-blocks with equal size. The sub-blocks partitioning are converted to the time domain by using IFFT operation. The output of all these IFFT results was rotated by a set of rotating phase factors and finally combined to achieve the minimum PAPR. The mathematical process is represented by:

**[IFFT]**

\[ x_0(n) = \text{IFFT}(P_0) \]  

(13a)

\[ x_1(n) = \text{IFFT}(P_1) \]  

(13b)

\[ x_{M-1}(n) = \text{IFFT}(P_{M-1}) \]  

(13c)

**[Rotation]**

\[ x_0^{(r_0)}(n) = x_0(n) \]  

(14a)

\[ x_1^{(r_1)}(n) = x_1(n) \]  

(14b)

\[ x_{M-1}^{(r_{M-1})}(n) = x_{M-1}(n) \]  

(14c)
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4. Enhanced PTS Method

Our PTS approach reduces PAPR through the enhanced combination of partitioning and signal generation techniques, which are provided in the section below.

This technique is employed to combine the adjacent and interleaved sub-block partitioning schemes. The enhanced PTS technique is similar to the other sub-blocks partitioning such as adjacent partitioning (AP), which it begins during frequency domain data frame as an input into \( v \) adjacent blocks.

Then, the blocks are divided into sub-blocks of size \( s \). Finally, blocked interleaved partitions \( P \) are constructed by appointment the sub-blocks into the partitions, as follows:

\[
P_i \left( \frac{q}{v} \right) = S_{br}(q)
\]

Where, \( R_i \left( \frac{q}{v} \right) \) represents the \( q \)-th element of the sub-block \( r \) within the partition \( P_i \), and \( S_{br}(q) \) represents the \( q \)-th element of the sub-block \( i \) within the block \( r \) of the original data. A blocked interleaved partition consists of a sub-block from each of the \( v \) blocks. Each sub-block has a size of \( s \), and then, the size of the partition is \( s \cdot v \).

Now, each of the blocked interleaved partitions contains \( s \cdot v \) elements in the enhanced PTS approach. Then IDFT of each of the partitions are computed independently. The output of IDFT in each of the partitions \( P \) is given by:

\[
x^{(i)}_n = \sum_{q=0}^{v-1} \sum_{r=0}^{v-1} P_i \left( \frac{q}{v} \right) e^{2\pi i (qr+is+q)n/N} (17)
\]

Where \( x^{(i)}_n \) represents the \( n \)-th sample in the PTS sequence corresponding to the partition \( P_i \), \( N \) is the total number of subcarriers, and \( l \) is the number of blocks \( (l = N/v) \). In addition, \( r \) is the sub-block index within the partition, and \( q \) is the index within the sub-block.

The PTS sequences \( x^{(i)}_n \) are phase rotated with a rotation factor \( w_i \), except that the first sequence \( x^{(0)}_n \) is kept constant, that is, \( w_0 = 1 \). The phase factors \( w_i \) are given by the exponentials:

\[
w_i = e^{i\phi_i} \quad i = 0,1,\ldots, (z-1) (18)
\]

Where, \( \phi_i \) are randomly selected numbers in the range of \( 0 \leq \phi_i \leq 2\pi \), \( z \) is the number of block interleaved partition. The rotated sequences \( x^{(i)}_n = w_i x^{(i)}_n \) are then combined to generate a transmit signal candidate \( x_n \) that contains the same information within a phase factor.

\[
\tilde{x}_n = \sum_{i=0}^{v-1} x^{(i)}_n (19)
\]

The procedure is repeated with various sets of phase rotation values. Through each repetition, PAPR of the candidate transmit signal is computed. The candidate OFDM symbol with the lowest PAPR is transmitted.

5. Simulation Results

In this section, we present some simulations to evaluate and compare the performance of the enhanced PTS method with the traditional PTS algorithm of two different types of phase sequences (interleaved and adjacent) for fixed length is carried out through simulation compared to the variable length adjacent PTS (VL-AP). Some simulations were conducted to assess the performance of the PTS techniques for PAPR reduction, with input data blocks of...
length 64 (N = 64) are random partitioned into 8 sub-blocks (M=8), and different types of modulation techniques including QPSK, 8PSK, 16QAM and 64QAM. In order to generate the complementary cumulative distribution function (CCDF) of the PAPR, 2000 OFDM blocks are generated randomly. Also, we used MATLAB version 7.8 for the simulation of the PTS techniques and the computation of CCDF.

In Figure 2, by using QPSK modulation, some results of the CCDF are simulated for the OFDM system. When CDDF = 10⁻³, the PAPR₀ of the original OFDM signals (signal without PTS) was 10.4 dB, interleaved partitioning (IP-PTS) was 7.6 dB, VL-AP was 7.2 dB, adjacent partitioning for fixed length (AP-PTS) was 6.9 dB, and the enhanced PTS technique was 6.6 dB. Therefore, the enhanced PTS technique reduced PAPR by around 3.8 dB, fixed length AP by 3.5 dB, VL-AP by 3.2 dB, and IP by 2.8 dB from the original signal. Obviously, the enhanced PTS algorithm has the best PAPR reduction performance.

