The Linear Model of a PV module

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ABSTRACT
This paper proposes a new approach to determine a linear mathematical model of a PV module based on an accurate nonlinear model. In this study, electrical parameters at only one operating condition are calculated based on an accurate model. Then, first-order Taylor series approximations apply on the nonlinear model to estimate the proposed model at any operating conditions. The proposed method determines the number of iteration times. This decreases calculation time and the speed of numerical convergence will be increased. And, it is observed that owing to this method, the system converged and the problem of failing to solve the system because of inappropriate initial values is eliminated. The proposed model is requested in order to allow photovoltaic plants simulations using low-cost computer platforms. The effectiveness of the proposed model is demonstrated for different temperature and irradiance values through conducting a comparison between result of the proposed model and experimental results obtained from the module data-sheet information.

1. INTRODUCTION
The current-voltage characteristics of photovoltaic is an important role in solar industry because it exactly reflects the module performance [1]. The one-diode model parameters of PV panels from a single I-V curve is identified in [2] by converting nonlinear fitting to a linear system identification. Paper [3] presents analytical solutions for the parameters of a five-parameter double-diode model of PV cells and modules which only require the coordinates of three key points of the I-V curves, i.e., the open-circuit (0, Voc), the short circuit (Isc, 0) and the maximum power point (MPP) (Im, Vm). These analytical solutions are successfully used in Newton–Raphson numerical iterations to achieve convergence and obtain more accurate solutions. Paper [4] presents a novel approach using the shuffled frog leaping algorithm (SFLA) to determine the unknown parameters of the single diode PV model. The validity of the proposed PV model is verified by the simulation results which are performed under different environmental conditions. However, some disadvantages are also existed in the original algorithm, such as nonuniform initial population, slow convergent rate, limitations in local searching ability and adaptive ability and premature converge. Paper [5] use particle swarm optimization (PSO) with inverse barrier constraint is proposed to determine the unknown PV model parameter. Disadvantages of the basic particle swarm optimization algorithm that the method easily suffers from the partial optimism, which causes the less exact at the regulation of its speed and the direction. Moreover, many evolutionary and swarm intelligence optimization techniques have been used to solve this problem such as genetic algorithms (GA) [6]-[7], differential evolution (DE) [8], particle swarm optimization (PSO) [9], simulated annealing (SA) algorithm [10], bacterial foraging (BF) algorithm [11], harmony search algorithm (HSA) [12], and artificial bee colony (ABC) algorithm [13]. A LABVIEW simulator for photovoltaic (PV) systems is presented in [15]-[16].
All of the previously mentioned models suffer from high computational time due to their dependency on complex transcendental implicit equations.

In this paper a five parameters extraction mainly based on a linearization method is presented. The proposed method estimate the parameter of a pv without any the conversion problem. And also, the number of iteration times is determined, so the calculation time is decreased. The predicted I-V and P-V curves are compared with experimental data to conclude on the validity of the model and the followed procedure.

2. NON LINEAR MODEL OF PHOTOVOLTAIC MODULE

Figure 1 shows the equivalent circuit for a PV cell. The output current of the equivalent circuit, Ipv, can be expressed as a function of the PV cell’s voltage, Vpv [1]:

![Figure 1. Equivalent circuit of a photovoltaic cell using](image)

The single exponential module

\[ F1 = I_{pv} = I_{ph} + I_0 \left( \frac{V_{pv} + IR_s}{nSV_t} - 1 \right) + \frac{V_{pv} + IR_s}{R_{sh}} = 0 \] (1)

In the above equation, \( V_t \) is the junction thermal voltage:

\[ V_t = \frac{nKT}{q} \] (2)

Where \( k \) is the Boltzmann constant (1.38 x 10^-23 J K^-1), \( q \) is the electronic charge (1.602 x 10^-19 C), \( T \) is the cell temperature (K); \( A \) is the diode ideality factor, \( R_s \) the series resistance (Ω) and \( R_s \) is the shunt resistance (Ω). \( nS \) is the number of cells connected in series. Equation (1) can be written for the three key-points of the V-I characteristic:

\[ I_{sc} = I_{ph} - I_0 e^{\frac{V_{sc}R_S}{nSV_t}} - \frac{I_{sc}R_s}{R_{sh}} \] (3)

\[ I_{mpp} = I_{ph} - I_0 e^{\frac{V_{mpp}R_S}{nSV_t}} - \frac{V_{mpp}R_S}{R_{sh}} \] (4)

\[ I_{ac} = 0 = I_{ph} - I_0 e^{\frac{V_{ac}R_S}{nSV_t}} - \frac{V_{ac}}{R_{sh}} \] (5)

An additional equation can be derived using the fact that is on the P-V characteristic of the panel, at the MPP, the derivative of power with voltage is zero.

\[ F2 = \left. \frac{dP}{dV} \right|_{V=V_{mpp}} = I_{mpp} - V_{mpp} \left( \frac{V_{mpp}R_S}{nSV_t} + \frac{1}{R_{sh}} \right) = 0 \] (6)

The fifth equation can be derived using the fact that is on the P-I characteristics of a PV system at the maximum power point, the derivative of power with respect to current is zero.

