Min Max Model Predictive Control for Polysolenoid Linear Motor

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ABSTRACT

The Polysolenoid Linear Motor (PLM) have been playing a crucial role in many industrial aspects because it provides a straight motion directly without mediate mechanical actuators. Some control methods for PLM based on Rotational Motor are applied to obtain several good performances, but position and velocity constraints which are important in real systems are ignored. In this paper, we analysis control problem of tracking position in PLM under state-independent disturbances via min-max model predictive control. The proposed controller brings tracking position error converge to zero and satisfies state including position and velocity and input constraints. The simulation results validity a good efficiency of the proposed controller.

Keyword: Min max model predictive Control

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1. INTRODUCTION

Linear Motor transmission systems are widely applied to provide directed straight motions in which, mechanical actuators are eliminated, resulting in better performance of motion systems. Generally, polysolenoid linear motor (PLM) has a durable structure [1], operations according to electromagnetic phenomenon with principles as shown in [2]-[6] and various applications such as CNC Lathe [7], sliding door [8]. Without the need of any gear box for motion transformation, the PLM system becomes sensitive due to external impacts such as frictional force, end – effect, changed load and non-sine of flux. These effects encounter both in the longitudinal and in the transversal direction, which along with saturation in supplied voltage make obtaining good control performance from the linear drive a difficult task.

There are several researches taking into account the position control of PLM in presence of external disturbances. The authors in [9] presented a control design method to regulate velocity based on PI – selftunning combining with appropriate estimation technique at slow velocity zone, but if load is changed, PI – selftunning controller will be not efficient. In order to overcome changed load, model reference control method based on Lyapunov stability theory was employed in [10]. Additionally, the compensation approaches were proposed in researches [11],[12] in which, the frictional force were estimated by Lugrie and Stribeck friction model respectively. In [13], the advantage of that the sliding mode control applied in Linear Motor is that real position value tracks set point. However, the disadvantages of this method are finding sliding surface and chattering. In the view of nonlinear systems, the study in [14],[15] apply linearization method to PLM system but this method is restricted by uncertain parameter and disturbances. The authors in [16] built a new mathematic model and use optimal control approach to result in linear quadratic regulation.
It is clear that the previous researches do not mention position, velocity and currents constraints as well as impact of external disturbance which is important properties of the control systems.

The contribution of this study is to develop a position control system for PLM in which, the the proposed control structure is based on separating a dynamic model into two subsystem including position-velocity and current. The output of position-velocity controller is reference of current control-ler. The position controller is designed based on a min-max model predictive control theory in [17] to ensure that position and velocity error being in their constraints and converging to a small ball neighborhood of origin under state-independent disturbance. The current controller is designed based on a PI-controller with cross-current compensation method.

2. DYNAMIC MODEL

Let us consider a dynamic model of PLM in [14],[15],[18]

\[
\begin{align*}
    \frac{di_d}{dt} &= -\frac{R_s}{L_{sd}}i_d + \left(\frac{2\pi p}{\tau}\right)\frac{L_{sq}}{L_{sd}}i_q + \frac{u_d}{L_s}, \\
    \frac{di_q}{dt} &= -\frac{R_s}{L_{sq}}i_q - \left(\frac{2\pi p}{\tau}\right)\frac{L_{sd}}{L_{sq}}i_d - \left(\frac{2\pi p}{\tau}\right)\frac{\psi_p}{L_{sq}} + \frac{u_q}{L_s}, \\
    \frac{dv}{dt} &= \frac{2\pi p}{\tau} \left(\psi_p + (L_{sd} - L_{sq})i_d\right)i_q - \frac{1}{m}F_v, \\
    \frac{dx}{dt} &= v.
\end{align*}
\]

where \(i_d, i_q, v, x\) are current, velocity and position respectively, \(R_s\) is resistance, \(L_{sd}, L_{sq}\) is inductor \(p\) is pole pair \(\tau\) is pole step \(U_{sd}, U_{sq}\) is voltage \(\psi_p\) is flux \(m\) is massive \(F_v\) is unmeasured external force.

In the dynamic model (1) is same as that of permanent magnet rotation synchronization motor. When it comes to PLM, \(L_{sd}, L_{sq}\) have the approximate similar values and reference current \(i_{dr} = 0\) in the current controller; therefore, term \(\left(L_{sd} - L_{sq}\right)i_d i_q\) can be ignored in the third equation in (1), leading to that current \(i_q\) relates position-velocity by linear equations.

