Improvement of Fuzzy Based Practical Controller for Continuous Motion Control

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Abstract
This article presents a development of a fuzzy based nominal characteristic trajectory following (NCTF) controller for continuous motion control. A new structure is proposed in order to achieve excellent performance of tracking to a continuous reference input and also for point-to-point positioning task. The proposed structure maintains the NCTF controller simple configuration which is composed of a nominal characteristic trajectory (NCT) and a compensator. The compensator is based on a Mamdani type fuzzy controller. Its membership functions are designed according to the available information provided by NCT and the hardware specification. Controller performance was evaluated through simulation by comparing it with the existing method previously proposed for the fuzzy based NCTF controller. The tracking performance was evaluated by measuring responses of the system providing continuous sinusoidal signal input. The result indicates that substantial improvement is achieved in tracking of continuous reference inputs. Moreover, a better result is also obtained in performing point-to-point positioning task.

Keywords: nctf, fuzzy control, tracking, trajectory following, continuous motion

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1. Introduction
Motion control systems are applied in various industrial applications such as machine tools, robotic systems and measuring machines. Industrial machines are required to perform high performance tasks such as positioning, tracking and contouring. These tasks are often used to evaluate performance of such systems. In order to realize high performance for all the types of motions, a general-purpose servo controller is desired. The controller is also expected to satisfy such requirements as high accuracy, fast response and robust. For motor servo systems, the position tracking precision is considered as the main index of evaluation [1]. Inaccurate tracking may lead to a failure of a system. Moreover, the practical controller design method is also required for practical applications.

Various controllers have been developed such as a PID control system which provides the simplest and yet most efficient solution to many control problems [2, 3]. Various PID tuning have been patented and many software packages are available. However, accurate model of the system is indispensable to perform satisfactory performance. Moreover, complexity of the model increases due to the different behaviour of the macro-region to the micro-region. Two-step controllers have been extensively studied in order to overcome the problem, particularly the object parameters difference of that two regions [3]. Since fuzzy control is less sensitive to the changes of parameters, fuzzy control approach is combined with PID control. The less accurate model is then required. Thus, it offers advantages in practical applications and implementation of fuzzy control [4-6].

For practical applications, a nominal characteristic trajectory following (NCTF) controller has been proposed [7]. It has advantages of simple structure and ease of design procedures. The structure consists of a nominal characteristic trajectory (NCT) and a PI compensator (NCTF-PI). NCT is constructed from a simple open loop experiment and PI compensator parameters are taken from NCT information. However, the drawback of the NCTF-PI is in the process of deciding PI parameters. Since an unlimited combination of PI parameters, this lead to designer judgement or trial and error approach [8].
NCTF controller with fuzzy compensator (NCTF-Fuzzy) has been proposed to replace PI compensator. The intention is to avoid trial and error approach and make the NCTF controller more practical. For point-to-point positioning task, NCTF-Fuzzy controller is effective since it gives a similar response compared with the NCTF-PI controller [9]. However, the performance of NCTF-Fuzzy controller performing continuous motion tasks require deep investigations.

This study presents the investigation of NCTF-Fuzzy controller dealing with continuous motion tasks. The concept of the NCTF controller is reviewed and the proposed design method is explained in section 2. In section 3, the controlled object is described and its controller is designed. The performance evaluation through simulation is then presented in section 4 by comparing with the existing method. Finally, the conclusion and future works are described in section 5.

2. NCTF Controller

Concept and design procedure of the NCTF control system have been explained in [7-9]. It is comprised of three steps: (i) The controlled object is driven with an open-loop step input and its displacement and velocity are measured. (ii) Using the displacement and velocity of the mechanism during the deceleration, the NCT is constructed on the phase-plane. (iii) The compensator is then designed using the open-loop response and the NCT information.

The structure of fuzzy based NCTF control system consists of an NCT and a fuzzy compensator. Figure 1 shows the proposed fuzzy based NCTF control system. The structure slightly different in feeding the inputs. The first input is the error and second is the object motion in the original NCTF structure instead of the error rate in the proposed structure. This is considering that in the continuous motion, controllers usually act near the reference and the action far from the reference is not important [10].

