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ABSTRACT

Fragile and conflict-affected states (FCAS) are those in which the government lacks the political will and/or capacity to provide the basic functions necessary for poverty reduction, economic development, and the security of human rights of their populations. Until recent history, unfortunately, the majority of research conducted and universal health care debates have been centered around middle income and emerging economies. As a result, FCAS have been neglected from many global discussions and decisions. Due to this neglect, many FCAS do not have proper vaccinations and antibiotics. Seemingly, well estimated health care costs are a necessary stepping stone in improving the health of citizens among FCAS. Fortunately, developments in statistical learning theory combined with data obtained by the WBG and Transparency International make it possible to accurately model health care cost among FCAS. The data used in this paper consisted of 35 countries and 89 variables. Of these 89 variables, health care expenditure (HCE) was the only response variable. With 88 predictor variables, there was expected to be multicollinearity, which occurs when multiple variables share relatively large absolute correlation. Since multicollinearity is expected and the number of variables is far greater than the number of observations, this paper adopts Zou and Hastie’s method of regularization via elastic net (ENET). In order to accurately estimate the maximum and expected maximum HCE among FCAS, well-known risk measures, such as Value at Risk and Conditional Value at Risk, and related quantities were obtained via Monte Carlo simulations. This paper obtained risk measures at 95 security level.

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1. INTRODUCTION

The right to health is an exclusive part of our human rights, yet people in certain countries are denied this fundamental right as a result of fragility and conflict [1]. According to the World Bank group (WBG), almost half of the world’s poor population is projected to live in countries swamped by fragility, conflict, and violence (FCV) by 2030. Because of these projections, the group may struggle to eradicate poverty and promote shared prosperity [2]. The WBG has been at the forefront of providing assistance to countries pursuing to address the problem of FCV. Unfortunately, the reality remains that countries trapped in FCV continue to have the worst health indicators in the world combined with very low and often deteriorating economic growth as well as high rates of reversions into conflict [3].

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Of equal importance is the Universal Health Coverage (UHC) agenda for global health to replace the United Nations (UN) Millennium Development Goals for health after 2015. For a long time global dialogs on UHC concentrated mainly on health coverage in middle income countries and emerging economies at the neglect of fragile and transitional states. It turns out that very little is known about the debate regarding UHC among fragile and conflict affected states (FCAS) [4]. The WHO published a research paper entitled “Neglected Health Systems Research: Health Policy and Systems Research in Conflict-Affected Fragile States,” which goes in further depth on the importance of supporting FCAS on a global scale. Evidence presented in [2] indicated that FCAS had not been receiving the appropriate amount of support and had been neglected in various topics of research.

Development assistance for health (DAH) is an indispensable financing program for health systems in fragile countries [5]. DAH in 2013 was worth $31.3 billion and in 2015 it was over five folds than it was in the 1990s [6], [7]. The importance of monitoring the performance of DAH in fragile countries cannot be overstated. However, prescribing a model rooted in risk and statistical learning theory to predict the limits of health care cost would provide a more appropriate and reliable avenue to help sustain the health sector of FCAS.

This paper contributes to the literature by using statistical learning theory to predict HCE where the number of predictors exceeds the number of observations as is the case for the FCAS data. Another contribution is the specification of risk-based measures rooted in actuarial and financial risk literature to predict maximum and expected maximum HCE for FCAS via simulation. These models are extremely important not only to development agencies and health policy makers, but also for governments, aid organizations, international financial institutions, investors, and all other stakeholders in health because they offer realistic approximations of health care costs. This paper is partitioned into seven sections. Section 2 commences with a review of literature on FCAS, current health care spending economics, predictors, and methodology. Section 3 discusses statistical learning theory in the context of a general linear model of the Gaussian family and regularization including coordinate descent algorithm. In Section 4, risk-based measures and Monte Carlo methods are highlighted. Section 5 provides a brief description of the FCAS data and its processes. Extensive empirical results are reported in section 6. Lastly, Section 7 provides the research conclusions.