3. PROPOSED METHOD

In this paper, let us separate dynamic model (1) into current subsystem and position-velocity subsystem. The previous chapter shows position subsystem can be considered as a linear system and applied algorithm in [17]. In current subsystem, the proposed method is cross-current compensation method between \(i_d, i_q\) to change dynamic model to a linear state space model to apply a controller based on PI – controller.

3.1. Control of current – subsystem

Min Max Model Predictive Control for Polysolenoid Linear Motor (Nguyen Hong Quang)
Applying decoupling control:

\[ u_{sd} = \left( \frac{2\pi p}{\tau} \right) L_{sd} i_{sd} + u_s, \]
\[ u_{sq} = \left( \frac{2\pi p}{\tau} \right) L_{sd} i_{sd} + \left( \frac{2\pi p}{p\tau} \right) \psi_p + u_s. \]  

(2)

Current systems is transformed to:

\[ \frac{di_{sd}}{dt} = -\frac{R_s}{L_{sd}} i_{sd} + \frac{u_s}{L_{sd}}, \]
\[ \frac{di_{sq}}{dt} = -\frac{R_s}{L_{sq}} i_{sq} + \frac{u_s}{L_{sq}}. \]  

(3)

Using current controller (PI controller):

\[ u_s = L_{sd} \frac{d}{dt} i_{sd} + \frac{R_s}{L_{sd}} i_{sd} + L_{sd} \left[ k_{i1} i_{sd} + k_{i2} \int_0^\tau i_{sd} (\tau) d\tau \right]. \]
\[ u_s = L_{sq} \frac{d}{dt} i_{sq} + \frac{R_s}{L_{sq}} i_{sq} + L_{sq} \left[ k_{i1} i_{sq} + k_{i2} \int_0^\tau i_{sq} (\tau) d\tau \right]. \]  

(4)

where \( i_{ad} = i_{ad} - i_{ad} \) and \( i_{aq} = i_{aq} - i_{aq} \). We obtain current closed loop:

\[ \frac{di_{ad}^e}{dt} + k_{i1} i_{ad}^e + k_{i2} \int_0^\tau i_{ad}^e (\tau) d\tau = 0 \]
\[ \frac{di_{aq}^e}{dt} + k_{i1} i_{aq}^e + k_{i2} \int_0^\tau i_{aq}^e (\tau) d\tau = 0 \]  

(5)

By turning coefficients \( k_{i1}, k_{i2}, k_{i1}, k_{i2} \), the controller (4) guarantee global exponential stability of closed loop (5).

Remark 1: The current reference \( i_{ad}^e = 0 \) and coefficients \( k_{i1}, k_{i2}, k_{i1}, k_{i2} \) is choosen such that closed loop (5) become undamped second order system and its transient time is small than horizon prediction in position subsystems.

3.2. Control of Position-Velocity subsystem

The dynamic of position subsystem is significantly slower than current subsystems. In the control design of position, we assumed that the desired current equals to actual current. From (1) and remark 1, we have model of position system

\[ \frac{dx}{dt} = v, \]
\[ \frac{dv}{dt} = \frac{2p p}{t m} y \dot{y} - \frac{1}{m} F_c. \]  

(6)

The equation (6) can be rewrite in state space model of tracking errors by setting \( i_{aq} = u + \frac{m\tau}{2\pi p\psi_p x_p} \)

\[ \frac{dz}{dt} = Az + Bu + Hd, \]
\[ y = Cz, \]  

(7)
where
\[ e_s = x - x_r, \quad e_v = v - v_r, \]
\[ z = [e_s, e_v], \quad d = F_s, \quad C = [1,0], \]
\[ A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 2\pi p \\ \frac{\psi_r}{m} \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ -1 \end{bmatrix}. \]

(8)

To obtain a discrete state space model, let us apply the forward Euler method to equation (7)
\[ z_{k+1} = A_z z_k + B_z u_k + D_z d_k, \]
\[ y_k = C z_k. \]

(9)

