Improved UFIR Tracking Algorithm for Maneuvering Target

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Abstract
Maneuvering target tracking is a target motion estimation problem, which can describe the irregular target maneuvering motion. It has been widely used in the field of military and civilian applications. In the maneuvering target tracking, the performance of Kalman filter (KF) and its improved algorithms depend on the accuracy of process noise statistical properties. If there exists deviation between process noise model and the actual process, it will generate the phenomenon of estimation error increasing. Unbiased finite impulse response (UFIR) filter does not need priori knowledge of noise statistical properties in the filtering process. The existing UFIR filters have the problem that generalized noise power gain(GNPG) does not change with measurement of innovation. We propose an improved UFIR filter based on measurement of innovation with ratio dynamic adaptive adjustment at adjacent time. It perfects the maneuvering detect-ability. The simulation results show that the improved UFIR filter has the best filtering effect than KF when process noise is not accurate.

Keywords: Maneuvering target tracking, UFIR, KF, GNPE, Adaptive adjustment

1. Introduction
Maneuvering target tracking [1] is very difficulty in the radar target tracking process. KF is widely used in the state estimation which uses process noise of state equation to adapt target maneuvering. In fact, target maneuvering state is unknown which results in difficulty of determining process noise of state equation. Finally, it has an effect on the accuracy of filtering. For this question, many researchers have represented some improved schemes. Wu C, et al [2] proposed current statistic maneuvering target tracking strong tracking filter. The new scheme kept the merits of high tracking precision that the current statistical model and strong tracking filter (STF) had in tracking maneuvering target. Liu Y, et al [3] presented a tracking algorithm based on spline fitting. The assumption was that prediction was without dynamic motion model and it was only based on the curve fitting over the measured data. Fan E, et al [4] showed a fuzzy logic-based recursive least squares filter in situations of observations with unknown random characteristics. It used fuzzy logic in the standard recursive least squares filter by the design of a set of fuzzy if-then rules. Shmaliy [5] proposed UFIR filter which could ignore noise statistical characteristics in the filtering process based on optimal finite impulse response (OFIR) filter and embedded unbiased OFIR filter. In order to obtain the best filter performance, the window length of UFIR filter must be optimal. The window length need not to assume like KF [6-7] by priori knowledge. It can be obtained by measurement calculation. UFIR filter can be expressed as an iterative form which reduces the calculation. However, the existing UFIR filter’s information gain only changes with space equation. We propose an improved UFIR filter algorithm. It uses the deviation between measurement results and filtering results to amend information gain dynamically. Improved UFIR filter has the adaptive ability for maneuvering target tracking. We apply it into the tracking process. Simulation results show that the improved UFIR filter has the best performance.

2. Linear System Model
Nonlinear system can make linearization as the process of extended Kalman filter (EKF) [9]. The advantage of UFIR filter can not be affected. So we only consider a linear system model. Discrete time-varying linear system model can be expressed as :
State equation : \( x_n = F_n x_{n-1} + B_n w_n \) \( (1) \)

Observation equation : \( z_n = H_n x_n + v_n \) \( (2) \)

Where \( x_n \in \mathbb{R}^N \) and \( z_n \in \mathbb{R}^M \) is state vector and observation vector at \( n \) time respectively. \( F_n \in \mathbb{R}^{N \times N} \), \( B_n \in \mathbb{R}^{N \times P} \) and \( H_n \in \mathbb{R}^{M \times N} \) is state-transition matrix, process noise gain matrix and measurement matrix at \( n \) time respectively. Assuming process noise vector is \( w_n \in \mathbb{R}^P \) at \( n \) time. Measurement noise vector is \( v_n \in \mathbb{R}^M \) at \( n \) time. There is no relationship between the two noise vectors. And \( E\{w_n\} = 0, \ E\{v_n\} = 0, \ E\{w_n v_n^T\} = 0 \). \( Q_n = E\{w_n w_n^T\} \) is the process noise covariance matrix. \( R_n = E\{v_n v_n^T\} \) is the measurement noise covariance matrix.

3. The Improved UKIR Filter

3.1. The Improved UKIR Filter Model

Assuming the window length of UKIR filter is \( N \). When we get the measurement value at \( n \) time. \( N \) measurement values are usable from \( m = n-N+1 \) to \( n \) time. And \( m \geq 0 \).

The estimated value \( \hat{x}_n \) of target state can be expressed as:

\[
\hat{x}_n = A_{n,0}^{m+1} B_{n,m}^{-1} Z_{n,m} \]

\( (3) \)

Where

\[
A_{r,h}^{m+1} = \prod_{i=0}^{m} F_n^{r-i} H_n^{r-i} \ldots F_n^{r-h} H_n^{r-h} \]

\( (4) \)

And \( H_n = [H_n H_{n-1} \ldots H_m] \), \( F_n = [A_n^{m+1} A_n^{m+1} F_n^{m+1}] \).

