Multi Dimension of Coarse to Fine Search Method Development for Solving Economic Dispatch

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Abstract
Economic dispatch problem has grown along with the development of electric power business, for example in a competitive electric power business that offers electrical energy in the form of the step function, non-differentiable function. This is not a continuous function so there is no guarantee that those methods can execute the optimization problem well, especially the Lagrange and Direct methods. There are the non-differentiable functions within the optimization will become a challenge that should be solved. This paper proposes Coarse to Fine Search method development to solve the problem. The Coarse to Fine Search is able to work for differentiable or non-differentiable functions, but is only limited maximum three dimensions. The development is done to multi dimension so that it can solve the economic dispatch problem. We named it Multi Dimension of Coarse to Fine Search. The simulation results of eight power plants show the developed method can work well, it is always convergent and fast with the execution time of 2.63 - 38.30 seconds for 25 - 200 population and 50 - 200 delta search.

Keywords: Optimization, Multi dimension, non-differentiable, always convergent, accuracy.

1. Introduction
Improving efficiency of the electrical power is a challenge for experts due to the electric prices that tend to rise steadily. The fuel cost is the main component, which generally the fuel cost is the 60-80% of total cost. So, minimizing the fuel cost of power plant is essential to be done by economic dispatch (ED) [1]. Especially in the competitive electric power business in order to determine the auction winner of the installed capacity should be based on a fair method, the accurate economic dispatch, the time process should be short, and the system must be robust [2]. Besides that, ED is also applied in the integrated system for scheduling power plants. A few methods have been published to solve the ED problem and Optimal Power Flow (OPF). Researchers have published a few methods to solve ED and OPF problems. Direct method is accurate and very simple but limited by the quadratic objective function [3]. The Particle swarm method [4-15], are not accurate because they are easy to fall into local optimum, studied by [16].

Whereas the ED problem can have the differentiable or non-differentiable objective function. For the differentiable objective function, its completion is not problem. It can be solved accurately by the calculus method, like Direct or Lagrange methods. But for the non-differentiable, it has a problem when solved by those methods, because of inaccurate.

This paper propose Coarse to Fine Search method, CFS [17-20], because based on study, it can be applied to differentiable or non-differentiable objective function of the optimization problem, and it is very accurate. But the method has not been applied yet in the power system and only it is limited by three dimension, like applied in image processing.

The ED problem is the multi-dimension that is suitable the number of unit generator. This is challenge of the CFS method for solving the ED problem. The paper will develop the CFS for Multi Dimension CFS, MD-CFS. The methodology will be described clearly, where the dimensions are decomposed into two or three dimensions, for example 7 dimension will be decomposed into 2 part of two dimensions and 1 part of three dimensions. Furthermore, the method will be tested by 8 generators, 8 dimensions.
2. Research Method
2.1. CFS Method

2.1.1. CFS Concept

CFS can be described by Figure 1, the feasible area is reduced until it get smallest area, called convergent point. But every step reducing is certained the convergent point will always be in the reducing area.

![Figure 1. Feasibility Area](image)

The optimization process starts from a large area \(a_0, b_0, c_0, d_0\) to a smaller area \(a_1, b_1, c_1, d_1\) and up to very small area \(a_n, b_n, c_n, d_n\) which are considered as a point solution. In each area must be ensured that the convergent point to be in it. It is stated by the point X is always in each of the areas. To get a smaller area, the previous area is divided into several areas, and from them is determined the area with the smallest objective value by testing a few points that exist within each area. With the above description, the CFS method is able to work in a variety of objective function of optimization problems, whether differentiable or not, such as step function.