Control Objective:
\[ z_k \in \mathbf{Z}, \quad u_k \in \mathbf{U}, \]
\[ z_k \rightarrow \mathbf{Z}_0 \subset \mathbf{Z}. \]

(10)

where
\[ \mathbf{Z}_0 = \{ (z_1, z_2) \in \mathbb{R}^2 | e_{z1\text{min}} < z_1 < e_{z1\text{max}}, e_{z2\text{min}} < z_2 < e_{z2\text{max}} \}, \]
\[ \mathbf{Z}_0 = \mathbf{Z} = \{ (z_1, z_2) \in \mathbb{R}^2 | e_{z1\text{min}} < z_1 < e_{z1\text{max}}, e_{z2\text{min}} < z_2 < e_{z2\text{max}} \}, \]
\[ \mathbf{U} = \{ U_{\text{min}}, U_{\text{max}} \} \in \mathbb{R}. \]

To achieve control objective (10), we use min-max model predictive control proposed in (1)-(6). In position controller, we consider a dual-mode control law: an “inner” and an “outer” controller. The inner controller is active when the state is in the robust control invariant set \( \mathbf{Z}_0 \), and its role is to keep the state in this set under external disturbance \( F_s \). The outer controller operates when the state is outside the invariant set and steers the system state to the invariant set \( \mathbf{Z}_0 \).

The inner controller we use is linear feedback \( u_k = K z_k \), which is obtained by different way in compare with [17]. This property of the inner controller is important in the construction of the control robust invariant set. For the outer controller, we use min–max MPC, which form the focus of this paper and consider a fixed horizon formulation.

Algorithm 1:
Data: \( z_k \)
If \( z_k \in \mathbf{Z}_0 \), set \( u_k = K z_k \). Otherwise, find the solution of (12) and set \( u_k \) to the first control in the optimal sequence calculated.

Figure 2. Control Structure
3.2.1. Design of inner controller

In research [6], the robust control invariant set \( Z_0 \) is selected as simple based on property (\( A - BK \)) = 0, \( s \) is a positive integer number and \( Z_0 = \sum_i (A - BK)_i \mathbf{W} \) with \( d \in \mathbf{W} \). In this selection, set \( Z_0 \) is not evaluated to be arbitrarily small to ensure the performances of system. Moreover in some cases, we can not find \( K \) such that (\( A - BK \)) = 0 hold for any \( s \). In this subchapter, \( Z_0 \) and matrix \( K \) is found based on Lyapunov’s direct method and LMIs technique

\[
V(z_k) = z_k^T P_z z_k, P = P^T > 0, \quad V(z_k) - V(z_{k+1}) = z_k^T P_z - (A_z + B_z u_k + D_z d_k)^T P (A_z + B_z u_k + D_z d_k).
\]

Substituting \( u_k = K z_k \) into equation (12) we have

\[
V(z_k) - V(z_{k+1}) = z_k^T P_z - (A_z z_k + B_z u_k + D_z d_k)^T P (A_z z_k + B_z u_k + D_z d_k)
\]

\[
= z_k^T (P - (A_z + B_z K)^T P (A_z + B_z K) z_k - 2z_k^T (A_z + B_z K)^T P D_z d_k - D_z^T P D_z d_k^2), \quad (13)
\]

With assumption that the disturbance is bounded: \( |d| \leq d_{\text{max}} \), we take the following matrix inequalities

\[
P - (\gamma + 1)(A_z + B_z K)^T P (A_z + B_z K) \succeq M,
\]

\[
M = \text{diag} [q_1, q_2] > 0, \quad (14)
\]

\[
V(z_k) - V(z_{k+1}) \geq z_k^T M_z k - \left(1 + \frac{1}{\gamma}\right) D_z^T P D_z d_{\text{max}}^2 = q_1 z_1^2 + q_2 z_2^2 - \left(1 + \frac{1}{\gamma}\right) D_z^T P D_z d_{\text{max}}^2.
\]

The robust control invariant set can be chosen as

\[
Z_0 \cup \{ z_k = (z_{k1}, z_{k2}) \in \mathbb{R}^2 : |z_{k1}| < d_{\text{max}} \sqrt{\frac{(1+1/\gamma) D_z^T P D_z}{q_1}}, |z_{k2}| < d_{\text{max}} \sqrt{\frac{(1+1/\gamma) D_z^T P D_z}{q_2}} \}, \quad (17)
\]