For time-invariant system, \( (3) \) can be simplified into:

\[
\hat{x}_n = F_n^{N-1} B_{n,N-1}^{-1} Z_{n,m} \]

\( (5) \)

Where

\[
B_{n,0}^{-1} = (\overline{H}_n^{-1} \overline{H}_n^{-1})^{-1} \overline{H}_n^{-1} \]

\( (6) \)

And \( \overline{H}_n = \hat{H}_{N-1}^{F_n^{-1}} \), \( \hat{H}_{N-1} = diag(H \ H \ldots H) \), \( F_n^{-1} = [(F_n^{-1})^T \ldots F_n^{-1}]^T \).

From the above equations, we can know that UKIR filter deals with filter for signal with ignoring noise statistical characteristics. When \( N \gg 1 \), UKIR filter is nearly the most optimal. But dimension of matrix and vector will increase with \( N \) increasing. It can result in the computation increasing. Iteration UKIR filter solves this problem. Estimated formula is:

\[
\hat{x}_n = F_n \hat{x}_{n-1} + K_n \left(z_n - H_n F_n \hat{x}_{n-1}\right) \]

\( (7) \)

Where

\[
K_n = G_n H_n^T \]

\( (8) \)

\[
G_n = [H_n^T H_n + (F_n G_n^{-1} F_n^T)^{-1}]^{-1} \]

\( (9) \)

\( G_n \) is generalized noise power gain (GNPG) at \( n \) time. Initial conditions \( \hat{x}_1 \) and \( G_1 \) can be obtained by UFIR.
\[ \hat{x}_i = A^{m+1}_{i,0}B^{-1}_{i,m}Z_{i,m} \]  
\[ G_i = A^{m+1}_{i,0}(H_i^{T}H_{i,m})^{-1}(A_i^{m+1})^T \]  
\[ \hat{x} = F\hat{x}_{a-1} + K_a(z_a - HF\hat{x}_{a-1}) \]  
\[ K_a = G_aH^T \]  
\[ G_a = [H^TH + (FG_a^{-1}F^T)^{-1}]^{-1} \]  
\[ \hat{x}_i = F^{i-m}B^{-1}_{i,m}Z_{i,m} \]  
\[ G_i = F^{i-m}(H_i^{T}H_{i,m})^{-1}(F^{i-m})^T \]  
Where \( i=m+K-1 \). Iteration variable \( a : m+K \leq a \leq n \). When \( a=n \), we get the estimation value \( \hat{x}_n \). The time-invariant system can be simplified as:

3.2. Improved Iterative UKIR Filter

In the maneuvering target tracking, noise can cause a large deviation between measurement data and filter data. So deviation can reflect the maneuvering status. Formula (7) and (12) show that GNPG only has the relation with state transition matrix and measurement matrix. Each filtering result of iterative UKIR filter is obtained by the original independent iteration \( N-K \) measurement results. The GNPG filtering processes of different time are independent. So changing GNPG is unconstrained. Thus we define a generalized noise power gain adjustment coefficient \( \partial \) to adaptive adjust GNPG by deviation between measurement data and filter data in this paper. That can further improve the filtering effectiveness of UKIR filter.

This paper selects root mean square \( \eta_k \) of deviation as description of maneuvering target. And \( \eta_k \) is expressed by:

\[ \eta_k = \sqrt{\frac{(Z_k - H_k\hat{x}_k)^T(Z_k - H_k\hat{x}_k)}{\nu}} \]  

Where \( \nu \) is dimensions of the target motion. \( Z_k \) is the measurement data at \( k \) time. \( \hat{x}_k \) is the filtering result at \( k \) time. \( H_k \) is measurement matrix.

In order to reflect the obvious maneuvering target tracking, we set a deviation ratio \( \lambda \).

Based on root mean square, the \( \lambda \) can be set:

\[ \lambda_k = \sqrt[\kappa]{\eta_k / \eta_{k-1}} \]  

In fact, noise can result in filtering divergence. So we select the mean value for \( \lambda \) to remove the effect of noise. The range of mean value is half of window length.

At \( k \) time, the generalized noise power gain adjustment coefficient is:
\[ \hat{p}_k = \frac{1}{N/2} \sum_{j=k_0}^{k} \lambda_j \]  

(17)

Where \( k_0 = k - \lfloor N/2 \rfloor + 1 \), \( \lfloor N/2 \rfloor \) denotes the integer part of \( N/2 \).

And \( k_0 \geq m + K + 1 \), \( k \geq m + K + \lfloor N/2 \rfloor \). So the time of generalized noise power gain adjustment starts at \( m + K + \lfloor N/2 \rfloor \). Formula (1) and (12) can be represented as:

\[ G_k = \hat{p}_k \left[ H_k^T H_k + (F_k G_{k-1} F_k^T)^{-1} \right]^{-1} \]  

(18)

\[ G_k = \hat{p}_k \left[ H_k^T H_k + (FG_{k-1} F^T)^{-1} \right]^{-1} \]  

(19)

Along with the iterative process, the effect of initial batch processing length will be reduced. \( \hat{p} \to 1 \) is able to ensure filtering convergence. With the increase of the number of iterations, the covariance of target location tends to decrease. We can get the explanation of the improved UKIR filter, when the deviation between measurement and filtering result is bigger than previous time. We should increase the weight of innovations in GNPG to reduce the filtering error. When the deviation is smaller than previous time, it shows that the filtering result is accurate. So it reduces the weight of innovations appropriately which does not affect the filter result. At the same time, the decrease of the GNPG will be able to improve the convergence rate of the filter.