2. LITERATURE REVIEW
2.1. Definition of FCAS

Fragile and conflict affected states (FCAS) are those in which the government lacks the political will and/or capacity to provide the basic functions necessary for poverty reduction, economic development, and the security and human rights of their populations [8]. While fragile states often fall into conflict over time, it is not necessarily true that all fragile states need be conflict affected [2], [8], [9], [10]. Trivially, conflict exists all over the world, but the concept of conflict affected takes into consideration the time period, area, and nature of the conflict [10]. The flexibility of this definition allows for the adaptation of time for past, present, and future situations. This kind of flexibility is necessary because many indicators of FCAS are strongly time dependent. For example, under-five mortality rates are much higher in FCAS than the vast majority of countries [11]. The majority of the nonviolent deaths in FCAS are preventable and often caused by the spread of infectious diseases, which is a result from poor infrastructure and little or no access to vaccinations, medical supplies, and clean water. For example, between 2003 and 2008, it was estimated that 87% of the excess civilian deaths in Darfur were nonviolent [11]. With well-established health services, FCAS could promote state legitimacy and demote state conflict. While some policy makers may be skeptical about the positive ramifications of well-established health care systems in FCAS, it is imperative to remember well estimated health investments can improve the lives and well-being of people throughout the world [11]. A starting point for establishing an effective health system would be estimating the costs for a sustainable health system as well as an investment strategy for future health care reconstruction.

2.2. Predictors of Healthcare Expenditure

Studies regarding the predictors of health care expenditure (HCE) are not always direct. In the literature, gross national income (GNI) has been documented as a vital predictor of HCE. However, there is no unanimity on which other predictor variables may be akin to the remaining largely inexplicable disparity in HCE. In what follows, there is an overview of literature on factors driving HCE from the demand side and the supply side.
2.2.1. Demand Side Factors
a. Income
Many cross-country and single-country studies have identified GNI as a key basis of rising HCE. Studies such as [12], [13], [14], [15], [16], and [17] concluded that GNI takes on a crucial role in determining HCE. Estimation of income elasticity of HCE has been the focus of many studies in the literature. To estimate income elasticity is quite challenging due to omitted variable bias [18] and the omission of the effect of GNI on HCE could lead to biased estimator of income elasticity [19].

b. Demographic Structure
Population growth and ageing, as well as their influences on HCE, have been studied by [15], [20], [21], and [22]. Empirical studies focusing on the positive relationship between time-to-death on the rising health care cost are well documented in the literature [23], [24], [25]. In [23] a panel data model was used to show that the ratio of population aged 65-74 and aged over 75 to the overall population had a significant positive impact on HCE.

c. Health Status of the Elderly Population
The growing number of elderly population has a recognizable effect on the demand for health care, for this reason, elderly health status has been measured in evaluating HCE. Most studies have used the health of the elderly and demographic structure to represent the same thing. For instance, [26] used the percentage of people over 65 in the population as a proxy for population health.

2.2.2. Supply Side Factors
a. Public Financing of Health Care
The largest proportion of HCE is covered by the public purse. Hence the ratio of public health care cost to total health care cost may be related to changes in HCE. There are two schools of thought on the role of public funding on HCE. One school is of the view that public financing of health care increases total HCE [27], [28]. The argument given in [27] explained that, due to less competition in the public sector relative to the private sector, incentives to reduce expenditures might be lower. He further concluded that government run health insurance schemes reduce the cost of health care to consumers; therefore, it becomes habitual for people to misuse health care services. The other school holds the opinion that the government’s participation in health care delivery and financing is good and any growth in HCE can be curbed [29], [21].

b. Technological Advancement
Advancement in technology has impacted and changed HCE from two angles. First, the proliferation of new and costly medical technologies causes escalations in HCE [30], [31]. Second, medical technology reduces HCE due to availability of better technology to deal with more inpatient situations thereby reducing cost for inpatient hospital stays. Also, new treatment leads to an improved health status as a consequence the demand for health care decreases [14]. In the literature there is no direct variable to capture technological advancements; therefore surrogate variables such as life expectancy, infant mortality, and the health of the elderly are typically used [20].