Letting \( U_{\text{max}} \) is saturation input bounded, the matrix state feedback control \( K \) can be obtained by solving matrix inequalities

\[
P - (1+\gamma)(A_z + B_z K)^T P (A_z + B_z K) \succeq M,
\]

\[
|K z_k| \leq U_{\text{max}}, \quad (18)
\]

\[
P^{-1} - (1+\gamma)(A_z P^{-1} + B_z K P^{-1})^T P (A_z P^{-1} + B_z K P^{-1}) - P^{-1} M P^{-1} \succeq 0.
\]

Setting \( P^{-1} = T_z K P^{-1} = L_z M^{-1} = \text{diag} [m_1, m_2] \), the inequalities (18) is converted to LMIs problem

\[
\text{Max} \sum_1^{m_1, m_2, \lambda_z} \xi
\]

Subject to
\[ T = T^T > 0, m_1 > 0, m_2 > 0, \lambda > 0, \]
\[ \begin{bmatrix} m_1 & \lambda \\ \lambda & \xi \end{bmatrix} < 0, \]
\[ \begin{bmatrix} T & \lambda \mathbf{1} \\ \lambda \mathbf{1} & \mathbf{1} \end{bmatrix} > 0, \]
\[ T (A_j T + B_j L)^T T \begin{bmatrix} (1 + \gamma)^{-1} T & 0 \\ 0 & M^{-1} \end{bmatrix} \geq 0, \]
\[ \begin{bmatrix} \lambda_k \xi_k^2 & (L z_k)^T \\ L z_k & P \end{bmatrix} \leq 0. \]

Remark 2: The optimization problem via LMIs (19) with constraints (20-24) can be solved by interior point with YALMIP toolbox. In that, \( \xi^{-1} \) is ball bounding origin and it is minimized when \( \xi \) is selected as maximum.

3.2.2. Design of outer controller

In this section, we consider quadratic cost function as
\[
L(z, u) = \sum_{i=0}^{N-1} \left( z_{k+i}^T Q z_{k+i} + u_{k+i}^T u_{k+i} \right),
\]
\[ Q \geq 0, R > 0. \]

Min – max optimization
\[
J = \min_{u_{k+i}} \max_{z_{k+i}} L(z, u),
\]
subject to
\[
\begin{align*}
\dot{z}_{k+i} &= A_j z_{k+i} + B_j u_{k+i} + D_j d_{k+i}, \\
z_{k+i} &= 0 < i < N, \\
z_{k+N} &= 0,
\end{align*}
\]
where:
\[ z = [z_j, z_{k+1}, \ldots, z_{k+N-1}]^T, \]
\[ u = [u_k, u_{k+1}, \ldots, u_{k+N-1}]^T, \]
\[ d = [d_{k+1}, d_{k+2}, \ldots, d_{k+N-1}]^T. \]

By setting:
\[ D_i = \begin{bmatrix} A_j^{i-1} D_j, A_j^{i-2} D_j, \ldots, D_j, O, \ldots, O \end{bmatrix}, D_i \in \mathbb{R}^{2 \times N}, \]
\[ B_i = \begin{bmatrix} A_j^{i-1} B_j, A_j^{i-2} B_j, \ldots, B_j, O, \ldots, O \end{bmatrix}, B_i \in \mathbb{R}^{2 \times N}. \]

We have:
$z_{k+i} = A_d^i z_k + B u + D d$

Substituting (27) into (28) and rewriting in quadratic form (35)

$L(z, u) = d^T \left( \sum_{i=1}^{N} D_i^T Q D_i \right) d + u^T \left( \text{diag} \left( [R_{ii}] \right) + \sum_{i=1}^{N} B_i^T Q B_i \right) u$

$$+ 2z_i^T \left( \sum_{i=1}^{N} (A_i')^T Q D_i \right) d + \sum_{i=1}^{N} z_i^T (A_i')^T Q A_i' z_i + \sum_{i=1}^{N} \left( 2z_i^T (A_i')^T Q B_i + 2d^T D_i^T Q B_i \right) u.$$