2.1.2. CFS Algorithm

The number of population spread over the area and each tested with the objective function. The one that has the best value from the number of the population is taken as the best population, \(P_{i}^{\text{best}}\). For the next step, in the area around of \(P_{i}^{\text{best}}\) that has a half side of previous area such that \(a_i, b_i, c_i, d_i\) is the same amount of population are spread and evaluated with the same objective function, so that the best population, \(P_{i}^{\text{best}}\), is obtained. Subsequently, we came up with a very small area and found the best population, \(P_{M}^{\text{best}}\) where delta cost less-than \(\varepsilon\), and \(P_{M}^{\text{best}}\) is a point of convergence. CFS optimization can be derived in four steps as follows:

1) Determine a feasible area and spread the number of population, \(N_{\text{pop}}\). The feasible area can be line, 2-dimensional field, or 3-dimensional space depends on the number of plants are involved.

2) Find the best populations were determined from the minimum objective value expressed by the following equation:

\[
P_{ij}^{\text{best}} = \min: f_j(P_{i}^{\text{pop}}), \quad i=1,2,...,N_{\text{pop}}, \quad j=1,2,...,M
\]

(1)

With,

\[
f_j(P_{i}^{\text{pop}}) = \sum_{i=1}^{N} f_i(P_{i})
\]

(2)

\[
f_i(P_{i}) = a_i + b_i P_{i} + c_i P_{i}^2
\]

(3)
Where \( f_j(P_{opf}) \) is Total cost of all plant, \( f_i(P_i) \) is Cost function of each plant, \( P_i \) is Power generated by the plant\(-i\), \( N \) is Number of Plant, \( N_{pop} \) is Number of population, \( M \) is Number of iteration, and \( a, b, c \) is Characteristic coefficient of plant\(-i\) of \( a, b, \) and \( c \) respectively.

3) Spread the \( N \) of population, \( N_{pop} \) in the around of the best population with the feasible area has side length a half of previous feasible area's side length, and find it's the best population.

4) Repeat the Step 3, so that its feasible area is very small and the best population can be considered as the point of convergence.

The iteration process is stopped if the difference of the best population value in the iteration\(-i\) of \( P_{best} \) with the best population value in iteration \( i-1 \) is satisfy to the Equation (4).

\[
\Delta \text{Cost} = f_j(P_{opf}) - f_{j-1}(P_{opf}) < \varepsilon, \tag{4}
\]

Where \( \varepsilon \) is the value that is previously determined, the value of \( \varepsilon \) is relatively not significant to the total generation cost.

2.1.3. CFS Application

Two power plants have the constraints as the following expression.

\[
\begin{align*}
10 & \leq P_1 \leq 50 \\
20 & \leq P_2 \leq 80 \\
10 & \leq P_1 \leq 100 \\
100 & \leq P_2 \leq 100
\end{align*}
\tag{5}
\]

Figure 2 depicts the feasibility area of the constraints, where horizontal axis represents active power of the power plant 1, vertical axis represents active power of the power plant 2 and feasibility area is stated by strike line satisfying power balance.

![Figure 2. Feasibility Area of Two Power Plants](image)

2.1.3.1. For Differentiable Case

Suppose the problem is a quadratic function, namely:

\[
\begin{align*}
f_1(P1) &= 200 + 10P1 + 0.028P1^2 \tag{6} \\
f_2(P2) &= 300 + 11P2 + 0.011P2^2 \tag{7}
\end{align*}
\]

Furthermore, the calculation process until three iteration are described by Figure 3.
Figure 3 shows that finding the best population is done from feasible area with spread the \( N \) population in along the line \( a_0c_0 \) so that we obtain the best population candidate \( P_{1}^{\text{best}} \). In the around of \( P_{1}^{\text{best}} \), a fraction of the population are also spread along the line of \( a_1c_1 \) and we obtain \( P_{2}^{\text{best}} \). The process continues until the best candidate is obtained and Equation (4) is satisfied.