2.3. Methodology for Studying HCE
The type of data determines which methodology to follow and for this reason different modeling approaches have been used to model the relationship between HCE and its predictors. For example, crosssectional models are used to analyze a multi subject-single period (cross-sectional) data [18], [27]. Panel data models are used to study a multi-subject multi-period (panel) structure [19], [32] while single subject multi period (time series) data is analyzed with unit root and co-integrated models [33], [34]. The literature emphasizes that, apart from income, which has been recognized as a crucial predictor of HCE, there is no agreement on what other variables may be connected to the remaining, largely unexplained, variation in HCE [33], [35]. Also, available empirical studies on HCE using different types of methodology aforementioned mostly have originated from the organization for economic co-operation development (OECD) countries [19], [36], [37]. Very little research has been conducted for FCAS, which may be due to data availability. Moreover, most early studies use parametric techniques that assume a functional form, such as a linear with number of predictors far less than number of observations. Reliable estimates for HCE can be obtained by applying concepts from statistical learning theory to FCAS data. Currently, [38] and [39] were among few studies that used statistical learning theory to analyze the link between HCE and many important predictors.
3. STATISTICAL LEARNING THEORY

Statistical learning theory refers to a set of methods used to interpret data by comparing multiple models based upon data behavior such as different regression models and is often called data-based statistical inference [40]. Statistical learning concepts are growing in application as both theory and technology advance [41]. Advancements in statistical learning theory have led to methods that out perform well known cross-sectional modeling techniques such as the least squares (LS) method.

Within this field, supervised learning and unsupervised learning are the two most outstanding topics. Supervised learning seeks to fit an accurate model based upon data with a known response variable. Unsupervised learning is best used for clustering data according to their features. Using unsupervised learning techniques along with modern technology is referred to as machine learning (ML). When exploring data, it is important to consider the entire statistical learning toolbox. Every tool inside the toolbox may help solve a problem or at least contribute to the solution. There is no one-size-fits-all tool that is independent of the data because every unique data set can act differently, it is important to keep in mind the whole lot of statistical learning techniques [42], [43].

3.1. Linear Models and Regularization

Consider the general linear model of the Gaussian family where \( i \) represents countries.

\[
y_i = \beta^T x_i + \epsilon_i,
\]

where \((x_i, y_i), i = 1, \cdots, N\) are a sample of \( N \) independent and identically distributed (i.i.d) random vectors, where \( x_i = (x_{i1}, x_{i2}, \cdots, x_{ik}) \in \mathbb{R}^k \) is the random vector of observations about \( k \) predictors for the \( i \)th sample unit and \( y_i \in \mathbb{R} \) is the corresponding response vector. Note that \( \epsilon_i \) is a stochastic term capturing all features that affect health expenditure per capita (HCE) but are not taken into consideration explicitly. The vector of \((k + 1)\) estimates \((\hat{\beta}_0, \hat{\beta})\) of regression coefficients were obtained by applying the coordinate descent to solve the optimization problem whose objective function is given by:

\[
\min_{(\hat{\beta}_0, \hat{\beta}) \in \mathbb{R}^{k+1}} \sum_{i=1}^{N} (y_i - \beta_0 - x_i^T \beta)^2 + \lambda[(1-\alpha) ||\beta||_2^2/2 + \alpha ||\beta||_1]
\]

where \( \lambda \geq 0 \) and \( 0 \leq \alpha \leq 1 \). The Ridge regression coefficients are obtained by setting \( \alpha = 0 \) which is not a subject of consideration in this paper. When \( \alpha = 1 \), the optimized problem produces the LASSO regression coefficients and \( 0 < \alpha < 1 \) results in the ENET coefficients.

3.1.1. Coordinate Descent Algorithm

Many algorithms have been proposed for finding the solution to the LASSO and ENET. The algorithm of choice for this research paper is the coordinate descent algorithm, which was developed by [19] [44] and [45]. The motivation behind the coordinate descent algorithm is efficiency and simplicity, especially for large scale problems [46]. The coordinate descent algorithm works as follows:

1. Initialize all \( \beta \) values
2. Cycle over \( j = 1, 2, \ldots, p; \ 1, 2, \ldots \) till convergence
   a) Compute partial residuals \( r_{ij} = y_i - \sum_{k \neq j} x_{ik} \beta_k \)
   b) Regress \( r_{ij} \) on \( x_{ij} \) to obtain ordinary LS estimate \( \hat{\beta}_j \)
   c) Update \( \beta_j \) using \( S(z, \gamma) \) with \( z = \hat{\beta}_j \) and \( \gamma = \alpha \lambda: \beta_j \leftarrow \frac{s(\hat{\beta}_j, \alpha \lambda)}{1+\lambda(1-\alpha)} \)
where

\[
S(z, \gamma) = \begin{cases} 
z - \gamma & : \text{if } z < 0 \text{ and } \gamma < |z|, \\
0 & : \text{if } \gamma \geq |z|,
\end{cases}
\]