In Figure 3, the performance of the enhanced PTS algorithm is also analyzed, to compare the performance of PAPR reduction with the traditional PTS methods for fixed and variable length. When CDDF=10⁻³ for 8PSK, the IP scheme gets much 2.7 dB of PAPR reductions than the original OFDM signal. Moreover, the enhanced PTS technique compared with variable and fixed length AP-PTS reduced PAPR 0.4 dB and 0.3 dB, respectively. It is evident that the enhanced PTS technique can provide better performance for PAPR reduction.

Also, in Figure 4, the complementary cumulative distribution functions (CCDF) of the PAPR for the comparison between the enhanced PTS technique with the ordinary PTS methods for fixed and variable length. At CDDF=10⁻³, it can be seen that the PAPR₀ of the original OFDM signal is 10.3 dB, interleaved partitioning (IP) was 7.8 dB, VL-AP was 7.2 dB, adjacent partitioning for fixed length (AP) was 6.8 dB, and enhanced PTS technique was 6.1 dB, respectively. The VL-AP is better performance than the IP by 2.5 dB. As well as, AP and enhanced PTS technique reduced the PAPR related to VL-AP around 0.4 dB and 1.1 dB, respectively with 16QAM modulation.

From Figure 5, as can see the CCDF of the PAPR performance of the conventional PTS schemes with fixed and variable length that compared to the enhanced PTS technique. In the simulations, the modulation technique is 64QAM. The enhanced PTS technique reduced PAPR by around 4.4 dB, fixed length AP by 3.5 dB, VL -AP by 3.1 dB, and fixed length IP by 2.8 dB from the original signal.

It can be observed that the PAPR reduction for the enhanced PTS technique compared to traditional PTS schemes with fixed and variable length is listed in Table 1. From this table, it can be observed that the enhanced PTS technique can achieve the best PAPR reduction performance as the ordinary PTS methods.

<table>
<thead>
<tr>
<th>Modulation</th>
<th>CDDF</th>
<th>PAPR of Enhanced PTS (dB)</th>
<th>PAPR of fixed AP-PTS (dB)</th>
<th>PAPR of VL-AP PTS (dB)</th>
<th>PAPR of IP-PTS (dB)</th>
<th>PAPR of original OFDM (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>QPSK</td>
<td>10⁻³</td>
<td>6.6</td>
<td>6.9</td>
<td>7.2</td>
<td>7.6</td>
<td>10.4</td>
</tr>
<tr>
<td>8PSK</td>
<td>10⁻³</td>
<td>6.4</td>
<td>6.7</td>
<td>6.8</td>
<td>7.3</td>
<td>10</td>
</tr>
<tr>
<td>16QAM</td>
<td>10⁻³</td>
<td>6.1</td>
<td>6.8</td>
<td>7.2</td>
<td>7.8</td>
<td>10.3</td>
</tr>
<tr>
<td>64QAM</td>
<td>10⁻³</td>
<td>5.9</td>
<td>6.8</td>
<td>7.2</td>
<td>7.5</td>
<td>10.3</td>
</tr>
</tbody>
</table>

Therefore, from all the figures, it can be observed that the PAPR reduction with enhanced PTS technique outperform performance than the conventional original OFDM signal and the other types of PTS partitioning schemes of fixed and variable length. On the other hand, the PAPR reduction performances for the PTS scheme using variable length adjacent subsequence partitioning can achieve the best PAPR performance against to the traditional interleaved sub-block partitioning, but it is a little worse than those uses the fixed length of adjacent and enhanced PTS technique subsequence partitioning for any different type of modulations.
Figure 2. PAPR Reduction Performance of the Enhanced PTS Method with the Ordinary Fixed and Variable Length Based on a PTS Scheme for QPSK Modulation

Figure 3. PAPR Reduction Performance of the Enhanced PTS Method with the Ordinary Fixed and Variable Length Based on a PTS Scheme for 8PSK Modulation

Figure 4. PAPR Reduction Performance of the Enhanced PTS Method with the Ordinary Fixed and Variable Length Based on a PTS Scheme for 16QAM Modulation
6. Conclusion

This paper presents an enhanced PTS approach for PAPR reduction in OFDM system that combines two PTS partitioning schemes (adjacent and interleaved) in order to effectively reduce the PAPR of OFDM systems. With an aim of determining the effects of length variability of disjoint sub-blocks of adjacent partitions, we performed an investigation into the performances of a variable length adjacent partitioning (VL-AP) and fixed length adjacent partitioning in comparison with the enhanced PTS scheme. From the various computer simulation results with different types of modulation with the same CCDF, we confirmed that the enhanced PTS approach offers better PAPR reduction performance compared to adjacent partitioning for fixed and variable lengths which itself is based on PTS scheme considered efficient in PAPR reduction.

References