<table>
<thead>
<tr>
<th>Item</th>
<th>Iteration ( i_1 )</th>
<th>Iteration ( i_2 )</th>
<th>Iteration ( i_3 )</th>
<th>Iteration ( i_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>38.00</td>
<td>42.00</td>
<td>41.00</td>
<td>41.00</td>
</tr>
<tr>
<td>P2</td>
<td>62.00</td>
<td>58.00</td>
<td>59.00</td>
<td>59.00</td>
</tr>
<tr>
<td>Total Power</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
<td>100.00</td>
</tr>
<tr>
<td>Total Cost</td>
<td>1,644.716</td>
<td>1,644.396</td>
<td>1,644.359</td>
<td>1,644.359</td>
</tr>
</tbody>
</table>

From Table 1, Delta cost from two plants for the differentiable function case are defined in Table 2.

<table>
<thead>
<tr>
<th>Item</th>
<th>Iteration ( i_2-i_1 )</th>
<th>Iteration ( i_3-i_2 )</th>
<th>Iteration ( i_4-i_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delta Cost</td>
<td>0.32</td>
<td>0.037</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 1 shows the trend of the total cost for the above case is decreasing toward the converging point. If \( \varepsilon \) is set at 0.5 the process stops at the 2\textsuperscript{nd} iteration. When \( \varepsilon \) is set to 0.05, the process stops at the 3\textsuperscript{rd} iteration. When \( \varepsilon \) is set to 0.005, the process stops at the 4\textsuperscript{th} iteration. The comparison between CFS Method and Direct Method is shown by Table 3. It shows that the results of CFS optimization relatively equal with the result of the Direct Method.

<table>
<thead>
<tr>
<th>Item</th>
<th>CFS Method</th>
<th>Direct Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>41.00</td>
<td>41.03</td>
</tr>
<tr>
<td>P2</td>
<td>59.00</td>
<td>58.97</td>
</tr>
<tr>
<td>Total Cost</td>
<td>1,644.359</td>
<td>1,644.359</td>
</tr>
</tbody>
</table>
2.1.3.2. For Non-differentiable Case

The general form of the step function can be expressed by,

$$B(P) = C_i + \text{step}_i \cdot k,$$

(8)

Where $C_i$, $\text{step}_i$, and $k$ are power plant constant $i^{th}$, size function of $i^{th}$ power plant, and Multiplier factor respectively. $C_i$ and $\text{step}_i$ can have different values, and do not affect the optimization process, so that $\text{step}_i$ can use the same value to simplify the problem.

While the process to obtain the objective function is defined by:
1. Divide the $i^{th}$ plant capacity ($\text{Cap}_i$) with $N_{\text{step}}$, for example $N_{\text{step}}=10$, means the $i^{th}$ plant only generate 10 different values ($P_i$).
2. Compute the cost value with substitute $P_{\text{best}}$ and $\text{step}_i$ to obtain multiplier factor value, where:
   $$K = j \times \frac{\text{Cap}_i}{N_{\text{step}}}$$
3. IF $K \geq P_{\text{best}}$, THEN factor = $j$;
4. So that $B(P_i)$ can be computed for all candidates ($P_{\text{best}}$).
5. $P_{\text{best}}$ that give lowest $B(P_i)$ is used as a candidate for the next iteration.

Two power plants that have cost as the step function are shown in Figure 4. Step function are used for spot price decision [21], for load and the power plant limit equal to the case of differentiable functions.

![Figure 4. Step Function of Cost Characteristic](image)

With the limiting function given in Equation (5), and the cost function given in Figure 4, both of the power plants provide the power generation value towards the convergent value. With $\varepsilon = 0.005$, the process will stop in the 19th iteration, where the value of delta cost is between the value in the 18th and 19th iteration equivalent with zero $< 0.005$, $P1 = 30.20$ and $P2 = 69.80$ as convergence point.

2.2. Development for Multi Dimensions (MD-CFS)
Figure 6. $N$-dimensional CFS for $N=4$ those are $P_1, P_2, P_3$ and $P_4$

The $N$-dimensional CFS can be explained easily by means of three dimensional geometry, where the candidates are spread in the feasible area evenly, as shown in Figure 5. The feasible area in the triangular shape is shrinking to the point of convergence.

This multidimensional solution can be explained through the partition as shown in Figure 6.