![Figure 1. The structure of NCTF control system with fuzzy compensator for continuous motion](image)

The typical NCT is shown in Figure 2. Important NCT parameters are maximum error rate, $h_{\text{max}}$, NCT maximum error, $A$, and gradient at origin, $m$.

![Figure 2. Typical NCT and its parameters](image)
2.1. Compensator Structure

The structure of the fuzzy compensator is shown in Figure 3. The fuzzy compensator is a Mamdani type fuzzy compensator with two inputs, $u_p$ which is the difference between the object motion and NCT, and $u_i$ which is the integral of $u_p$. The output is the control signal $u$.

![Figure 3. The structure of the Mamdani type fuzzy compensator](image)

2.2. Rule Design

Construction of the rule base is designed according to the object motion in reaching and following NCT as shown in Figure 4. The reaching phase region is when the object motion approaches the NCT and the following phase region is when the object follows the NCT within bounded specified accuracy.

![Figure 4. The controlled object motion](image)

Based on the object motion in reaching and following the NCT, the fuzzy rules are summarized as shown in Table 1.

<table>
<thead>
<tr>
<th>$u_i$</th>
<th>N</th>
<th>Z</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_p$</td>
<td>N</td>
<td>N</td>
<td>Z</td>
</tr>
<tr>
<td></td>
<td>Z</td>
<td>N</td>
<td>Z</td>
</tr>
<tr>
<td></td>
<td>P</td>
<td>Z</td>
<td>P</td>
</tr>
</tbody>
</table>

2.2. Membership Function Design

In the fuzzification process, crisp signal $u_p$ and $u_i$ is converted into fuzzy variables. The membership function of input $u_p$ is shown in Figure 5(a).
There are three triangular-shaped membership functions which are Negative (N), Zero (Z) and Positive (P). Signal $u_p$ only varies in the range of $\pm h_{\text{max}}$. The membership function of $u_p$ Zero (Z) has the value of error rate of sensor resolution, $h_{s\text{-res}}$.

The membership function of input $u_i$ is shown in Figure 5(b). There are also three triangular-shaped membership functions which are N, Z and P. The range of this input is $\pm u_{i\text{-max}}$ calculated based on the following equation:

$$u_{i\text{-max}} = \int_{-h}^{h} u_i \, de \leq 0.5 h_{\text{max}}$$

(1)

In the following phase, object motion oscillates within $\pm$ sensor resolution, $a_{s\text{-res}}$. Thus the membership function of $u_i$ Zero (Z) can be simplified based on the following equation:

$$u_{i\text{-res}} = \pm a_{s\text{-res}} \| h_{s\text{-res}} = 4 a_{s\text{-res}} h_{s\text{-res}}$$

(2)

In the defuzzification process, fuzzy variables are converted into a crisp signal $u$. The membership function of output $u$ are shown in Figure 5(c). There are three membership functions which are z-shaped N, singleton Z and s-shaped P. The range of fuzzy variable output is $\pm u_r$, which is the rated motor input.

The fuzzy output variable, $u_i$, is converted to the crisp control output using center of area (COA) defuzzification method. The combination of COA method, z-shaped and s-shaped makes the crisp output never reach the maximum rated motor input. However, since the controllers act near the reference, maximum control signal value may not necessary.

3. Results and Analysis

3.1. System Description

In order to evaluate the effectiveness of the proposed controller design, a series of simulation is carried out based on the dynamic model of the experimental linear positioning system as a controlled object as shown in Figure 6(a). The system consists of a DC motor coupled with ballscrew to drive a linear slide moving table. Dynamic model of the system is derived according to the information from NCT to construct simplified model of the system [7].

$$G(s) = K \frac{\alpha}{s(s + \alpha)}$$

(3)

Where $K = \frac{h_{\text{max}}}{u_r}$ and $\alpha = -m$.

Following the procedure given in [7], the system is driven by a stepwise input and the system responses are measured as shown in Figure 6(b). The stepwise input value is the rated motor input, $u_r$. 