4. RISK MEASURES AND MONTE CARL METHODS

4.1. VaR and CVaR

Risk measurement, based on proper risk measures, is one of the central pillars of risk management. A probability-based model is often used in order to provide a description of risk exposure. This level of risk exposure is often represented by a single number or small set of numbers. The VaR is a well-known and often used risk measure that provides a quantile measure of the total loss distribution. The VaR provides key information on risk exposure, such as the necessary amount of capital required to withstand an adverse event.
The VaR, however, provides very little detail when an enterprise is experiencing an adverse event because the amount of capital needed to recover exceeds the VaR. In this case, other risk measures, such as the conditional VaR (CVaR), are considered for further information.

4.1.1. VaR
If \( X \) is a loss random variable and \( p \) is a given security level, then the VaR is a real number value corresponding to the \((100 \cdot p)^{th}\) quantile of loss distribution of \( X \) denoted \( \text{VaR}_p(X) \) or \( \pi_p \).

Mathematically, the \((100 \cdot p)^{th}\) quantile is given by:

\[
P(X > \pi_p) = 1 - p.
\]

In dire times, the VaR is no longer beneficial because it lacks the necessary information for an enterprise to recover. In these cases, the CVaR provides an estimate that exceeds the VaR and satisfies the properties of a coherent risk measure.

4.1.2. CVaR
The CVaR is another well-known risk measure and has been referred to as the expected short-fall (ES), tail value at risk (TVaR), and conditional tail expectation (CTE). The CVaR of a loss random variable \( X \), at the \((100p)\%\) security level, is the expected value of \( X \) given that it lies above some security level \( p \) denoted \( \text{CVaR}_p(X) \).

The CVaR can be simplified using the probability distribution function, \( f(x) \), and cumulative distribution function, \( F(x) \), into terms of VaR as follows:

\[
\text{CVaR}_p(X) = E(X | X > \pi_p) = \frac{\int_{\pi_p}^{\infty} x \cdot f(x)dx}{1 - F(\pi_p)}
\]

The CVaR is also expressed in the form:

\[
\text{CVaR}_p(X) = \pi_p + \frac{\int_{0}^{\pi_p} (x - \pi_p) f(x)dx}{1 - p}
\]

4.2. Monte Carlo Simulation
Since the random loss distribution is rarely known, a MC simulation provides an unbiased estimate and simplifies complex computations. Below, we discuss crude Monte Carlo (CMC) and antithetic variate.

4.2.1. CMC
Let the output of a simulation run be of the form \( Y = h(U) \) where \( h \) is a real-valued function and \( U = (U_1, U_2, \ldots) \) is a random vector of \( iid \) random variables. The CMC estimate for \( \theta = Eh(U) \) is given by:

\[
\hat{\theta} = \frac{1}{n} \sum_{k=1}^{n} h(U_k), \quad \text{where} \quad \hat{\theta} = \frac{1}{n} \sum_{k=1}^{n} Y_k \quad \text{each of } Y_k \text{ is distributed as } Y \quad \text{and} \quad \text{Var}(\hat{\theta}) = \frac{\text{Var}(Y)}{n}.
\]

If \( \text{Var}(Y) = \sigma_2 \), then \( \text{Var}(\hat{\theta}) = \frac{\sigma^2}{n} \) and the usual estimate of \( \sigma_2 \) is given by \( s^2 = \frac{\sum_{k=1}^{n} (h(U_k) - \hat{\theta})^2}{n-1} \).

For large \( n \), the central limit theorem (CLT) could be used to construct approximate confidence interval for \( \theta \) as follows:

\[
(\hat{\theta} - z_p \cdot \frac{s}{\sqrt{n}}, \hat{\theta} + z_p \cdot \frac{s}{\sqrt{n}}),
\]

where \( z_p \) is the \((100p)^{th}\) quantile of the standard normal distribution, \( N(0,1) \). The accuracy of \( \hat{\theta} \) is proportional to \( \frac{1}{n} \) and depends on \( s^2 \).