In the simple method, this study which uses $N$ power plants, there are $N/2$ pairs of 2-D or multi-dimensional CFS, as shown in Figure 6.

In the implementation, the system does not divide $N$ dimension to $N/2$, but the system tries to find one point in $(P_1^{\text{best}}, P_2^{\text{best}}, P_3^{\text{best}}, ..., P_n^{\text{best}})$ in $N$-dimension, so that the cheapest cost is achieved and also the load requirement is satisfied.

3. Simulation and Result

The CFS system testing is done with some real data. For the differentiable case, CFS uses generator plant cost function as shown in Table 4.

<table>
<thead>
<tr>
<th>Generator</th>
<th>Characteristic of Generator Cost Function [9]</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>$65.94P_1^2 + 395668.05P_1 + 3163021$</td>
</tr>
<tr>
<td>P2</td>
<td>$690.98P_2^2 + 32478064.47P_2 + 107892572.17$</td>
</tr>
<tr>
<td>P3</td>
<td>$0 + 6000.00P_3 + 0$</td>
</tr>
<tr>
<td>P4</td>
<td>$0 + 5502.00P_4 + 0$</td>
</tr>
<tr>
<td>P5</td>
<td>$21.88P_5^2 + 197191.76P_5 + 1636484.18$</td>
</tr>
<tr>
<td>P6</td>
<td>$132.15P_6^2 + 777148P_6 + 13608770.96$</td>
</tr>
<tr>
<td>P7</td>
<td>$52.19P_7^2 + 37370.67P_7 + 6220765.38$</td>
</tr>
<tr>
<td>P8</td>
<td>$533.92P_8^2 + 2004960.63P_8 + 86557397.40$</td>
</tr>
</tbody>
</table>

Table 5. Parameters of MD-CFS Simulation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Candidate</td>
<td>25 - 200</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>5,000,000</td>
</tr>
<tr>
<td>Delta Search</td>
<td>5 - 150</td>
</tr>
<tr>
<td>Number of Plant</td>
<td>8</td>
</tr>
</tbody>
</table>
The simulation results of MD-CFS method for differentiable and non-differentiable cases are shown in the Table 6 and Table 7. Table 6 is for differentiable function case and Table 7 is for non-differentiable function case.

Table 6. The Comparison of PSO Method, Direct Method and MD-CFS Method Simulation Results

<table>
<thead>
<tr>
<th>Generator</th>
<th>PSO Method Power (MW)</th>
<th>After Optimization</th>
<th>MD-CFS Method Power (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>2,999.75</td>
<td>3,332.18</td>
<td>3,332.18</td>
</tr>
<tr>
<td>P2</td>
<td>1,174.14</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>P3</td>
<td>960.673</td>
<td>948.00</td>
<td>948.00</td>
</tr>
<tr>
<td>P4</td>
<td>770.906</td>
<td>698.40</td>
<td>698.40</td>
</tr>
<tr>
<td>P5</td>
<td>696.889</td>
<td>1,321.60</td>
<td>1,321.60</td>
</tr>
<tr>
<td>P6</td>
<td>530.184</td>
<td>883.57</td>
<td>900.00</td>
</tr>
<tr>
<td>P7</td>
<td>2,796.59</td>
<td>3,100.00</td>
<td>3,084.92</td>
</tr>
<tr>
<td>P8</td>
<td>454.61</td>
<td>100.00</td>
<td>100.85</td>
</tr>
</tbody>
</table>

Total (MW) 10,383.75 10,383.75 10,385.95
Total Cost (IDR/Hour) 8,002,149,695.65 4,141,852,694.99 4,205,656,273.08

By using the objective function in Table 3 and the parameters in Table 5, for non-differentiable function case: we obtain the generation result is 10,385.95 MW with a total cost of IDR 4,205,656,273.08. For the varying number of steps, the results are shown in Table 7.