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Equations. (3) and (5) can be inserted into Equation (4), which will take the form

\[ F4 = I_{mpp} - I_{sc} + \frac{V_{mpp} + I_{mpp} R_s - V_{oc}}{R_{sh}} + \left( I_{sc} - \frac{V_{oc} - I_{sc} R_s}{R_{sh}} \right) e^{-\frac{V_{mpp} + I_{mpp} R_s - V_{oc}}{n_2 V_T}} = 0 \]

The first equations when constructing the model are the expressions of \( I_o \) from Equation (3) and \( I_{ph} \) from Equation (5), in STC

\begin{align*}
F5 &= I_o - (I_{sc} - \frac{V_{oc} - I_{sc} R_s}{R_{sh}}) e^{-\frac{V_{oc}}{n_2 V_T}} = 0 \\
F6 &= I_{ph} - I_o e^{n_2 V_T} - \frac{V_{oc}}{R_{sh}} = 0
\end{align*}

The effects of the environment, e.g. temperature and irradiance on the values of \((I_{sc}, V_{oc}, I_m, \text{and } V_m)\) are include with differen methods [13]-[14].

### 3. LINEAR MODEL OF PHOTOVOLTAIC MODULE

The main objective of linearization is to transform and elemainte the nonlinearity model of a pv moduel into a simple equivalent model. The linearized system of a pv moduel can be written corresponding to equations (1,6-10) respectively as follow

\[ A1 \ast X = B1 \ast U1 + B2 \ast U2 \]

Where

\[ X^T = [ \Delta I_o \ \Delta A \ \Delta I_{ph} \ \Delta R_s \ \Delta R_{ab} ] \]

\[ A1(i,j) = \frac{\partial F_i}{\partial x_j} \]

\[ U1^T = [ \Delta V_{pv} \ \Delta I_{pv} ] \]

\[ B1(i,j) = -\frac{\partial F_i}{\partial U_{ij}} \]

\[ U2^T = [ \Delta G \ \Delta T ] \]

\[ B2(i,j) = -\frac{\partial F_i}{\partial U_{ij}} \]

The Gauss–Jordan elimination method is a suitable technique for solving systems of linear equations of any size. This method involves a sequence of operations on a system of linear equations to obtain at each stage an equivalent system that is, a system having the same solution as the original system. The reduction is complete when the original system has been transformed so that it is in a certain standard form from which the solution can be easily read as follow.

\[ \Delta V_{pv} = CI \Delta I_{pv} + CG \Delta G + CT \Delta T \]

The constants \( CI, CG, CT \) are defined in the appendix.
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Table 2. Identification of first operating point

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V = V_{oc}$, $I = 0$</td>
<td>(arbitrary)</td>
</tr>
<tr>
<td>$T = 25 , ^\circ C$, and $G = 1000 \text{ W/m}^2$</td>
<td></td>
</tr>
<tr>
<td>$A = 1.5611$</td>
<td>(arbitrary)</td>
</tr>
<tr>
<td>$R_{sh} = 2.8860 \times 10^6$</td>
<td>determined using Newton-Raphson based on equations (6-8)</td>
</tr>
<tr>
<td>$R_s = 2.5241 \times 10^{-1}$</td>
<td></td>
</tr>
<tr>
<td>$I_n = 1.4352 \times 10^{-1}$</td>
<td>determined using equations (9-10)</td>
</tr>
<tr>
<td>$I_{ph} = 4.8$</td>
<td></td>
</tr>
</tbody>
</table>

Figures 2 and 3 illustrate the voltage-current characteristics of the solar module at different temperatures and irradiances, respectively.

To evaluate the accuracy of the proposed model, the corresponding normalized root mean square error percentage ($nRMSE(\%)$) are calculated at different conditions and compared with the $nRMSE(\%)$ of an accurate model as given in [18]-[19]. Table 3 gives the corresponding $nRMSE(\%)$ calculated by

$$nRMSE(\%) = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{E_i - T_i}{T_i} \right)^2 \times 100$$

where $E_i$ is the estimated value, and $T_i$ is the true value obtained from data sheet. Table 2 gives the evaluated $nRMSE(\%)$ from Ref. [19] and evaluated $nRMSE(\%)$ corresponding to the theoretical 1–5 curves (Figure 2–3). It can be observed that the linear model has presented the best accuracy under different conditions. In addition, the number of tunable parameters are lowered.