4.2.2. Antithetic Variates
A pair of real-valued random variables \( (Y, Y') \) are an antithetic pair if \( Y \) and \( Y' \) have the same distribution and are negatively correlated. Now, if \( n \) is an even number and \( (Y_1, Y'_1), \ldots, (Y_n, Y'_n) \) are independent antithetic pairs of random variables, where each \( Y_k \) and \( Y'_k \) share the same distribution, say distribution \( Y \), then the antithetic estimator:

\[
\hat{\theta}(a) = \frac{1}{n} \sum_{k=1}^{n} (Y_k + Y'_k)
\]
is an unbiased estimator of \( \theta = E(Y) \) with variance \( \text{Var}(\hat{\theta}) = \frac{\text{Var}(Y)}{n} \left(1 + \rho_{Y,Y}^2\right) \) where \( \rho_{Y,Y} \) represents the correlation between \( Y \) and \( Y' \).

The antithetic estimation process is as follows:

1. Generate \( Y_1 = h(U_1), \ldots, Y_n = h \left( \frac{U_n}{n} \right) \) via independent simulations.
2. Let \( Y' = h(1 - U_1), \ldots, Y'_n = h \left( 1 - \frac{U_n}{n} \right) \).
3. Compute the sample covariance matrix for each pair \( (Y_k, Y'_k) \):

\[
\text{Cov}(Y_k, Y'_k) = \begin{bmatrix}
\frac{1}{n^2} \sum_{k=1}^{n} (Y_k - \bar{Y})^2 & \frac{1}{n^2} \sum_{k=1}^{n} [(Y_k - \bar{Y})(Y'_k - \bar{Y})] \\
\frac{1}{n^2} \sum_{k=1}^{n} [(Y_k - \bar{Y})(Y'_k - \bar{Y})] & \frac{1}{n^2} \sum_{k=1}^{n} (Y'_k - \bar{Y})^2
\end{bmatrix}
\]  

(8)

4. Mean estimate, \( \hat{\theta} \) using the antithetic estimator \( \hat{\theta}^{(a)} \) determines the confidence interval for some desired security level \( p \): \( \hat{\theta}^{(a)} - z_p \cdot \text{SE}, \hat{\theta}^{(a)} + z_p \cdot \text{SE} \).

where the SE term represents the standard error given by: \( \text{SE} = \sqrt{\frac{C_{1.1} + C_{2.2} + 2C_{1.2}}{2n}} \) and \( z_p \) is the \( (100 \cdot p)^{th} \) quantile of the standard normal distribution, \( N(0,1) \) [47].

5. **DATA DESCRIPTION**

This research was based on data acquired from the World Bank [48] and Transparency International data bank [49]. The data set encompassed 35 countries and 89 variables as shown in Tables 7 (see Appendix A) respectively. Apart from corruption perceptions index (CPI) which was obtained from Transparency International data bank, the rest came from the World Bank. Of the 89 variables, health HCE was used as the response variable and the other 88 variables were used as predictors of HCE. In total, there were 131 missing data points from the original data set. To overcome this difficulty, unsupervised learning was used and validated via the Kolmogorov-Smirnov (KS) test. The KS-test is a goodness of fit test that checks whether two samples come from the same distribution [50]. Each of the 89 KS-test performed concluded that, there was no statistically significant difference between the original and the imputed data set when the level of significance was set at 5%. It can therefore be concluded that the two data sets came from a common distribution. For the rest of the paper, the imputed data were used. Having cleaned the data, all variables were transformed into the natural logarithm scale. The imputed data set was randomly partitioned into two sets: a training set (24 observations) and a testing set (11 observations), prior to modeling.

6. **DETAILED ANALYSIS**

Among the FCAS data there were 88 predictor variables and 1 response variable with descriptions found in the appendix. During correlation analysis, it was observed that there was a vast amount of multicollinearity, which indicated that the ENET may be an appropriate model. Unfortunately, due to the size of the correlation matrix (88×88), it has been omitted from this paper in its entirety. During analysis, however, it was found that 20 variables shared an absolute correlation of at least 0.95 and 37 variables had an absolute correlation of at least 0.90.