4. Analysis

Table 6 shows that the MD-CFS method has high accuracy, this is indicated by the results of the Direct method is not much different with error rate that is determined as:

\[
\frac{4,312,750,827.02 - 4,141,852,694.99}{4,141,852,694.99} \times 100\% = 4.126\%.
\]  

(9)

The MD-CFS method is more accurate when compared with the PSO method, and will be more superior when the cost function that is used is non-differentiable function as previously described in the two-dimensional case and eight-dimensional case.

Table 7. The Optimization Results for Non-Differentiable Case

<table>
<thead>
<tr>
<th>Plant</th>
<th>Power (MW)</th>
<th>Cost (IDR/Hour)</th>
<th>Power (MW)</th>
<th>Cost (IDR/Hour)</th>
<th>Power (MW)</th>
<th>Cost (IDR/Hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>3,332.18</td>
<td>2,053,315,795.95</td>
<td>3,332.18</td>
<td>2,053,315,795.95</td>
<td>3,332.18</td>
<td>2,053,315,795.95</td>
</tr>
<tr>
<td>P2</td>
<td>0.00</td>
<td>107,892,572.17</td>
<td>0.00</td>
<td>107,892,572.17</td>
<td>0.00</td>
<td>107,892,572.17</td>
</tr>
<tr>
<td>P3</td>
<td>948.00</td>
<td>5,688,000.00</td>
<td>948.00</td>
<td>5,688,000.00</td>
<td>948.00</td>
<td>5,688,000.00</td>
</tr>
<tr>
<td>P4</td>
<td>698.40</td>
<td>3,842,596.80</td>
<td>698.40</td>
<td>3,842,596.80</td>
<td>698.40</td>
<td>3,842,596.80</td>
</tr>
<tr>
<td>P5</td>
<td>1,321.60</td>
<td>300,461,303.33</td>
<td>1,321.60</td>
<td>300,461,303.33</td>
<td>1,321.60</td>
<td>300,461,303.33</td>
</tr>
<tr>
<td>P6</td>
<td>890.57</td>
<td>820,083,470.96</td>
<td>890.57</td>
<td>820,083,470.96</td>
<td>890.57</td>
<td>820,083,470.96</td>
</tr>
<tr>
<td>P7</td>
<td>3,032.63</td>
<td>601,535,506.12</td>
<td>3,050.27</td>
<td>607,794,834.70</td>
<td>3,084.92</td>
<td>620,184,504.78</td>
</tr>
<tr>
<td>P8</td>
<td>163.89</td>
<td>429,491,450.00</td>
<td>135.59</td>
<td>368,225,940.54</td>
<td>100.85</td>
<td>294,188,029.09</td>
</tr>
</tbody>
</table>

Total 10,387.27 4,312,750,827.02 10,386.04 4,267,304,514.45 10,385.95 4,205,656,273.08
For the differentiable cost function with the order of polynomial is greater than two, the Direct method not work properly but the MD-CFS method can still work well. In general, non-differentiable functions require a greater cost than differentiable function. This is due to a differentiable function will provide cost on each generation value. While for non-differentiable function, each generation values are checked previously with the existing value in step of un-differentiable function. If the generation value is not in the non-differentiable function, then it will be rounded up or shifted to the nearest value in non-differentiable function level. This causes a shift of cost that becomes more expensive. As a solution, the number of steps (Nstep) of non-differentiable function can be increased so that the cost of generation is getting closer to a differentiable function as shown in Table 7.

5. Conclusion
1. The MD-CSF method has been successfully developed to solve Multi-dimensional Economic Dispatch problems, as shown for the case of eight power plants in Table 6 and Table 7.

2. The MD-CFS method has the advantage that is not affected by the both of differentiable and non-differentiable functions of cost generation. This is indicated by the case of two-dimensional non-differentiable function with the results in Table 3, and eight-dimensional case results in Table 7.

3. The proposed method can also work well for a variety of generation costs functions, such as the polynomial with order greater than two. As a comparison, the Direct Method only works for polynomial order of two.

4. From simulation results, it can be concluded that the MD-CFS method can solve the ED problem for both of Differentiable and non-differentiable function.

References