Putting the constraints in (26) into linear inequality form by bounded interval representing for robust control invariant set

$$Z_{\min} = [e_{\min}, e_{\min}], \quad Z_{\max} = [e_{\max}, e_{\max}],$$

$$Z_{0\min} = [e_{0\min}, e_{0\min}], \quad Z_{0\max} = [e_{0\max}, e_{0\max}].$$

We rewrite the constraints in (26) in linear forms. State constraints

$$Z_{\min} \leq z_{k+i} \leq Z_{\max}, \quad (1 \leq i \leq N-1)$$

$$\Leftrightarrow Z_{\min} \leq A_d^i z_k + B u + D d \leq Z_{\max}$$

$$\Leftrightarrow Z_{\min} - A_d^i z_k - D d \leq B u \leq Z_{\max} - A_d^i z_k - D d$$

$$\Leftrightarrow \begin{bmatrix} B_i & -B_i \end{bmatrix} u \leq \begin{bmatrix} Z_{\max} - A_d^i z_k - D d \\ -Z_{\min} + A_d^i z_k + D d \end{bmatrix}.$$  

Terminal state constraints

$$Z_{0\min} \leq z_{k+N} \leq Z_{0\max}$$

$$\Leftrightarrow Z_{0\min} \leq A_d^N z_k + B_N u + D_N d \leq Z_{0\max}$$

$$\Leftrightarrow Z_{0\min} - A_d^N z_k - D_N d \leq B_N u \leq Z_{0\max} - A_d^N z_k - D_N d$$

$$\Leftrightarrow \begin{bmatrix} B_N & -B_N \end{bmatrix} u \leq \begin{bmatrix} Z_{0\max} - A_d^N z_k - D_N d \\ -Z_{0\min} + A_d^N z_k + D_N d \end{bmatrix}.$$  

Input limitations:

$$\begin{bmatrix} U_{\min} \leq u_k \leq U_{\max} \\ \vdots \end{bmatrix}$$

Combining three constraints (30)-(32), we solve Min – Max optimization (26) by using QP method.

Remark 3: To make optimization problem (26) become simpler we assumption that $D = \{d_k \in \mathbb{D} \}$ has limited known bounded values. For instant, $D = \{d_1, d_2, \ldots, d_l \}$ the convergent of $z_k$ to robust control invariant set $Z_0$ is guaranteed by theorem 1 in [17].

4. RESULTS AND ANALYSIS

In this section, we simulate the position tracking of whole system under state, input constraint and external disturbance in Table 1. We use the same current controller and two different prediction horizons of position controller to compare quality of each controller.
Table 1. The parameter of Polysolenoid Linear Motor P01-23X80/80X140 and controller

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pole pair</td>
<td>1</td>
</tr>
<tr>
<td>Pole step</td>
<td>20 (mm)</td>
</tr>
<tr>
<td>Rotor mass</td>
<td>0.17 (kg)</td>
</tr>
<tr>
<td>Phase coil Resistance</td>
<td>10.3 (Ω)</td>
</tr>
<tr>
<td>d-axis inductance</td>
<td>1.4 (mH)</td>
</tr>
<tr>
<td>q-axis inductance</td>
<td>1.4 (mH)</td>
</tr>
<tr>
<td>Flux</td>
<td>0.035 (Wb)</td>
</tr>
<tr>
<td>K (inner control)</td>
<td>[−300, −5]</td>
</tr>
<tr>
<td>[k_{11}, k_{12}, k_{21}, k_{22}]</td>
<td>[100,800,100,800]</td>
</tr>
</tbody>
</table>

As can be seen in Figure 3-4, at initial time both position and velocity error stay outside of state constraints region and after smaller than 0.2s, they converges to small ball centered at origin. The currents is satisfies input constraint under time varying external forces.

Figure 3. The time evolution of position error
Figure 4. The time evolution of velocity error

Figure 5. The time evolution of current in dq coordinates
Figure 6. The time evolution of supplied voltage dq-coordinates

Figure 7. The time evolution of external disturbance force
5. CONCLUSION
This research proposed min-max model predictive control for polysolenoid linear motor. Our method not only addressed the position tracking problem of the linear motor in the presence of external disturbance and input saturations but also stabilized closed-loop system in comparison with classical model predictive control. The good performance of control method, working properly even at high speed was demonstrated by numerical simulation. Furthermore, the min-max controller can be implemented easily to hardware by using quadratic programming method.

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