6.1. **Model Selection: NET**

The coordinate descent optimization algorithm described under section 3 was implemented for a chosen \( \alpha \). Next, the tuning parameter, \( \lambda \) from the ENET solution was selected by minimizing the CV-MSE given by the testing set. Since no formal technique for the choice of \( \alpha \) has been well developed, this research paper chooses \( \alpha \) as follows: First the closed interval [0,1] was evenly partitioned to achieve equally spaced value for \( \alpha \) in [0,1], where the ridge regression and the LASSO act as special cases at the end points. For each value, say \( \alpha_k \), obtained from this partitioning, ENET models were fitted based on \( \alpha = \alpha_k \). Now, for every \( \alpha \), the tuning parameter, \( \lambda \), was selected via CV algorithm and yielded a CV-MSE. Lastly, within each partition group, the chosen value of \( \alpha \) was selected by choosing the value \( \alpha_k \) that corresponds to the minimum CV-MSE. In hopes to find a pattern, the study tested four different increment lengths (P): 0.2, 0.1, 0.01, and 0.001. The four \( \alpha \) choices are shown in Table 1 and Figure 1.
Table 1 provides the values of $\alpha$ based upon lowest CV-MSE, as well as the number of variables selected for each chosen value of $\alpha$, for partition lengths 0.200, 0.100, 0.010, and 0.001. Consider Figure 1, for partition length $P=0.200$, the $\alpha_k$ with the smallest CV-MSE was $\alpha=1.000$. Similarly, for partition lengths of 0.100, 0.010, and 0.001, the chosen values for $\alpha$ were 0.800, 0.640, and 0.991, respectively. Moreover, these four models each conducted variable selection as shown in Table 2.

Table 1. $\alpha$ Values with Lowest CV-MSE

<table>
<thead>
<tr>
<th>Partition length ($P$)</th>
<th>0.2</th>
<th>0.1</th>
<th>0.01</th>
<th>0.001</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of variables chosen</td>
<td>13</td>
<td>19</td>
<td>16</td>
<td>6</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1.000</td>
<td>0.800</td>
<td>0.640</td>
<td>0.991</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.019</td>
<td>0.0137</td>
<td>0.119</td>
<td>0.104</td>
</tr>
<tr>
<td>CV-MSE</td>
<td>0.269</td>
<td>0.247</td>
<td>0.143</td>
<td>0.190</td>
</tr>
</tbody>
</table>

Figure 1. Plot of minimum CV-MSE against $\alpha$

Table 2: Chosen Predictors of HCE among FCAS

<table>
<thead>
<tr>
<th>Chosen Variables</th>
<th>Model Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCS2, FTS4, UPG2, NOF1, GDP, BST</td>
<td>4</td>
</tr>
<tr>
<td>PED2, LSE, USA, LRI, SPN, MTH, TRB</td>
<td>3</td>
</tr>
<tr>
<td>AGL2, TXP</td>
<td>2</td>
</tr>
<tr>
<td>FTS2, AUS, NOF3, MD4, SAS</td>
<td>1</td>
</tr>
</tbody>
</table>

6.2. Results of MC Method

Using the values of $\alpha$ obtained at each partition level in section 6.1, MC simulations were conducted to produce a point (mean) estimate and an interval estimate (confidence interval for the mean) for HCE as well as the standard error of the point estimate, VaR and CVaR at 95% security level. In order to identify convergence, the MC simulations were compared using a varying number of simulations, $k$. In what follows the Crude MC (CMC) and Antithetic MC (AMC) were compared for multiple values of $k$. Moreover, Table 2 indicated that some variables recurred in the chosen models at each partition level. For instance, MCS2, FTS4, UPG2, NOF1, GDP, and BST were chosen at every chosen value of $\alpha$, however FTS2, AUS, NOF3, MD4, and SAS were only chosen at just one partition level.

6.3. Comparing CMC and AMC Estimates

Tables 3, 4, 5, and 6 give both CMC and AMC simulation results for HCE among FCAS. The risk quantities reported in these tables are the mean estimates ($\hat{\theta}$), 95% confidence interval (CI) for $\theta$, standard error of $\theta$ ($SE\theta$), VaR and CVaR values at a 95% security level. It can be observed, from all of the tables that, as $k$ increases, $SE\theta$ reduces for all four models chosen at $\alpha$ values 1.000, 0.800, 0.640, and 0.991. Another realization that can be observed is that the selected models with the smallest CV-MSE yielded lower estimates for VaR and CVaR.