![Figure 5. The membership functions design](image)
The NCT of the controlled object is then constructed according to the data within deceleration range, as shown in Figure 6(c). Required parameters to construct the simplified model and to design the compensator are $m$, $A$, $a_{s-res}$, $h_{max}$ and $h_{s-res}$. The simplified dynamic model of the controlled object derived from NCT information is calculated according to equation (3) is,

$$G(s) = \frac{0.2152}{s + 0.2152}$$

### 3.2. Compensator Design

Following the proposed design procedure, the membership function of the controlled object $u_p$ is shown in Figure 7(a). The range of $u_p$ is $\pm h_{max} = \pm 56.72$. The membership function of $u_p$ Zero (Z) has the value of $\pm h_{s-res} = \pm 0.2152$. The membership function of the controlled object $u_i$ is shown in Figure 7(b). The range of the membership function follows equation (1), $\pm u_{i-max} = \pm 38.4$. The membership function of $u_i$ Zero (Z) has the value of $\pm u_{i,s-res} = \pm 0.0008$ according to the equation (2). The membership function of the controlled object output $u$ are shown in Figure 7(c). The range of $u$ is $\pm u_i = \pm 10V$. 

![Diagram](image.png)
4. Evaluation

4.1. Positioning Performance

In order to evaluate its performance, the proposed controller is compared with the existing proposed method in designing fuzzy based NCTF controllers. The performance of the previous method of fuzzy based NCTF (F-NCTF-exst) controller performing a point-to-point task has been reported overcome the normal NCTF controller [9]. In this evaluation, the performance of the proposed controller (F-NCTF-prop) is compared to that of the controller through simulation using dynamic model of an experimental linear positioning system.

The F-NCTF-exst controller is strictly designed according to the procedure mentioned in [9]. It has the object velocity as the NCT second input. The structure of its fuzzy compensator includes a gain $K_f$ to amplify the control signal $u$, and its fuzzy membership functions may provide maximum actuator rated by using singletons as fuzzy output variables.

Figure 8 shows the responses of the F-NCTF-exst and the F-NCTF-prop controllers to a 1mm step input. It is shown that both controllers provide same rise time, but significant amount of overshoot. The F-NCTF-exst controller produce overshoot ten times higher than F-NCTF-prop. The F-NCTF-exst overshoot is 10%, while the F-NCTF-prop overshoot is only 1% of a reference input. Since the overshoot is high, F-NCTF-exst reaches 2% steady state condition slower than the F-NCTF-prop of about 0.03 s.

![Figure 8. Positioning performance to a 1 mm step input](image)

The positioning performance to a 5 mm step input, which is far from NCT, are shown in Figure 9. As shown, the F-NCTF-prop controller outperformed the F-NCTF-exst. The rise time achieves by F-NCTF-prop is slightly faster. Moreover, the overshoot is only 0.2%, less than 4.6% of F-NCTF-exst. Similar to the previous result for a small input, the settling time response of F-NCTF-prop controller is faster than F-NCTF-exst of about 0.16s.

From Figure 8 and 9, it is shown that significant overshoot reduction and faster settling time has been achieved while the proposed controller maintain the rise time. This shows that the positioning performance improvement has been achieved.

![Figure 9. Positioning performance to a 5 mm step response.](image)
4.1. Tracking Performance

The tracking performance evaluation is conducted using sinusoidal input with varying amplitude and frequency. Responses to a 1 mm amplitude at frequency of $\pi$ rad/s sinusoidal input is shown in Figure 10. It is shown that the F-NCTF-prop immediately forced the object to follow the reference input, but this is not the case for the F-NCTF-exst controller. By varying the amplitude, Figure 11, 12 and 13 show the position tracking error to a sinusoidal input at $\pi$ rad/s frequency with amplitude of 1 mm, 1.5 mm and 5 mm respectively. The numerical results are summarized in Table 2.