4. Maneuvering target tracking simulation.

The simulation scene is: Acceleration-Turn.

Accelerating zone: \( 0 \leq t \leq 20 \). Uniform zone: \( 20 \leq t \leq 40 \). Turning zone: \( 40 \leq t \leq 70 \). Uniform zone: \( 70 \leq t \leq 100 \). As in figure 1.

![Simulation scene](image)

Figure 1. Simulation scene

Simulation system is described as white noise model under rectangular coordinate system. In the formula (1) and (2), \( x \) is defined as displacement, velocity and acceleration 9-dimensional vector at \( x-y-z \) direction.
Improved UFIR Tracking Algorithm for Maneuvering Target (Shoulin Yin)

\[ F = \begin{bmatrix}
1 & 0 & 0 & T & 0 & 0 & 0.5T^2 & 0 & 0 \\
0 & 1 & 0 & 0 & T & 0 & 0 & 0.5T^2 & 0 \\
0 & 0 & 1 & 0 & 0 & T & 0 & 0 & 0.5T^2 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & T & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\end{bmatrix}, \quad H = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\end{bmatrix}, \quad B = I. \\

Process noise covariance \( Q \) and measurement noise covariance \( R \):

\[ Q = \begin{bmatrix}
0.25T^4 & 0 & 0 & 0.5T^3 & 0 & 0 & 0.5T^2 & 0 & 0 \\
0 & 0.25T^4 & 0 & 0 & 0.5T^3 & 0 & 0 & 0.5T^2 & 0 \\
0 & 0 & 0.25T^4 & 0 & 0 & 0.5T^3 & 0 & 0 & 0.5T^2 \\
0.5T^3 & 0 & 0 & T^2 & 0 & 0 & T & 0 & 0 \\
0 & 0.5T^3 & 0 & 0 & T^2 & 0 & 0 & T & 0 \\
0 & 0 & 0.5T^3 & 0 & 0 & T^2 & 0 & 0 & T \\
0.5T^3 & 0 & 0 & T & 0 & 0 & 1 & 0 & 0 \\
0 & 0.5T^3 & 0 & 0 & T & 0 & 0 & 1 & 0 \\
0 & 0 & 0.5T^3 & 0 & 0 & T & 0 & 0 & 1 \\
\end{bmatrix}, \quad R = \begin{bmatrix}
400 & 0 & 0 \\
0 & 400 & 0 \\
0 & 0 & 400 \\
\end{bmatrix}. \\

\( T \) is sample interval, \( T=0.1s. \)

The existing maneuvering target tracking algorithms based on KF can also design the corresponding tracking algorithms based on UFIR filter. The advantage of ignoring noise statistical characteristics in UFIR filter cannot change. So we compare the performance of KF, UFIR and improved UFIR in this paper.

When target trajectory is attached zero mean gaussian white noise (its covariance is 100), Kalman filter may achieve the optimal solution. The optimal window length of UFIR filter \( N_{opt} = 55 \) [10] and length of the batch processing \( K=5 \) by calculating. Position mean square error (PMSE) is as figure 2, 3, 4 with known noise statistical characteristics.

![Figure 2. PMSE at x-axis](image1)

![Figure 3. PMSE at y-axis](image2)
From figure 2, 3, 4, we can know that filter performance of KF is superior to UKIR and improved UKIR when noise statistical characteristics is known. The improved UKIR is similar to UKIR. It proves that the KF is optimal when noise statistical characteristics is obtained. However, we cannot accurately predict the target moving condition. We have assumed that the process noise statistical properties may be wrong. In order to verify the robustness under the unknown process noise statistical properties, we adjust the process noise covariance as 16. The simulation results are as figure 5, 6, 7 with unknown noise statistical characteristics.

From figure 5, 6, 7, we can know that the estimation error of KF significantly increases when the process noise statistical properties is known. However, UFIR filter does not need prior information of process noise statistical properties in the filtering process. It represents stronger robustness for the inaccurate process noise statistical properties. Moreover, the improved UFIR filter can better adapt to the target maneuvering and get better filtering performance than existing UFIR filter by using the deviation between measurement and filtering results to adaptive adjust new rate gain matrix.

5. Conclusion

In this paper, we propose the improved UFIR filter to offset the deficiency of existing UFIR filter. We use it for maneuvering target tracking. The simulation experiments show that:
1. when the known initial values and noise statistical distribution follow ideal conditions, KF is slightly better than existing UFIR filter;
2. when noise statistical distribution is unknown, UFIR filter shows the stronger robustness than KF;
3. this paper’s new scheme can adaptive adjust new rate gain matrix which shows a better filtering performance than existing UFIR filter;
4. though the new UFIR filter can improve the filter precision, its amount of computation is $N_{opt}$ times greater than the KF.

In the future, we will evaluate our scheme with other UFIR filter algorithms and design a UFIR filter with small amount of computation for using it in the maneuvering target tracking.

References