Table 3 gives risk measures and its related quantities when $\alpha=1.000$. From this table, it is obvious that as $k$ increases $SE\theta^*$ decreases for both the CMC and AMC. Moreover, at a 95% security level, the VaR
indicated the AMC estimate was smaller relative to the CMC at all values of k. In Tables 4, 5, and 6, the same conclusions can be drawn, however the estimates were different.

### Table 3. Estimates of α=1.000

<table>
<thead>
<tr>
<th>α = 1.000</th>
<th>Estimate</th>
<th>SE of Estimate</th>
<th>95% CI for θ</th>
<th>95% VaR</th>
<th>95% CVaR</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>CMC</td>
<td>AMC</td>
<td>CMC</td>
<td>AMC</td>
<td>CMC</td>
</tr>
<tr>
<td></td>
<td>($)</td>
<td>($)</td>
<td>($)</td>
<td>($)</td>
<td>($)</td>
</tr>
<tr>
<td>10000</td>
<td>175.24</td>
<td>175.63</td>
<td>1.92</td>
<td>1.25</td>
<td>(171.47, 179.01)</td>
</tr>
<tr>
<td>20000</td>
<td>174.71</td>
<td>176.88</td>
<td>1.35</td>
<td>0.90</td>
<td>(172.07, 177.35)</td>
</tr>
<tr>
<td>50000</td>
<td>177.06</td>
<td>175.17</td>
<td>0.87</td>
<td>0.56</td>
<td>(175.34, 178.75)</td>
</tr>
<tr>
<td>100000</td>
<td>175.82</td>
<td>175.60</td>
<td>0.61</td>
<td>0.40</td>
<td>(174.63, 177.01)</td>
</tr>
</tbody>
</table>

### Table 4. Estimates for α=0.800

<table>
<thead>
<tr>
<th>α = 0.800</th>
<th>Estimate</th>
<th>SE of Estimate</th>
<th>95% CI for θ</th>
<th>95% VaR</th>
<th>95% CVaR</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>CMC</td>
<td>AMC</td>
<td>CMC</td>
<td>AMC</td>
<td>CMC</td>
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### Table 5. Estimates for α=0.640

<table>
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<th>α = 0.640</th>
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<th>95% VaR</th>
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### Table 6. Estimates for α=0.991

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<th>SE of Estimate</th>
<th>95% CI for θ</th>
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### 7. CONCLUSIONS

This research paper found four potential ENET models for HCE among FCAS (including the LASSO as a special case). These chosen models were fitted using the following values of α: 1.000 (LASSO), 0.800, 0.640, and 0.991 and their corresponding CV-MSEs were 0.269, 0.247, 0.143, and 0.190 respectively. Therefore, based on CV-MSE, the ranking of these models was given by the α values 0.640, 0.991, 0.800, and 1.000, respectively.

These potential ENET models were used in the MC simulations to arrive at the risk measures and their related quantities. Results of these simulations indicated that, the AMC outperformed the CMC in terms of variance reduction for all the four models. However, maximum variance reduction in terms of AMC was observed in the model given by α=0.991, followed by the models with α given by 0.640, 0.800, and 1.000 in that order. Therefore, based on maximum variance reduction, the ranking of these ENET models are respectively given by the α values 0.991, 0.640, 0.800, and 1.000.

Under a comparison of the four models, it was evident that the models based on α=1.000 and α=0.800 were poorly ranked with respect to both CV-MSE and variance reduction. Because of this finding, the models given by the α=1.000 and α=0.800 were not taken into consideration. Therefore, the two candidate predictive models of HCE among FCAS were the ENET models based on α values 0.640 and 0.991. Although both models are deemed appropriate, the model based on α=0.640 may be more appropriate in terms of selecting key predictors of HCE among FCAS because it yielded the minimum CV-MSE. In a similar fashion, if the desire was to provide well-estimated risk measures and related estimates, then the α=0.991 may be more appropriate to use because it performed best in terms of variance reduction.
Nonetheless, it can finally be concluded that at a 95% security level, the maximum HCE among FCAS was approximately between $270.00 (α=0.991) and $359.53 (α=0.640) and the expected maximum HCE among FCAS was approximately between $327.94 (α=0.991) and $426.26 (α=0.640).

REFERENCES
### Table 7. FCAS

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*Estimating Health Care Costs Among Fragile and Conflict Affected States*... (Kevin Wunderlich)