Table 2. Tracking performances with F-NCTF-exst and F-NCTF-prop controllers to a sinusoidal reference input at frequency $\pi$ rad/s

| Amplitude | Controller     | $\max|x_r-x|$ (µm) | $\text{rms (x_r-x)}$ (µm) |
|-----------|----------------|--------------------|--------------------------|
| 1 mm      | F-NCTF-exst    | 34.3               | 19.4                     |
|           | F-NCTF-prop    | 4.1                | 1.2                      |
| 1.5 mm    | F-NCTF-exst    | 51.4               | 28.8                     |
|           | F-NCTF-prop    | 4.5                | 1.2                      |
| 5 mm      | F-NCTF-exst    | 169.6              | 90.1                     |
|           | F-NCTF-prop    | 27.9               | 4.2                      |

From the numerical result it is shown that the tracking error of the F-NCTF-exst tends to increase proportionally to the input amplitude. Its values are also much higher than the proposed controller. The tracking error of the proposed controller, F-NCTF-prop, relatively constant for an oscillation amplitude within and near NCT. There is a small variation of $\max|x_r-x|$, but the $\text{rms}(x_r-x)$ values confirm that there is no significant variation. However, for an input amplitude far from NCT, there is an indication that the tracking error tends to increase while the amplitude increases.

Figure 10. Responses to a 1 mm amplitude $\pi$ rad/s frequency sinusoidal signal input

Figure 11. Position tracking error to a 1 mm amplitude and $\pi$ rad/s frequency sinusoidal input
In order to investigate the influence of the input frequency, the system is driven by a 1 mm amplitude sinusoidal input at various frequencies. Responses to a 1 mm amplitude at frequency of $2\pi$ rad/s sinusoidal input is shown in Figure 14. The position tracking error is shown in Figure 11, 15 and 16 for frequency $\pi$, $2\pi$ and $4\pi$ respectively. The numerical results are summarized in Table 3.

Table 3. Tracking performances with F-NCTF-ext and F-NCTF-prop controllers to a sinusoidal reference input with amplitude of 1 mm

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Controller</th>
<th>$x-x_{\text{max}}$</th>
<th>$\text{rms (x-x)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$ rad/s</td>
<td>F-NCTF-ext</td>
<td>34.3 (µm)</td>
<td>19.4 (µm)</td>
</tr>
<tr>
<td></td>
<td>F-NCTF-prop</td>
<td>4.1</td>
<td>1.2</td>
</tr>
<tr>
<td>$2\pi$ rad/s</td>
<td>F-NCTF-ext</td>
<td>68.2</td>
<td>39.7</td>
</tr>
<tr>
<td></td>
<td>F-NCTF-prop</td>
<td>4.4</td>
<td>1.2</td>
</tr>
<tr>
<td>$4\pi$ rad/s</td>
<td>F-NCTF-ext</td>
<td>134.1</td>
<td>76.5</td>
</tr>
<tr>
<td></td>
<td>F-NCTF-prop</td>
<td>16.3</td>
<td>1.5</td>
</tr>
</tbody>
</table>
Similar to the result shown in Table 2, the tracking error of the F-NCTF-exst shows tendency to increase proportionally to the input frequency. Its tracking error increases about twice while the frequency increases by 2 times. For the proposed controller, F-NCTF-prop, the tracking error is relatively constant. However, at the higher frequency, max|x_r-x| is a little bit higher at initial object movement as shown in Figure 16, but the rms(x_r-x) value is only slightly increase. Thus, the tracking error performance of the proposed controller is less influenced by the input.

4. Conclusion
An improvement of a fuzzy based NCTF controller for continuous motion has been presented in this study. Minor structure change has significantly influenced the controller performance on performing continuous motion and point-to-point positioning tasks. The simulation result shows that the proposed controller is effective for tracking tasks to a sinusoidal input. The tracking error to inputs within and near NCT are not depending on the input amplitude and less influenced by the oscillation frequency. Moreover, for point-to-point positioning tasks, less overshoot, faster settling time and rise time were achieved. The proposed controller is effective for point-to-point positioning whether the input is close or far from NCT. Since the only minor change in the structure, the proposed controller maintains the NCTF simple structure and ease of design procedures. Overall, the proposed controller is effective and significantly outperformed the existing method as a practical controller. As the future works, the proposed controller will be implemented in a real mechanism to validate and evaluate its robustness to disturbance experimentally.

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References