Table 2. $nRMSE(\%)$ of the different PV models for KC200GT module

<table>
<thead>
<tr>
<th>G=1000</th>
<th>G=600</th>
<th>G=200</th>
</tr>
</thead>
<tbody>
<tr>
<td>T=25</td>
<td>T=25</td>
<td>T=25</td>
</tr>
<tr>
<td>NRMSE(% OF MODELE IN [19])</td>
<td>1.12 [18]</td>
<td>2.15 [18]</td>
</tr>
<tr>
<td>NRMSE(% OF THIS WORK)</td>
<td>0.47</td>
<td>0.57</td>
</tr>
</tbody>
</table>

Table 3 gives the evaluated $nRMSE(\%)$ corresponding to the theoretical 1–5 curves based linear model and experimental data for KC200GT at different temperatures. It can be observed that the theoretical 1–5 curves are sufficiently accurate for the experimental data. This proves the validity of the proposed parameter identification technique for PV modules.
Table 2. NRMSE(%) of the linear model at different temperature

<table>
<thead>
<tr>
<th>G</th>
<th>T</th>
<th>NRMSE(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>25</td>
<td>0.47</td>
</tr>
<tr>
<td>1000</td>
<td>50</td>
<td>0.39</td>
</tr>
<tr>
<td>1000</td>
<td>75</td>
<td>0.69</td>
</tr>
</tbody>
</table>

6. CONCLUSION
The target of this study is to obtain an linear PV model which plays an important role in linear control approach and simulation studies of the PV power systems. The mathematical model of the PV module is a nonlinear I-V characteristic that includes several unknown parameters because of the limited information provided by the PV manufacturers. So, a linear model of photovoltaic panels has been developed and implemented. From the present analysis, one can draw the following main conclusions:
1. The proposed method estimate the parameter of a pv without any the conversion problem .
2. The calculated (I-V) curves based on proposed model are in good agreement with the experimental data of KC200GT module for different effects of the environment (temperature and irradiance). Also, the maximum value of corresponding normalized root mean square error percentage [nRMSE(%)] less than 1%
3. The proposed model can be used for linear control approach and simulate large-scale PV systems with low-cost computer platforms

APPENDIX
By simplification of equations (13-17), CI, CG, CT can be defined as follow

\[ C_l = \frac{C_{18}}{c_{17}} \]
\[ C_T = \frac{C_{15}}{c_{17}} \]
\[ C_G = \frac{C_{16}}{c_{17}} \]

where

\[ C_1 = \frac{l_{sc ref}}{A_{ref}}, C_3 = \frac{A_K}{q} \ln \left( \frac{g_0}{A_{ref}} \right) + K_v, C_{4} = \frac{1}{g_0} A_K T_0, C_5 = - \frac{V_{oc}}{R_{sh}} \]
\[ c_6 = \left( 1 + \frac{R_s}{R_{sh}} \right) e^{-\frac{V_{oc}}{n_A T_0}}, c_7 = - \left( I_{sc o} - \frac{V_{oc}}{R_{sh}} \right) e^{-\frac{V_{oc}}{R_{sh} K T_0}} \]
\[ c_8 = \left( I_{sc o} - \frac{V_{oc}}{R_{sh}} \right) e^{-\frac{V_{oc}}{R_{sh} K T_0}} \]
\[ c_9 = c_3(c_5 + c_7) + c_6 K_i + c_8 c_{10} = c_4(c_5 + c_7) + c_6 C_1 \]
\[ c_{11} = c_9 e^{\frac{V_{oc}}{k T_0}} + \frac{I_{sc ref}}{k T_0} - \frac{V_{oc}}{k T_0} \]
\[ c_{12} = C_{10} + \frac{V_{oc}}{k T_0}, c_{13} + c_4 \]
\[ C_{13} = -l_0 \left( e^{-\frac{V_{pp} + l_{pp} R_s}{k T}} \right), C_{14} = -l_0 \left( e^{-\frac{V_{pp} + l_{pp} R_s}{k T}} \right) \]
\[ C_{15} = \left( c_{11} + \frac{V_{pp}}{(k T)^2} \right) + \frac{-l_{pp} R_s}{k T} + C_14 C_9 \]

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\[ C_{16} = (C_{12} + C_{14}C_{10}) \]
\[ C_{17} = \frac{TC_{13}}{(kT)} - \frac{1}{R_{sh}} \]
\[ C_{18} = 1 - \left( \frac{R_{C_{13}}}{R_{sh}} \right) \]

REFERENCES


